The property of James Looney Junr Chester

Decr — 1829

A NEW AND COMPLETE

SYSTEM OF ARITHMETICK.

COMPOSED FOR THE

USE OF THE CITIZENS

OF THE

United States.

By NICOLAS PIKE, A. M. A. A. S.

QUID MUNUS REIPUBLICÆ MAJUS MELIUSVE AFFERRE POSSUMUS, QUAM SI JUVENTUTEM DOCEMUS, ET BENE ERUDIMUS?
—— E VARIIS SUMENDUM EST OPTIMUM....*CICERO.*

THIRD EDITION.

REVISED, CORRECTED AND IMPROVED, AND MORE PARTICULARLY ADAPTED TO THE FEDERAL CURRENCY,

By NATHANIEL LORD, A. M.

Boston.

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April, 1808.

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RECOMMENDATIONS.

Dartmouth University, A. D. 1786.

AT the requeſt of Nicolas Pike, Eſq. we have inſpected his Syſtem of Arithmetick, which we cheerfully recommend to the publick, as eaſy, accurate, and complete. And we apprehend there is no treatiſe of the kind extant, from which ſo great utility may ariſe to Schools.

B. WOODWARD, Math. and Phil. Prof.
JOHN SMITH, Prof. of the Learned Languages.

I do moſt ſincerely concur in the preceding recommendation.

J. WHEELOCK, Preſident of the Univerſity.

Providence, State of Rhode-Iſland, 1785.

WHOEVER may have the peruſal of this treatiſe on Arithmetick may naturally conclude I might have ſpared myſelf the trouble of giving it this recommendation, as the work will ſpeak more for itſelf than the moſt elaborate recommendation from my pen can ſpeak for it: But as I have always been much delighted with the contemplation of mathematical ſubjects, and at the ſame time fully ſenſible of the utility of a work of this nature, was willing to render every aſſiſtance in my power to bring it to the publick view: And ſhould the ſtudent read it with the ſame pleaſure with which I peruſed the ſheets before they went to the preſs, am perſuaded he will not fail of reaping that benefit from it which he may expect, or wiſh for, to ſatisfy his curioſity in a ſubject of this nature. The author, in treating on numbers, has done it with ſo much perſpicuity and ſingular addreſs, that I am convinced the ſtudy thereof will become more a pleaſure than a taſk.

The arrangement of the work, and the method by which he leads the *tyro* into the firſt principles of numbers, are novelties I have not met with in any book I have ſeen. Wingate, Hatton, Ward, Hill, and many other authors, whoſe names might be adduced, if neceſſary, have claimed a conſiderable ſhare of merit; but when brought into a comparative point of view with this treatiſe, they are inadequate and defective. This volume contains, beſides what is uſeful and neceſſary in the common affairs of life, a great fund for amuſement and entertainment. The Mechanick will find in it much more than he may have occaſion for; the Lawyer, Merchant and Mathematician, will find an ample field for the exerciſe of their genius; and I am well aſſured it may be read to great advantage by ſtudents of every claſs, from the loweſt ſchool to the Univerſity. More than this need not be ſaid by me, and to have ſaid leſs, would be keeping back a tribute juſtly due to the merit of this work.

BENJAMIN WEST.

Univerſity in Cambridge, A. D. 1786.

HAVING, by the deſire of Nicolas Pike, Eſq. inſpected the following volume in manuſcript, we beg leave to acquaint the publick, that in our opinion it is a work well executed, and contains a complete ſyſtem of Arithmetick. The rules are plain, and the demonſtrations perſpicuous and ſatisfactory; and we eſteem it the beſt calculated, of any ſingle piece we have met with, to lead youth, by natural and eaſy gradations, into a methodical

and thorough acquaintance with the ſcience of figures. Perſons of all deſcriptions may find in it every thing, reſpecting numbers, neceſſary to their buſineſs; and not only ſo, but if they have a ſpeculative turn, and mathematical taſte, may meet with much for their entertainment at a leiſure hour.

We are happy to ſee ſo uſeful an American production, which, if it ſhould meet with the encouragement it deſerves, among the inhabitants of the United States, will ſave much money in the country, which would otherwiſe be ſent to Europe, for publications of this kind.

We heartily recommend it to ſchools, and to the community at large, and wiſh that the induſtry and ſkill of the Author may be rewarded, for ſo beneficial a work, by meeting with the general approbation and encouragement of the publick.

JOSEPH WILLARD, D. D. Preſident of the Univerſity.
E. WIGGLESWORTH, S. T. P. Hollis.
S. WILLIAMS, L. L. D. Math. et Phil. Nat. Prof. Hollis.

Yale College, 1786.

UPON examining Mr. Pike's Syſtem of Arithmetick and Geometry, in manuſcript, I find it to be a work of ſuch mathematical ingenuity, that I eſteem myſelf honoured in joining with the Rev. Preſident Willard, and other learned gentlemen, in recommending it to the publick as a production of genius, interſperſed with originality in this part of learning, and as a book, ſuitable to be taught in ſchools: of utility to the merchant, and well adapted even for the Univerſity inſtruction. I conſider it of ſuch merit, as that it will probably gain a very general reception and uſe throughout the republick of letters.

EZRA STILES, Preſident.

Boſton, 1786.

FROM the known character of the Gentlemen who have recommended Mr. Pike's Syſtem of Arithmetick, there can be no room to doubt, that it is a valuable performance; and will be, if publiſhed, a very uſeful one. I therefore wiſh him ſucceſs in its publication.

JAMES BOWDOIN.

PREFACE

TO THE FIRST EDITION.

IT may, perhaps, by some, be thought needless, when Authors are so multiplied, to attempt publishing any thing further on Arithmetick, as it may be imagined there can be nothing more than the repetition of a subject already exhausted. It is however the opinion of not a few, who are conspicuous for their knowledge in the Mathematicks, that the books, now in use among us, are generally deficient in the illustration and application of the rules; of the truth of which, the general complaint among Schoolmasters is a strong confirmation. And not only so, but as the United States are now an independent nation, it was judged that a System might be calculated more suitable to our meridian, than those heretofore published.

Although I had sufficient reason to distrust my abilities for so arduous a task, yet not knowing any one who would take upon himself the trouble, and apprehending I could not render the publick more essential service, than by an attempt to remove the difficulties complained of, with diffidence I devoted myself to the work.

I have availed myself of the best Authors which could be obtained, but have followed none particularly, except Bonnycastle's Method of Demonstration.

Although I have arranged the work in such order as appeared to me the most regular and natural, the student is not obliged to pay a strict adherence to it; but may pass from one Rule to another, as his inclination or opportunity for study, may require.

The Federal Coin, being purely decimal, most naturally falls in after Decimal Fractions.

I have given several methods of extracting the Cube Root, and am indebted to a learned friend, who declines having his name made publick, for the investigation of two very concise Algebraick Theorems for the extraction of all Roots, and of a particular Theorem for the Sursolid.

Among the Miscellaneous Questions, I have given some of a philosophical nature, as well with a view to inspire the pupil with a relish for philosophical studies, as to the usefulness of them in the common businesses of life.

The short introduction to Algebra, which is subjoined, was abstracted principally from Bonnycastle, and that of Conick Sections, from Emerson's Works.

Being sensible the following Treatise will stand or fall, according to its real merit or demerit, I submit it to the judgment of the candid.

With pleasure I embrace this opportunity, to express my gratitude to those learned Gentlemen, who have honoured this Treatise with their approbation, as well as to such Gentlemen, as have encouraged it by their subscriptions; and to request the reader to excuse any errours he may meet with; for although great pains have been taken in correcting, yet it is difficult to prevent errours from creeping into the press, and some may have escaped my own observation; in either case, a hint from the candid will much oblige their

Most obedient,

And humble Servant,

THE AUTHOR.

PREFACE

TO THIS NEW IMPROVED (THIRD) EDITION.

THE demand for this work still continuing, notwithstanding the publication of other works on Arithmetick and the higher branches of the Mathematicks, is evidence of its intrinsick merit, and has induced the Proprietors of the copy-right to present the publick with a new and improved Edition.

Application was made to the Author, requesting him to revise and improve the work for a new Edition; but he declined on account of want of health, and the Gentleman, whom we employed, was engaged by the Author's consent, and improved and corrected the work agreeably to his directions and advice.

The most important improvement in this Edition, is the introduction of examples in the Federal Currency under each rule; and while this was considered necessary, in order to extend the knowledge and use of that currency, it was thought important not to omit examples in pounds, shillings and pence, which are, and will continue to be, the basis of many arithmetical questions; and therefore an acquaintance with them will always be useful.

Mr. NATHANIEL LORD, 3d. *of Ipswich, the Gentleman employed to correct and improve the work, has bestowed much attention upon it, and has received from Mr.* PIKE *all the information and advice he desired. The manner in which Mr.* LORD *has executed the task entrusted to him, will, we hope, gain additional reputation for the work, and entitle him to the thanks of the publick.*

THOMAS & ANDREWS.

Boston, April, 1808.

☞ *Some errours, which escaped correction, are noticed in an* errata, *at the end of the work.*

CONTENTS.

Difcount

An

Article 21.

EXPLANATION

EXPLANATION OF THE CHARACTERS MADE USE OF IN THIS TREATISE.

$=$	THE sign of equality: as 12 pence $=$ 1 shilling, signifies that 12 pence are equal to one shilling; and, in general, that whatever precedes it is equal to what follows.
$+$	The sign of Addition: as $5+5=10$, that is, 5 added to 5 is equal to 10. Read 5 plus 5, or 5 more 5 equal to 10.
$-$	The sign of Subtraction: as, $12-4=8$, that is, 12 lessened by 4 is equal to 8, or 4 from 12 and 8 remains. Read 12 minus 4, or 12 less 4 equal to 8.
$\times$	The sign of Multiplication: As $6\times 5=30$, that is, 6 multiplied by 5 is equal to 30. Read 6 into 5 equal to 30.
$\div$ or 6)30(	The sign of Division: as, $30\div 5=6$, that is, 30 divided by 5 is equal to 6. Read 30 by 5 equal to 6.
$\frac{875}{25}$	Numbers placed fractionwise, do likewise denote division, the numerator or upper number being the dividend, and the denominator or lower number, the divisor; thus, $\frac{875}{25}$ is the same as $875\div 25=35$.
: :: :	The sign of proportion, thus, $2:4::8:16$, that is, as 2 is to 4 so is 8 to 16.
$\div\!\!\div$	Signifies Geometrical Progression.
$\overline{9-2}+6=13$	Shews that the difference between 2 and 9 added to 6 is equal to 13. Read 9 minus 2 plus 6 equal to 13. And that the line atop (called a *Vinculum*) connects all the numbers over which it is drawn.
$12-\overline{3+}=5$	Signifies that the sum of 3 and 5 taken from 12 leaves or is equal to 4.
$\overline{\quad}\|^{2}$	Signifies the second power or Square.
$\overline{\quad}\|^{3}$	Signifies the third power, or Cube.
$\overline{\quad}\|^{m}$	Signifies any power in general, as $6\|^{2}=$ square of 6; and $\overline{50}\|^{3}=$ cube of 50, &c thus m signifies either the square or cube, or any other power.
$\surd$, or $\overline{\quad}\|^{\frac{1}{2}}$	Prefixed to any number or quantity, signifies that the square root of that number is required. It likewise (as also the character for any other root) stands for the expression of the root of that number or quantity to which it is prefixed. As $\surd\,36=6$, and $\sqrt{108+36}=12$, or $\overline{36}\|^{\frac{1}{2}}=6$, &c.

$\sqrt[3]{}$, or $|^{\frac{1}{3}}$ — Prefixed to any number, signifies that the cube root of that number is required, or expressed. As $\sqrt[3]{216} = 6$, and $\sqrt[3]{513+216} = 9$, &c.—or $\overline{216}|^{\frac{1}{3}} = 6$, &c.

$\sqrt[m]{}$, or $|^{\frac{n}{m}}$ — Signifies any root in general. As $\overline{36}|^{\frac{1}{2}}$ = square root, $\overline{216}|^{\frac{1}{3}}$ = cube root, &c. Thus, $\frac{n}{m}$ signifies either the square root, cube root, or any other root whatever.

a b c d — When several letters are set together, they are supposed to be multiplied into each other; as those in the margin are the same as $a \times b \times c \times d$, and represent the continual product of quantities or numbers.

$\frac{1}{a}$ — Is the reciprocal of a, and $\frac{a}{b}$ is the reciprocal of $\frac{b}{a}$.

If a be the root, then $a \times a = aa$ or a^2 is the square of a, and $a \times a \times a = aaa$ or a^3 is the cube of a, &c.

Note. The figure atop is called the index of the power.

It is usual to write shillings at the left hand of a stroke, and pence at the right; thus, 13/4 is thirteen shillings and four pence.

Note. The use of these characters must be perfectly understood by the pupil, as he may have occasion for them.

A
NEW AND COMPLETE
SYSTEM OF ARITHMETICK.

ARITHMETICK is the Art or Science of computing by numbers, and consists both in Theory and Practice. The Theory considers the nature and quality of numbers, and demonstrates the reason of practical operations. The Practice is that, which shews the method of working by numbers, so as to be most useful and expeditious for business, and is comprised under five principal or fundamental Rules, viz. NOTATION or NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; the knowledge of which is so necessary, that, scarcely any thing in life, and nothing in trade, can be done without it.

NUMERATION

TEACHES the different value of figures by their different places, and to read or write any sum or number by these ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.—0 is called a cypher, and all the rest are called figures or digits. The names and significations of these characters, and the origin or generation of the numbers they stand for, are as follow; 0 nothing; 1 one, or a single thing called an unit; 1+1=2, two; 2+1=3, three; 3+1=4, four; 4+1=5, five; 5+1=6, six; 6+1=7, seven; 7+1=8, eight; 8+1=9, nine; 9+1=10, ten; which has no single character; and thus, by the continual addition of one, all numbers are generated.

2. Beside the simple value of figures, as above noted, they have, each, a local value, according to the following law; viz. In a combination of figures, reckoning from right to left, the figure in the first place represents its primitive simple value; that in the second place, ten times its simple value, and so on; the value of the figure, in each succeeding place, being ten times the value of it, in that immediately preceding it.

3. The

3. The values of the places are estimated according to their order: The first is denominated the place of units; the second, tens; the third, hundreds; and so on, as in the table. Thus in the number—5293467: 7, in the first place signifies only seven; 6, in the second place, signifies 6 tens, or sixty; 4, in the third place, four hundred; 3, in the fourth place, three thousand; 9, in the fifth place, ninety thousand; 2, in the sixth place, two hundred thousand; 5, in the seventh place, is five millions; and the whole, taken together, is read thus; five millions, two hundred and ninety three thousand, four hundred and sixty seven.

4. A cypher, though it is of no signification, itself, yet, it possesses a place, and, when set on the right hand of figures, in whole numbers, increases their value in the same tenfold proportion; thus, 9 signifies only nine; but if a cypher is placed on its right hand, thus, 90, it then becomes ninety; and, if two cyphers be placed on its right, thus, 900, it is nine hundred; &c.

To enumerate any parcel of figures, observe the following Rule.

First, commit the words at the head of the table, viz. units, tens, hundreds, &c. to memory, then, to the simple value of each figure, join the name of its place, beginning at the left hand, and reading towards the right.—*More particularly* —1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over such dot will, universally, have the name of thousands.—2. Place the figures, 1, 2, 3, 4, &c. as indices over the 2d, 3d, 4th, &c. period. These indices will then shew the number of times the millions are involved—The figure under 1, bearing the name of millions, that under 2, the name of billions (or millions of millions) that under 3, trillions (or millions of millions of millions.)

EXAMPLE.

Sextillions.	Quintilli.	Quatrill.	Trillions.	Billions.	Millions.	Units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u
6	5	4	3	2	1	
913,208;	000,341;	620,057;	219,356;	809,379;	120,406;	129,763
Thousands	Thousands	Thousands	Thousands	Thousands	Thousands	Thousands

Note 1. Billions is substituted for millions of millions: Trillions, for millions of millions of millions: Quatrillions, for millions of millions of millions of millions.

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c. answer to millions so often involved as their indices respectively denote.

Note 2. The right hand figure of each half period has the place of units of that half period; the middle one, that of tens, and the left hand one, that of hundreds.

The

The Application.

Write down, in proper figures, the following numbers.

Fifteen. - - - - - - - - - - - -	15
Two hundred and ſeventy nine. - - - - - - -	279
Three thouſand four hundred and three. - - - - - -	3403
Thirty ſeven thouſand, five hundred and ſixty-ſeven. - - -	37567
Four hundred, one thouſand and twenty eight. - - - -	401028
Nine millions, ſeventy two thouſand and two hundred. - -	9072200
Fifty five millions, three hundred, nine thouſand and nine. - -	55309009
Eight hundred millions, forty four thouſand, and fifty five. - -	800044055
Two thouſand, five hundred and forty three millions, four hundred and thirty one thouſand, ſeven hundred and two. -	2543431702

Write down in words at length the following numbers.

8	437	709040	3476194	7584397647
17	3010	879066	84094007	49163189186
129	76506	4091875	690748591	500098400700

Notation by Roman Letters.

I. One.	XV. Fifteen.	CC. Two hundred.
II. Two.	XVI. Sixteen.	CCC. Three hundred.
III. Three.	XVII. Seventeen.	CCCC. Four hundred.
IV. Four.	XVIII. Eighteen.	D. or IↃ. Five hundred.
V. Five.	XIX. Nineteen.	DC. Six hundred.
VI. Six.	XX. Twenty.	DCC. Seven hundred.
VII. Seven.	XXX. Thirty.	DCCC. Eight hundred.
VIII. Eight.	XL. Forty.	DCCCC. Nine hundred.
IX. Nine.	L. Fifty.	M. or CIↃ. One thousand.
X. Ten.	LX. Sixty.	IↃↃ. Five thousand.
XI. Eleven.	LXX. Seventy.	IↃↃↃ. Fifty thousand.
XII. Twelve.	LXXX. Eighty.	IↃↃↃIↃↃↃ. Five hund. tho.
XIII. Thirteen.	XC. Ninety.	MDCCCVIII. One thousand,
XIV. Fourteen.	C. Hundred.	eight hundred and eight.

A less literal number placed after a greater, always augments the value of the greater; if put before, it diminishes it. Thus, VI. is 6; IV. is 4; XI. is 11; IX. is 9, &c.

ADDITION

IS the putting together of two or more numbers, or sums, to make them one total, or whole sum.

SIMPLE ADDITION

Is the adding of several integers or whole numbers together, which are all of one kind, or sort; as 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or sum total, is 39 pounds.

Rule.

Rule.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units; after adding up every figure in that column, consider how many tens are contained in their sum, and, placing the excess under the units, carry so many as you have tens, to the next column, of tens: Proceed in the same manner through every column, or row, and set down the whole amount of the last row.*

Proof. Begin at the top of the sum and reckon the figures downwards, in the same manner as they were added upwards, and, if it be right, this aggregate will be equal to the first. Or, cut off the upper line of figures, and find the amount of the rest; then, if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

Addition

* This Rule as well as the method of proof, is founded on the known axiom, "the whole is equal to the ſum of all its parts." The method of placing the numbers, and carrying for the tens, is evident from the nature of notation; for any other diſpoſition of the numbers would alter their value; and carrying one, for every ten, from an inferiour to a ſuperiour column, is, evidently, right, becauſe one unit in the latter caſe is equal to the value of ten units in the former.

Beſides the method of proof, here given, there is another, by caſting out the nines; thus:

1. Add the figures in the upper row together, and find how many nines are contained in their ſum.
2. Reject the nines, and ſet down the remainder, directly even with the figures in the row.
3. Do the ſame with each of the given numbers, and ſet all the exceſſes of nines in a column, and find their ſum; then, if the exceſs of nines in this ſum, found, as before, is equal to the exceſs of nines in the ſum total; the queſtion is ſuppoſed to be right.

Example.

	Excess of nines
5738	5
9156	3
8471	2
5324	5
23689	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; viz, that any number, divided by 9, will leave the ſame remainder, as the ſum of its figures, or digits, divided by 9: which may be thus demonſtrated.

Demonſtration. Let there be any number, as 5432; this, ſeparated into its ſeveral parts, becomes $5000+400+30+2$; but $5000=5\times1000=5\times\overline{999+1}=\overline{5\times999}+5$. In like manner $400=\overline{4\times99}+4$, and $30=\overline{3\times9}+3$. Therefore, $5432=\overline{5\times999}+5$, $+\overline{4\times99}+4$, $+\overline{3\times9}+3+2=\overline{5\times999}+\overline{4\times99}+\overline{3\times9}+5+4+3+2$.

And $\dfrac{5432}{9}=\dfrac{\overline{5\times999}+\overline{4\times99}+\overline{3\times9}+5+4+3+2}{9}$; but $\overline{5\times999}+\overline{4\times99}+\overline{3\times9}$ is diviſible by 9; therefore, 5432, divided by 9, will leave the ſame remainder, as $5+4+3+2$, divided by 9; and the ſame will hold good of any other number whatever.

The ſame property belongs to the number 3: However, this inconveniency attends this method, that, although the work will always prove right, when it is ſo; it will not, always, be right, when it proves ſo; I have, therefore, given this demonſtration more for the ſake of the curious, than for any real advantage.

Addition and Subtraction Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column and the other atop, and in the common angle of meeting, or, at the right hand of the first, and under the second, you will find the sum—as, 5 and 8 is 13.

When you would subtract: Find the number to be subtracted in in the left hand column, run your eye along to the right hand till you find the number from which it is taken, and right over it, atop, you will find the difference—as 8, taken from 13, leaves 5.

Examples.

1.	2,	3.	4.	5.	6.
£.	℔.	Cwt.	Miles.	Yards..	£.
1	12	123	1234	12345	987654321
2	34	456	5678	67890	123456789
3	56	785	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	62	67890	567879287
7	23	901	4713	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4567	2102865
4567890	4566	89093	4321234
5678209	333	654321	5682098
6789098	90	1234567	6543218

SUBTRACTION

TEACHES to take a less number from a greater, to find a third, shewing the inequality, excess or difference between the given numbers; and it is both simple and compound.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of a like kind.

RULE.

Place the larger number uppermost, and the less underneath, so that units may stand under units, tens under tens, &c. then, drawing a line underneath, begin with the units, and subtract the lower from the upper figure, and set down the remainder; but if the lower figure be greater than the upper, borrow ten, and subtract the lower figure therefrom: To this difference, add the upper figure, which, being set down, you must add one to the ten's place of the lower line, for that which you borrowed; and thus proceed through the whole.*

PROOF.

In either simple, or compound Subtraction, add the remainder and the less line together, whose sum, if the work be right, will be equal to the greater line: Or subtract the remainder from the greater line, and the difference will be equal to the less.

EXAMPLES.

	1.	2.	3.	4.	5.	6.
	£.	£.	Miles.	Yards.	Feet.	Cwt.
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
Rem.						
Proof.						

7.	8.	9.
100200300400500600700800900	10000	1000000
98076054032011023045067089	9999	1

MULTIPLICATION

* *Dem.* When all the figures of the less number are less than their correspondent figures in the greater, the difference of the figures, in the several like places, must, all taken together, make the true difference sought; because, as the sum of the parts is equal to the whole; so must the sum of the differences, of all the similar parts, be equal to the difference of the whole.

2. When any figure in the greater number is less than its correspondent figure in the less, the ten, which is added by the Rule, is the value of an unit in the next higher place, by the nature of notation; and the one which is added to the next place of the less number, is to diminish the corrrespondent place of the greater, accordingly; which is only taking from one place, and adding as much to another, whereby the total is never changed: And, by this mean, the greater is resolved into such parts, as are, each, greater than, or equal to, the similar part of the

MULTIPLICATION

MAY be accounted the most serviceable Rule in Arithmetick. It teaches how to increase the greater of two numbers given, as often as there are units in the less; performs the work of many additions in the most compendious manner; brings numbers of great denominations into small, as pounds into shillings, pence or farthings, &c. and, by knowing the value of one thing, we find the value of many.

It consists of three parts.

1. The Multiplicand, or number given to be multiplied, and, commonly, the largest number.

2. The Multiplier, or number to multiply by, commonly, the least number.

3. The Product, which is the result of the work, or the answer to the question.

SIMPLE MULTIPLICATION

Is the multiplying of any two numbers together, without having regard to their signification; as 7 times 8 is 56, &c.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table for Multiplication: Find your multiplier in the left hand column, and your multiplicand atop, and in the common angle of meeting, or against your multiplier, along at the right hand, and under your multiplicand, you will find the product, or answer.

To learn it for Division: Find the divisor in the left hand column, and run your eye along the row to the right hand until you find the dividend; then, directly over the dividend, atop, you will find the quotient, shewing how often the divisor is contained in the dividend.

CASE

the lefs; and the difference of the correfpondent figures, taken together, will, evidently, make up the difference of the whole.

The truth of the method of proof is evident; for the difference of two numbers added to the lefs, is, manifeftly, equal to the greater.

CASE I.

When the multiplier is not more than 12 : Always placing the greater number uppermost, set the multiplier underneath, units under units, &c. and begin as the Table directs, setting down the unit figure under units, and carrying the tens to the next place, in all respects as in simple addition.*

Proof.

Multiply the multiplier by the multiplicand.

Examples.

	1.	2.	3.	4.
	37934	769308	4980076	763896
	2	3	4	5
Prod.				

5.	6.	7.	8.
67589	503764	3918295	9164785
6	7	8	9

9.	10.	11.
4879567	5864794	8583478646
10	11	12

CASE II.

When the multiplier is more than 12 : Multiply each figure in the multiplicand by every figure in the multiplier, beginning with the units, and placing the first figure of each product exactly under its multiplier : Lastly, add these several products together, in the same order as they stand, and their sum will be their total product.†

Examples.

* *Dem.* When the multiplier is a single digit, it is plain that we find the product ; for, by multiplying every figure, that is, every part of the multiplicand, we multiply the whole: and, the writing down of the products, which are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens to the product of the next place, is only gathering together the similar parts of the respective products, and is, therefore, the same, in effect, as though we wrote down the multiplicand as often as the multiplier expresses, and added them together ; for the sum of every column is the product of the figures in the place of that column ; and the products, collected together, are evidently equal to the whole required product.

† If the multiplier be a number, made up of more than one figure ; after we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and, after the same manner, find the product of the multiplicand by the second figure of the multiplier ; but as the figure, by which we are multiplying, stands in the place of tens, the product must be ten times its simple value ; and, therefore, the first figure in this product

must

EXAMPLES.

1.	2.	3.
6357534	8324629	46293845
47	59	76
44502738		
25430136		
Prod. 298804098		

4.	5.	6.
647906	760483	91867584
4873	9152	6875
3157245938	6959940416	631589640000

CASE

must be noted in the place of tens, or, which is the same, directly under the figure we are multiplying by. And, proceeding in the same manner with all the figures of the multiplier, separately, it is evident we shall multiply all the parts of the multiplicand by all the parts of the multiplier; therefore, these several products, being added together, will be equal to the whole required product.

The reason of the method of proof, depends upon this proposition, that if two numbers are to be multiplied together, either of them may be made the multiplier or multiplicand, and the product will be the same.

A small attention to the nature of numbers will make this truth evident; for $5\times9 = 45 = 9\times5$; and, in general, $2\times3\times4\times5\times6$, &c. $= 3\times2\times6\times5\times4$, &c. without any regard to the order of the terms; and this is true of any number of factors whatever.

N. B. By factors are meant the multiplier and multiplicand.

The following examples are subjoined, to make the reason of the rule appear as clearly as possible.

```
  64753
      5
  -----
     15=     3×5
     25 =   50×5
    35  =  700×5
   20   = 4000×5
  30    =60000×5
  ------  -------
  323765=64753×5
```

```
   237956
     3728
   ------
  1903648 = 8 times the multiplicand.
  475912 = 20 times ditto.
 1665692 = 700 times ditto.
 713868 = 3000 times ditto.
 ---------
887099968=3728 times ditto.
```

Multiplication may also be proved, by casting out the nines; but is liable to the inconvenience before mentioned.

It may likewise be, very naturally, proved by division; for the product, being divided by either of the factors, will, evidently, give the other; and it might not be amiss for the pupil to be taught division, at the same time with multiplication; as it not only serves for proof; but also gives him a readier knowledge of them both. But it would have been contrary to good method to have given this rule in the text, because the pupil is supposed, as yet, to be unacquainted with division.

CASE III.

When the multiplier is a composite number, that is, when it is produced by the multiplication of any two numbers in the Table, multiply the multiplicand by one of those figures, first, and that product by the other: And the last product will be the total required.*

EXAMPLES.

1.	2.	3.
Mult. 59375 by 35.	39187 by 48.	91632 by 56.
7		
7X5 = 35 —— 415625		
5		
2078125		

CASE IV.

When there are cyphers on the right hand of either the multiplicand, or multiplier, or both: Neglect those cyphers; then place the significant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand as many cyphers as there are in both the factors.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380296000	306144000	8836301500000

CASE V.

To multiply by 10, 100, 1000, *&c.:* Set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.†

EXAMPLES.

	1.	2.	3.	4.
	57935	84935	613975	8473965
	10	100	1000	10000
Prod.	579350			

CASE

* The reason of this method is obvious: For any number, multiplied by the component parts of another number, must give the same product, as though it were multiplied by that number at once: Thus, in example first, 5 times the product of 7, multiplied into the given number, makes 35 times that given number, as plainly, as 5 times 7 makes 35.

† This is evident from the nature of numbers, since every cypher annexed to the right of a number increases the value of that number in a tenfold proportion.

CASE VI.

To multiply by 99, 999, *&c. in one line:* Place as many dots at the right hand of the multiplicand, as there are figures of 9 in your multiplier, which dots suppose to be cyphers; then, beginning with the right hand dot, subtract the *given* multiplicand from the *new* one, and the remainder will be the product.*

EXAMPLES.

1.	2.	3.
6473..	857389...	5384976....
99	999	9999
640827	856531611	53844375024

That these examples may appear as clear as possible, I will illustrate them by giving another.

Mult. 371967...	According to the rule,	371967...	Minuend.
by 999	it will stand thus,	371967	Subtrah.
371595033		371595033	Rem. or total Prod.

CASE VII.

To multiply by 13, 14, 15, *&c. to* 19; *also from* 101 *to* 109, *from* 1001 *to* 1009, *&c.:* Multiply with the unit figure only, of the multiplier, removing the product one place to the right hand of the multiplicand, and so many places further as there may be cyphers between the significant figures; then add all together, and their sum will be the product.

EXAMPLES.

* Here it may eaſily be ſeen that, if you multiply any ſum by 9, the product will be but 9 tenths of the product of the ſame ſum, multiplied by 10; and as the annexing of a dot or cypher, to the right hand of the multiplicand, ſuppoſes it to be increaſed tenfold; therefore, ſubtracting the given multiplicand from the tenfold multiplicand, it is evident that the remainder will be ninefold the ſaid given multiplicand, equal to the product of the ſame by 9; and the ſame will hold true of any number of nines.

Note, When the multiplicand has a fraction added to it, as one fourth, one half, &c. add ſuch a part of the multiplier as the fraction makes, to the laſt product: But when ſuch fraction belongs to the multiplier, add to the laſt product ſuch a part of the multiplicand as the fraction denotes.

EXAMPLES.

1.	2.	3.
75964 × 13	7598 × 104	6735 × 1005
227892	30392	33675
Prod. 987532		

CASE VIII.

To multiply by 21, 31, 41, *&c. to* 91, *also by the same figures with any number of cyphers between them:* Multiply by the left hand figure, only, of the multiplier, and set the unit figure of the product one place to the left, and as many places further as there are cyphers between the significant figures; and add the numbers together for the product.

EXAMPLES.

	1.	2.	3.
	73918 × 21	56934 × 301	45936 × 4001
	147836	170802	183744
Prod.	1552278	17137134	183789936

CASE IX.

To multiply any number, viz. whole or decimal, by any number, giving only the Product: Put down the product figure of the first figure of the multiplicand by the *first* of the multiplier. To the product of the *second* of the multiplicand by the *first* of the multiplier, add the number to be carried, and the product of the *first* of the multiplicand by the *second* of the multiplier; then, carrying for the tens in the sum, put down the rest. To the product of the *third* of the multiplicand by the *first* of the multiplier, add the number to be carried, and the product of the *second* of the multiplicand by the *second* of the multiplier, also the product of the *first* of the multiplicand by the *third* of the multiplier, carry the tens, and put down the rest, and so proceed till you have multiplied the *highest* of the multiplicand by the *lowest* of the multiplier. Then multiply the *highest* of the multiplicand by the *second* of the multiplier: Add the number to be carried, and the product of the *last but one* of the multiplicand by the *third* of the multiplier, and the product of the *last but two* of the multiplicand by the *fourth* of the multiplier, &c. Then to the product of the *last but one* of the multiplicand by the *fourth* of the multiplier; and so proceed till you have multiplied the *last* of the multiplicand by the *last* of the multiplier, which finishes the work.

Example.

Example.

Mult. 5321415
By 2354

Prod. 12526610910

Explanation.

$$\overline{5\times4}=20$$
$$\overline{1\times4}+2+\overline{5\times5}=31$$
$$\overline{4\times4}+3+\overline{1\times5}+\overline{5\times3}=39$$
$$\overline{1\times4}+3+\overline{4\times5}+\overline{1\times3}+\overline{5\times2}=40$$
$$\overline{2\times4}+4+\overline{1\times5}+\overline{4\times3}+\overline{1\times2}=31$$
$$\overline{3\times4}+3+\overline{2\times5}+\overline{1\times3}+\overline{4\times2}=36$$
$$\overline{5\times4}+3+\overline{3\times5}+\overline{2\times3}+\overline{1\times2}=46$$
$$\overline{5\times5}+4+\overline{3\times3}+\overline{2\times2}=42$$
$$\overline{5\times3}+4+\overline{3\times2}=25$$
$$\overline{5\times2}+2=12$$

DIVISION

TEACHES to separate any number, or quantity given, into any number of parts assigned; or to find how often one number is contained in another; or from any two numbers given, to find a third, which shall consist of so many units, as the one of those given numbers is comprehended in the other; and is a concise way of performing several Subtractions.

There are four principal parts to be noticed in Division, viz.

1. The Dividend, or number given to be divided.
2. The Divisor, or number given to divide by.
3. The Quotient, or answer to the question, which shews how often the divisor is contained in the dividend.
4. The Remainder (which is always less than the divisor, and of the same name with the dividend) is very uncertain, as there is sometimes a remainder, and sometimes none.

Division is both simple and compound.

Proof.

Multiply the divisor and quotient together, and add the remainder, if there be any, to the product; if the work be right, that sum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values: As 56, divided by 8, produces 7 in the quotient: That is, 8 is contained 7 times in 56.*

CASE

* According to the rule, we reſolve the dividend into parts, and find, by trial, the number of times the diviſor is contained in each of thoſe parts; and the only thing which remains to be proved, is, that the ſeveral figures of the quotient, taken as one number, according to the order, in which they are placed, is the true quotient of the whole dividend by the diviſor; which may be thus demonſtrated.

Dem. The complete value of the firſt part of the dividend, is, by the nature of notation, 10, 100, 1000, &c. times the ſimple value of what is taken in the operation; accordingly, as there are 1, 2, or 3, &c. figures ſtanding before it; and, conſequently, the true value of the quotient figure, belonging to that part of the dividend, is alſo 10, 100, 1000, &c. times its ſimple value; but the true value of the quotient figure, belonging to that part of the dividend, found by the rule, is alſo 10, 100, 1000, &c. times its ſimple value; for there are as many figures ſet before it, as the number of remaining figures in the dividend; therefore, the firſt quotient figure, taken in its complete value from the place it ſtands in, is the true quotient of the diviſor in the complete value of the firſt part of the dividend. For the ſame reaſon, all the reſt of the figures of the quotient, taken according to their places, are, each, the true quotient of the diviſor, in the complete value of the ſeveral parts of the dividend belonging to each; becauſe, as the firſt figure, on the right hand of each ſucceeding part of the dividend, has a leſs number of figures ſtanding before it, ſo ought their quotients to have; and ſo they are actually ordered; conſequently, taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the ſum of the true quotients of all the ſeveral parts of the dividend; and is, therefore, the true quotient of the wholedividend by the diviſor.

That no obſcurity may remain, in this demonſtration, it is illuſtrated by the following example.

Diviſor 25)74503 Dividend
1ſt part of the dividend is = 74000
25×2000 = 50000 — — 2000 the 1ſt quotient.
1ſt remainder = 24000
add 500
2d part of the dividend = 24500
25×900 = 22500 — — 900 the 2d quotient.
2d remainder = 2000
add 00
3d part of the dividend = 2000
25×80 = 2000 — — 80 the 3d quotient.
00
add 3
4th part of the dividend = 3
25×0 0 — — 0 the 4th quotient.
Laſt remainder = 3 — 2980 = Sum of all the quotients, or, the Anſwer.

Explan. It is evident the dividend is reſolved into theſe parts, 74000+500+00+3; for the firſt part of the dividend is conſidered only as 74; but yet it is, truly, 74000; and therefore its quotient, inſtead of 2, is 2000, and the remainder 24000; and ſo of the reſt; as may be ſeen in the operation.

CASE I.

Rule.—First, seek how many times the divisor is contained in a competent number of the first figures of the dividend ; when found, place

When there is no remainder to a diviſion, the quotient is the abſolute and perfect anſwer to the queſtion ; but where there is a remainder, it may be obſerved, that it goes ſo much towards another time as it approaches the diviſor ; thus, if the remainder be half the diviſor, it will go half of a time more, and ſo on ; in order, therefore, to complete the quotient, put the laſt remainder to the end of it, above a line, and the diviſor below it.

It is ſometimes difficult to find how often the diviſor may be had in the numbers of the ſeveral ſteps of the operation : The beſt way will be to find how often the firſt figure of the diviſor may be had in the firſt, or two firſt figures of the dividend, and the anſwer, made leſs by one or two, is, generally, the figure wanted ; but, if, after ſubtracting the product of the diviſor and quotient from the dividend, the remainder be equal to, or exceed the diviſor, the quotient figure muſt be increaſed accordingly ; or, if the product of the diviſor and quotient figure exceed the dividend, then the quotient figure muſt be proportionably leſſened.

The reaſon of the method of proof is plain ; for, ſince the quotient is the number of times the dividend contains the diviſor, the product of the quotient and diviſor, muſt, evidently, be equal to the dividend.

There are ſeveral other methods made uſe of to prove diviſion ; as follow, viz.

Rule I.

Subtract the remainder from the dividend ; divide this number by the quotient, and the quotient, found by this diviſion, will be equal to the former diviſor, when the work is right.

Rule II.

Add the remainder and all the products of the ſeveral quotient figures multiplied by the diviſor together, according to the order in which they ſtand in the work, and the ſum, when the work is right, will be equal to the dividend.

Here, the numbers to be added are the products of the diviſor by every figure of the quotient, ſeparately ; and each, by its place, poſſeſſes its complete value ; therefore, the ſum of the parts, together with the remainder, muſt be equal to the whole. I will illuſtrate the whole by an example proved according to the ſeveral different methods.

```
79)9 8 7 6 5 4 3 2 1(12501953
   7 9*                         79+34 remainder.
   ---                        ---------
   1 9 7                        112517577
   1 5 8*                       87513671
   -----                             +34
   . 3 9 6                    ---------
   . 3 9 5*                   987654321 Proof by Multiplication.
   . -----
   . . . 1 5 4
   . . . . 7 9*                      987654321
   . . . . -----                           —34
   . . . . 7 5 3                     ---------
   . . . . 7 1 1*    12501953)987654287(79 Proof by Diviſion.
   . . . . -------            87513671
   . . . . . 4 2 2            ---------
   . . . . . 3 9 5*            112517577
   . . . . . -------           112517577
   . . . . . . . 2 7 1         ---------
   . . . . . . . 2 3 7*
   . . . . . . . -------
   . . . . . . . . 3 4*
   ---------------------
   9 8 7 6 5 4 3 2 1   Proof by Addition.
```

We need only to refer to the example ; except for the proof by addition ; where it may be remarked, that the Aſteriſms ſhew the numbers to be added, and the dotted lines their order.

place the figure in the quotient; multiply the divisor by this quotient figure, place the product under the left hand figures of the dividend; then subtract it therefrom, and bring down the next figure of the dividend to the right hand of the remainder: If, when you have brought down a figure to the remainder, it is still less than the divisor, a cypher must be placed in the quotient, and another figure be brought down: after which, you must seek, multiply and subtract, till you have brought down every figure of the dividend.

Examples.

1.

Divisor. Dividend Quotient.
3)175817(58605
15
25
24
18
18
17
15
2 Rem.

Proof.
58605 Quotient.
×3 Divisor + 2
175817

In this example, I find that 3, the divisor, cannot be contained in the first figure of the dividend; therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend, &c. I then subtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz, 5; then I inquire how often the divisor 3, is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when, finding I cannot have the divisor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the first.

Observe, that, in multiplying by 3, I add in the 2.

2.
29)153598(5296
145
85
58
279
261
188
174
14

3.
6493)91876375(14150
6493
26946
25972
9743
6493
32507
32465
425

4.
28)503775(

5.
35)197184(

6.
85)994466(

7.
236)3798567(

8.
3479)483956795(

9.
5679)19647394(

10.

10.
38473)119184693(

11.
641976)9187642959(

12.
5823789)791822376496(

13.
123456789)121932631112635269(

CASE II.

When there is one cypher, or more, at the right hand of the divisor: It or they must be cut off; also, cut off the same number of figures from the dividend, and then proceed as in Case first: But the figures which were cut off from the dividend must be placed at the right hand of the remainder.*

EXAMPLES.

1.

```
65|00)3794326|75(58374
      325
      ---
       544
       520
       ---
        243
        195
        ---
         482
         455
         ---
          276
          260
          ---
          1675 Rem.
```

2.
5193|000)8937643|893(

3.
917|0)47658|3(

4.
875|000)917647894301000(

5.
Quot. Rem.
1|0)9584|6

6.
Quot. Rem.
1|00)76495|80

7.
Quot. Rem.
1|000)93751839|462

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divisor, your work is done; those figures, cut off at the right hand, are the remainder, and those on the left, the quotient, as above.

CASE

* The reaſon of this contraction is eaſy to conceive; for cutting off the ſame figures from each, is the ſame as dividing each of them by 10, 100, 1000, &c. and it is evident, that as often as the whole diviſor is contained in the whole dividend, ſo often muſt any part of the diviſor be contained in the like part of the dividend; this method is only to avoid a needleſs repetition of cyphers, which would happen in the common way.

CASE III.

Short Division *is, when the divisor does not exceed* 12.

Rule.

First, seek how often the divisor can be had in the first figure, or figures of the dividend ; which, when found, place in the quotient ; then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left hand figures of your dividend, and the units which remain, must be accounted so many tens, which you must suppose to stand at the left hand of the next figure in the dividend, and to be reckoned with it ; then, seek how often you can have your divisor in those two figures ; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

Examples.

Divisor, Dividend.	2.	3.	4.	5.
2)71935	3)51903	5)633795	6)8471937	7)193847
Quot. 35967—1				

6.	7.	8.	9.
8)5437846	9)45963784	11)91843756	12)1196437847536

CASE IV.

When the divisor is such a number that any two, or more, figures in the Table, being multiplied together, will produce it : Divide the given dividend by one of those figures ; the quotient, thence arising, by the other, and so on ; and the last quotient will be the answer.*

Examples.

* This follows from the contraction in case 3d, of Simple Multiplication, of which it is only the reverse ; for the fourth part of the half of any thing is evidently the same as the eighth part of the whole ; and so of any other number.

As the learner at present is supposed to be unacquainted with the nature of fractions, and as the quotient is incomplete without the remainder ; I shall here give a rule for finding the true remainder, without having recourse to fractions.

Rule I.

Multiply the quotient by the divisor : Subtract the product from the dividend, and the result will be the true remainder.

The Rule, which is most commonly made use of, when the divisor is a composite number, is

Rule II.

Multiply the last remainder by the preceding divisor, or last but one, and to the product add the preceding remainder ; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder ; and so on, till you have gone through all the divisors and remainders, to the first.

Example.

EXAMPLES.

1st. method.	2d. method.	3d. method.
9)196473	8)196473	72)196473(2728 Quot.
8)21830	9)24559	144
Quot. 2728—57	Quot. 2728—57	524
		504
		207
		144
		633
		576
		57 Remainder.

I have wrought the above question three ways, that the learner may understand the method of finding the true remainder, according to this case. In the *first*, in dividing by 9, 3 remains, and by 8, 6 remains; which being the last remainder, I multiply it by the first divisor 9, and add in the first remainder 3, and they make 57, the true remainder. In the *second* method, dividing by 8, 1 remains, and by 9, 7 remains; I therefore, multiply 7, the last remainder, by 8, adding in the 1, and they make 57 as before. The *third* method is self evident, and shews that the other remainders are true.

2.	3.	4.	5.
36)79638	25)197835	8)93975	54)93738764

6.	7.	8.
121)75323939	132)38473692	144)891376429732

Supplement

EXAMPLE.

6)85397 divided by 150	1 the laſt remainder.
*5)14232—5	multiply by *5 the laſt diviſor but one.
5)2846—2	5
569—1	add 2 the ſecond remainder.
	7
	multiply by 6 the firſt diviſor.
Ans. $569\frac{47}{150}$	42
	add 5 the firſt remainder.
	47 the true remainder.

To explain this rule from the example, we may obſerve, that every unit in the firſt quotient may be looked upon as containing 6 of the units in the given dividend; conſequently, every unit which remains, will contain the ſame; therefore, this remainder muſt be multiplied by 6, to find the units it contains of the given dividend. Again, each unit in the next quotient will contain 5 of the preceding ones, or 30 of the firſt, that is, 6 times 5; therefore, what remains muſt be multiplied by 30, or, which is the ſame thing, by 6 and 5 continually: Now, this is the ſame as the Rule; for inſtead of finding the remainders, ſeparately, they are reduced from the bottom, upwards, ſtep by ſtep, to one another, and the remaining units, of the ſame claſs, taken as they occur.

Supplement to Contractions in Multiplication.

1. The shortest method of multiplication, when the multiplier is any even part of 100, 1000, &c is by division: For if the multiplicand be increased by a number of cyphers equal to the number of places in the multiplier, and a part of that product taken for the same proportion, which the multiplier bears to 1, and the same number of cyphers annexed to it, the quotient will be the true product

1. Multiply 39756 into 125.
$125=\frac{1}{8}$ of 1000, wherefore,
8)39756000

4969500 Product.

2. Multiply 57638 by $33\frac{1}{3}$
$33\frac{1}{3}=\frac{1}{3}$ of 100, therefore,
3)5763800

$1921266\frac{2}{3}$ Product.

3. Multiply 91378 by $333\frac{1}{3}$.
$333\frac{1}{3}=\frac{1}{3}$ of 1000, therefore,
3)91378000

$30459333\frac{1}{3}$ Product.

2. If any digit, with cyphers annexed, be divided by 9, the quotient will consist, wholly, of such digits, and so many 9ths of an unit over; hence the following method of multiplying by repetends of any of the digits.

	1.		2.		3.
	645 by 8888.		5394 by 66666.		3798 by 444
	80000		600000		4000
	9)51600000		9)3236400000		9)15192000
	5733333		359600000		1688000
Subtract	573	Subt.	3596	Subt.	1688
Product.	5732760	Prod.	359596404	Prod.	1686312

TABLES IN COMPOUND ADDITION.

1. Federal Money.*

		marked.	mills.				
10 Mills	make one	Cent m.c.	10=	1 cent.			
10 Cents		Dime d	100=	10=	1 dime.		
10 Dimes		Dollar D.	1000=	100=	10=	1 dollar.	
10 Dollars		Eagle E.	10000=	1000=	100=	10=	1 eagle.

2. English

* It may be proper to introduce here an account of the Federal Money, as ſettled by Congreſs, the 8th of Auguſt, 1786, when it was "*Reſolved*,

"That the ſtandard of the United States of America, for gold and ſilver, ſhall be eleven parts fine and one part alloy.

"That the *Money Unit* of the United States (being by the Reſolve of Congreſs, of the 6th July, 1785, a Dollar) ſhall contain, of fine ſilver, $375\frac{64}{100}$ grains.

"That the money of account, to correſpond with the diviſion of coins, agreeably to the above Reſolve, proceed in a decimal ratio, agreeably to the forms and manner following, viz. "Mill,

2. English Money.

			marked
4 Farthings	make one	Penny	*qrs. d.*
12 Pence		Shilling	*s.*
20 Shillings		Pound.	£.

Farthings.
4 = 1 Penny.
48 = 12 = 1 Shilling.
960 = 240 = 20 = 1 Pound.

Pence Tables.

d.	s.	d.	d.	s.	d.	s.	d.	s.	d.
20 =	1	8	120 =	10	0	1 =	12	11 =	132
30 =	2	6	130 =	10	10	2 =	24	12 =	144
40 =	3	4	140 =	11	8	3 =	36	13 =	156
50 =	4	2	150 =	12	6	4 =	48	14 =	168
60 =	5	0	160 =	13	4	5 =	60	15 =	180
70 =	5	10	170 =	14	2	6 =	72	16 =	192
80 =	6	8	180 =	15	0	7 =	84	17 =	204
90 =	7	6	190 =	15	10	8 =	96	18 =	216
100 =	8	4	200 =	16	8	9 =	108	19 =	228
110 =	9	2	240 =	20	0	10 =	120	20 =	240

3. Troy

" Mill, the loweſt money of account, of which 1000 ſhall be equal to the federal dollar, or money unit, - - - - - - - - 0,001.

" Cent, the higheſt copper piece, of which 100 ſhall be equal to the federal dollar, - - - - - - - - - - 0,010.

" Dime, the loweſt ſilver coin, of which 10 ſhall be equal to the dollar, 0,100.

" Dollar, the higheſt ſilver coin, - - - - - - - 1,000.

" That, betwixt the dollar and the loweſt copper coin, as fixed by a reſolve of Congreſs of the 6th of July, 1785, there ſhall be *three* ſilver coins, and *one* copper coin.

" That the ſilver coins ſhall be as follow : One coin containing $187\frac{82}{100}$ grains of fine ſilver, to be called a *Half Dollar :* One coin containing $75\frac{128}{1000}$ grains of fine ſilver, to be called a *Double Dime :* And one coin containing $37\frac{64}{1000}$ grains of fine ſilver, to be called a *Dime.*

" That the two copper coins ſhall be as follow : One equal to the one hundredth part of the federal dollar, to be called a *Cent :* and one equal to the two hundredth part of the federal dollar, to be called a *Half Cent.*

" That $2\frac{1}{4}$lb. Avoirdupois weight of copper, ſhall conſtitute 100 Cents.

" That there ſhall be two gold coins : One containing $264\frac{268}{1000}$ grains of fine gold, equal to 10 dollars, to be ſtamped with the impreſſion of the American Eagle, and to be called an *Eagle :* One containing $123\frac{134}{1000}$ grains of fine gold, equal to 5 dollars to be ſtamped in like manner, and to be called a *Half Eagle.*

" That the mint price of one pound Troy weight of uncoined gold, eleven parts fine, and one part alloy, ſhall be 9 dollars, 9 dimes and 2 cents.

" That the mint price of one pound Troy weight of uncoined gold, eleven parts fine and one part alloy, ſhall be 209 dollars, 7 dimes and 7 cents."

3. Troy Weight.*

24 Grains	make one	Pennyweight,	marked grs. pwt.
20 Pennyweights	- - - -	Ounce, - - - - -	oz.
12 Ounces	- - - -	Pound, - - - - - -	℔

Grains.
24 = 1 Pennyweight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

4. Avoirdupois Weight.†

16 Drams -	make 1	Ounce,	marked dr. oz.
16 Ounces	- - -	Pound, - - - - - - -	℔
28 Pounds	- - -	Quarter of a hundred wt. -	qr.
4 Quarters	- - -	Hundred wt. or 112 pounds, -	Cwt.
20 Hundred wt.	- - -	Ton, - - - - - - -	T.

Drams.
16 = 1 Ounce.
256 = 16 = 1 Pound.
7168 = 448 = 28 = 1 Quarter.
28672 = 1792 = 112 = 4 = 1 Hund. wt.
573440 = 35840 = 2240 = 80 = 20 = 1 Ton.

5. Apothecaries'

* By this weight are weighed Gold, Silver, Jewels, Electuaries, and all liquors.

An ounce of gold is divided into 24 parts, called carats, and an ounce of silver, into 20 parts, called pennyweights; therefore, to distinguish fineness of metals, such gold as will abide the fire without loss, is accounted 24 carats fine: If it lose 2 carats in trial, it is called 22 carats fine, &c.

A pound of silver, which loses nothing in trial, is 12 ounces fine; but, if it lose 3 pennyweights, it is 11 oz. 17 pwts. fine, &c.

Alloy is some base metal with which gold or silver is mixed to abate its fineness; 22 carats of gold, and 2 carats of copper, are esteemed the true standard for gold coin in England, the alloy being one eleventh part of the fine gold: and 11 oz. 2 pwts. of fine silver, melted with 18pwts. of copper, make the true standard for silver coin.

Note. 175 Troy ounces, are precisely equal to 192 Avoirdupois ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1 lb. Troy = 5760 grains, and 1 lb. Avoirdupois = 7000 grains.

† By Avoirdupois are weighed all coarse and drossy goods, grocery and chandlery wares; bread, and all metals, except gold and silver.

A barrel of pork weighs 220℔. A barrel of beef, 220℔. A quintal of fish, 1 Cwt. Avoirdupois. 12 particular things make one dozen; 12 dozen 1 gross, and 144 dozen 1 great gross. 20 particular things make 1 score,

		℔
A Firkin of Foreign	Butter	56
——— ———	Soap	94
A Barrel of ———	Anchovies	30
——— ———	Soap	256
——— ———	Raisins	112
A Punch. of———	Prunes	1120
A Fother of———	Lead	$19\frac{1}{2}$ Cwt.

	lb.
A Stone of Iron, shot, or horseman's weight,	14
——Butcher's Meat,	8
A gallon of Train Oil	$7\frac{1}{2}$
A Tod is - - - - - -	28
A Weigh - - - - -	182
A Sack - - - - - -	364
A Last - - - - - -	4368

5. Apothecaries' Weight.*

20 Grains	make one	Scruple,	marked gr. ℈
3 Scruples	- - -	Dram, - - - -	ʒ
8 Drams - - -		Ounce, - - - -	℥
12 Ounces	- -	Pound, - - - -	lb.

Grains.
20 = 1 Scruple.
60 = 3 = 1 Dram.
480 = 24 = 8 = 1 Ounce.
5760 = 288 = 96 = 12 = 1 Pound.

6. Cloth Measure.†

2 Inches, and one fourth - -	make 1	Nail, marked in.	na.
4 Nails, or 9 Inches - - - -		Quarter of a yard,	qr.
4 Quarters of a yard, or 36 Inches - -		Yard, - -	yd.
3 Quarters of a yard, or 27 Inches -		Ell Flemish,	E. Fl.
5 Quarters of a yard, or 45 Inches - -		Ell English, -	E. E.
6 Quarters of a yard, or 54 Inches - -		Ell French,	E. Fr.
4 Quarters, 1 Inch & one 5th, or 37 Inches and one fifth - - -		Ell Scotch, -	E. Sc.
3 Quarters and two thirds - - - -		Spanish Var.	

Nails, 4 = 1 Quarter.
16 = 4 = 1 Yard.
12 = 3 = 1 Flemish Ell.
20 = 5 = 1 English Ell.
24 = 6 = 1 French Ell.

7. Long Measure.‡

3 Barley corns -	make 1	Inch,	marked bar. in.
12 Inches - -		Foot,	ft.
3 Feet - -		Yard,	yd.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet -		Rod, Perch, or Pole,	pol.
40 Poles - -		Furlong,	fur.
8 Furlongs - -		Mile,	mile.
$69\frac{1}{2}$ Statute miles, *nearly*, -		Degree of a great Circle,	deg.
360 Degrees - - -		A great Circle of the Earth.	

Or,

* All the weights now used by Apothecaries, above grains, are Avoirdupois. The Apothecaries' pound and ounce, and the pound and ounce Troy are the same, only differently divided and subdivided,

† All Scotch and Irish linens are bought by the English or American yard, which is the same, and all Dutch linens by the Ell Flemish; but are all sold in America by the American yard: though the Dutch linens are sold in England by the Ell English, and the Scotch and Irish linens, as in America.
The Scotch allow one English yard in every score yards.

‡ The use of Long Measure is to measure the distance of places, or any other thing, where length is considered without regard to breadth.

Note. 60 geometrical miles make a degree. 4 inches a hand. 5 feet a geometrical pace. 6 points make 1 line, 12 lines an inch, 12 inches a foot, and 6 feet one French toise, or fathom, equal to 6 feet 4 inches, 8,812,875 lines, English measure. 1 English foot equal to 11 inches, 31154 lines French. 66 feet, or 4 poles, make a Gunter's chain. 3 miles make a league.

Or, in Measuring Distances.

$7\frac{92}{100}$ Inches	-	make 1 Link.
25 Links	- -	Pole.
100 Links	- -	Chain.
10 Chains	- -	Furlong.
8 Furlongs	- -	Mile.

Bar. corns, 3 =	1 Inch.					
36 =	12 =	1 Foot.				
108 =	36 =	3 =	1 Yard.			
594 =	198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 Pole.		
23760 =	7960 =	660 =	220 =	40 =	1 Furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 M.

Inches, $7\frac{92}{100}$ =	1 Link,				
198 =	25 =	1 Pole or Perch.			
792 =	100 =	4 =	1 Chain.		
7920 =	1000 =	40 =	10 =	1 Furlong.	
63360 =	8000 =	320 =	80 =	8 =	1 Mile.

8. Time.*

60 Seconds	- - - -	make 1 Minute,	marked s. m.
60 Minutes	- - - -	Hour,	h.
24 Hours	- - - -	Day,	d.
7 Days	- - - -	Week,	w.
4 Weeks	- - - -	Month,	mo.
13 Months, 1 day & 6 hours	- -	Julian year,	yr.

Seconds, 60 =	1 Minute.				
3600 =	60 =	1 Hour.			
86400 =	1440 =	24 =	1 Day.		
604800 =	10080 =	168 =	7 =	1 Week,	
2419200 =	40320 =	672 =	28 =	4 =	1 Month.

Sec	*Min.*	*h.*	*d. h.*	*w. d. h.*	
31557600 =	575960 =	8766 =	365 6 =	52 1 6 =	1 Julian year.†
31558154 =	505969 =	8766 =	365 6	9 14 =	1 Period. year.‡
31556957 =	525948 =	8765 =	365 5	48 57 =	Tropical year.§

9. Motion.

* By the Calendar, the year is divided in the following manner :
Thirty days hath September, April, June, and November ;
February twenty-eight alone, and all the reft have thirty-one.
When you can divide the year of our Lord by 4, without any remainder, it is then Biffextile, or Leap Year, in which February has 29 days.

† The civil, folar year of 365 days, being fhort of the true by 5h. 48m. 57f. occafioned the beginning of the year to run forwards through the feafons nearly 1 day in four years. On this account, Julius Cæfar ordained that one day fhould be added to February, every fourth year, by caufing the 24th day to be reckoned twice ; and becaufe this 24th day was the fixth, (fextilis) before the kalends of March, there were in this year, two of thefe fextiles, which gave the name of Biffextile to this year, which, being thus corrected, was from thence called the Julian year.

‡ A juft and equal meafure of the year is called the periodical year, as being the time of the earth's period about the fun ; in departing from any fixed point in the heavens, and returning to the fame again.

§ The feveral points of the ecliptick having a retrograde, or backward motion, the equinox will, as it were, meet the fun ; by which mean the fun will arrive at the Equinox, or firft point of Aries, before his revolution is completed, and this fpace of time is called the tropical year.

9. Motion.

60 Seconds - -	make 1	Prime minute, marked ″ ′
60 Minutes - -		Degree, •
30 Degrees - - -		Sign, s.
12 Signs, or 360 degrees -		The whole great circle of the Zodiack.*

Seconds, 60 =	1 Minute.			
3600 =	60 =	1 Degree.		
108000 =	1800 =	30 =	1 Sign.	
1296000 =	21600 =	360 =	12 =	Zodiack.

10. Land or Square Measure.

144 Inches - -	make 1	Square foot.
9 Feet - - -		——— Yard,
30¼ Yards, or 272¼ Feet -		——— Pole.
40 Poles - - -		——— Rood.
4 Roods, or160 Rods, or 4840 yards -		——— Acre.
690 Acres - - -		——— Mile.

Inches, 144 =	1 Foot.					
1296 =	9 =	1 Yard.				
39204 =	272¼ =	30¼ =	1 Pole.			
1568160 =	10890 =	1210 =	40 =	1 Rood.		
6272640 =	43560 =	4840 =	160 =	4 =	1 Acre.	
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1 Mile.

11. Solid Measure.†

1728 Inches - -	make 1	Foot.
27 Feet - - -		Yard.
40 Feet of round Timber, or 50 feet of hewn Timber, -		Ton or Load.
128 Solid Feet, i.e. 8 in length, 4 in breadth and 4 in height,		Cord of Wood.

12. Wine Measure.‡

2 Pints - -	make 1	Quart,	marked pts. qts.
4 Quarts - - -		Gallon,	gal.
10 Gallons - - -		Anchor of Brandy.	anc.
18 Gallons - - -		Runlet,	run.
31½ Gallons - - -		Half an Hogshead,	½hhd.
42 Gallons - - -		Tierce,	tier.
63 Gallons - - -		Hogshead,	hhd.
2 Hogsheads - - -		Pipe or butt,	P. or B.
2 Pipes - - -		Tun,	Tun.

Cubick

* The Zodiack is a great circle of the fphere, containing the 12 figns, through which the fun paffes.

† By Solid Meafure are meafured all things that have length, breadth and depth.

‡ All Brandies, Spirits, Perry, Cider, Mead, Vinegar, Honey and Oil, are meafured by Wine Meafure: Honey is, commonly, fold by the pound Avoirdupois.

Cubick Inches.								
$28\frac{1}{8}$ =	1 Pint.							
$57\frac{3}{4}$ =	2 =	1 Quart.						
231 =	8 =	4 =	1 Gallon.					
9702 =	336 =	168 =	42 =	1 Tierce.				
14553 =	504 =	252 =	63 =	$1\frac{1}{2}$ =	1 Hogshead.			
19404 =	672 =	336 =	84 =	2 =	$1\frac{1}{3}$ =	1 Puncheon.		
29106 =	1008 =	504 =	126 =	3 =	2 =	$1\frac{1}{2}$ =	1 Pipe.	
58212 =	2016 =	1008 =	252 =	6 =	4 =	3 =	2 =	1 Tun.

15. ALE or BEER MEASURE.*

2 Pints - - -	make 1	Quart,	marked pts. qts.
4 Quarts - - -		Gallon,	gal.
8 Gallons - - - -		Firkin of Ale in Lond.	A. fir.
$8\frac{1}{2}$ Gallons - - -		Firkin of Ale or Beer.	
9 Gallons - - - -		Firkin of Beer in Lond.	B. fir.
2 Firkins - - -		Kilderkin,	kil.
2 Kilderkins - - -		Barrel,	bar.
$1\frac{1}{2}$ Barrel, or 54 Gallons -		Hogshead of Beer,	hhd.
2 Barrels - - -		Puncheon,	pun.
3 Barrels or 2 Hogsheads - -		Butt,	butt.

BEER.

Cubick Inches.								
$35\frac{1}{4}$ =	1 Pint.							
$70\frac{1}{2}$ =	2 =	1 Quart.						
282 =	8 =	4 =	1 Gallon.					
2538 =	72 =	36 =	9 =	1 Firkin.				
5076 =	144 =	72 =	18 =	2 =	1 Kilderkin.			
10152 =	288 =	144 =	36 =	4 =	2 =	1 Barrel.		
15228 =	432 =	216 =	54 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.	
20304 =	576 =	288 =	72 =	8 =	4 =	2 =	1 Puncheon.	
30456 =	864 =	432 =	108 =	12 =	6 =	3 =	2 =	1 Butt.

ALE.

Cubick Inches.							
$35\frac{1}{4}$ =	1 Pint.						
$70\frac{1}{2}$ =	2 =	1 Quart.					
282 =	8 =	4 =	1 Gallon.				
2256 =	64 =	32 =	8 =	1 Firkin.			
4512 =	128 =	64 =	16 =	2 =	1 Kilderkin.		
9024 =	256 =	128 =	32 =	4 =	2	1 Barrel.	
13536 =	384 =	192 =	48 =	6 =	3	$1\frac{1}{2}$ =	1 Hogshead.

14. DRY

* Milk is fold by the Beer quart.

A barrel of Mackarel, and other barrelled fifh, by an act of this Commonwealth, is to contain not lefs than 30 gallons.

In England, a barrel of Salmon or Eels is 42 gallons, and a barrel of Herrings 32 gallons. The gallon, appointed to be ufed for meafuring all kinds of Liquors, in Ireland, is two hundred and feventeen cubick inches, and fix tenths.

16. Dry Measure.*

2 Pints - - -	make 1	Quart, marked	pts. qts.
2 Quarts - - - -		Pottle,	pot.
2 Pottles - - - -		Gallon,	gal.
2 Gallons - - - -		Peck,	pk.
4 Pecks - - - -		Bushel,	bu.
2 Bushels, - - - -		Strike,	str.
2 Strikes - - . - -		Coom,	co.
2 Cooms - - - -		Quarter,	qr.
4 Quarters - - - -		Chaldron,	ch.
$4\frac{1}{2}$ Quarters - - - - -		Chaldron in London.	
5 Quarters - - - -		Wey,	wey.
2 Weys - - - -		Last,	last.

Cubick Inches.

$268\frac{4}{5} =$ 1 Gallon.
$537\frac{3}{5} = 2 =$ 1 Peck.
$2150\frac{2}{5} = 8 = 4 =$ 1 Bushel.
$4300\frac{4}{5} = 16 = 8 = 2 =$ 1 Strike.
$8601\frac{3}{5} = 32 = 12 = 4 = 2 =$ 1 Coom.
$17203\frac{1}{5} = 64 = 32 = 8 = 4 = 2 =$ 1 Quarter.
$86016 = 320 = 160 = 40 = 20 = 10 = 5 =$ 1 Wey.
$172032 = 640 = 320 = 80 = 40 = 20 = 10 = 2 =$ 1 Last.

COMPOUND ADDITION

IS the adding of several numbers together, having different denominations as Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

Rule.†

1. Place the numbers so that those of the same denomination may stand directly under each other.

2. Add the first column or denomination together as in whole numbers; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the column added, and carry the quotient to the next superiour denomination, continuing the same to the last, which add as in simple addition.

Examples.

* This meaſure is applied to all dry goods, as Corn, Seed, Fruit, Roots, Salt, Sand, Oyſters and Coals.

A Wincheſter buſhel, is $18\frac{1}{2}$ inches diameter, and 8 inches deep.

† The reaſon of this rule is evident from what has been ſaid in Simple Addition: For, in addition of money, as 1, in the pence is equal to 4 in the farthings; 1, in the ſhillings, to 12 in the pence; and 1, in the pounds, to 20 in the ſhillings; therefore, carrying as directed, is the arranging the money, ariſing from each column, properly, in the ſcale of denominations; and this reaſoning will hold good in the addition of compuond numbers, of any denomination whatever.

EXAMPLES.

1. FEDERAL MONEY.

1.

E.	D.	d.	c.	m.
7	3	8	9	5
	2	1	2	5
9	0	0	5	
	3	6	2	5
7	1	4	0	8

2.

D.	c.	m.
49	18	7
25	32	1
93	7	5
13	25	
	97	2

3.

D.	c.	m.
375		
29	18	
7	12	5
199	18	7
30	01	

2. ENGLISH MONEY.

1.

£.	s.	d.
9	16	10
7	10	9
0	18	6
5	11	11
6	0	8
5	9	10

2.

£.	s.	d.	qr.
47	17	6	2
3	9	10	3
75	13	9	1
4	11	11	0
0	16	8	2
17	6	2	1

3.

£.	s.	d.	qr.
847	11	11	3
491	19	6	1
59	6	10	0
747	16	1	2
849	12	11	3
741	17	8	2

4.

£.	s.	d.	qr.
915	10	10	2
64	8	9	1
5	16	11	3
419	2	10	2
491	19	11	3
762	17	6	1

3. TROY WEIGHT.

1.

lb.	oz.	pwt.	gr.
767	10	17	22
39	6	9	17
417	11	16	18
935	9	17	19
478	10	17	22
387	9	16	15

2.

lb.	oz.	pwt.	gr.
649	11	19	20
32	9	6	5
841	10	11	19
473	9	17	23
764	11	8	9
165	6	10	19

3.

lb.	oz.	pwt.	gr.
859	9	15	20
437	10	17	22
640	11	6	0
738	9	12	18
49	0	16	17
584	10	0	9

4. AVOIRDUPOIS WEIGHT.

1.

lb.	oz.	dr.
19	13	12
21	9	6
4	15	15
22	10	5
18	13	12
6	11	10

2.

Cwt.	qrs.	lb.
17	3	19
18	1	27
9	2	9
14	3	16
12	0	6
15	2	0

3.

T.	Cwt.	qrs.	lb.
59	13	2	17
6	17	1	21
45	11	3	25
57	16	2	19
75	17	3	17
6	19	0	26

4.

T.	Cwt.	qrs.	lb.	oz.	dr
91	17	2	25	13	15
19	9	0	17	10	12
14	13	2	0	9	11
47	11	3	19	14	0
69	0	1	0	0	12
77	19	3	27	15	11

5. APOTHECARIES' WEIGHT.

1.

ʒ	℈	gr.
9	1	17
3	2	19
6	1	17
4	0	6
5	2	12
8	1	10

2.

℥	ʒ	℈	gr.
10	7	2	19
6	3	0	12
7	6	1	17
9	5	2	12
6	1	0	16
9	3	2	19

3.

℔	℥	ʒ	℈	gr.
12	11	6	1	15
4	9	1	0	12
91	10	7	2	16
4	8	1	2	19
6	0	0	1	10
4	9	2	1	6

4.

℔	℥	ʒ	℈	gr.
5	9	3	2	13
4	8	6	0	19
9	10	5	2	12
6	5	6	1	17
8	9	4	0	0
7	1	0	1	17

6. Cloth Measure.

1.			2.			3.			4.			5.		
Yd.	*qr.*	*n.*	*E.E.*	*qr.*	*n.*	*E.Fl.*	*qr.*	*n.*	*E.Fr.*	*qr.*	*n.*	*Yds.*	*qr.*	*n.*
76	2	3	91	3	2	75	2	1	49	3	3	914	2	3
3	3	1	49	4	3	7	1	3	19	5	2	49	2	1
42	3	3	6	2	3	84	0	2	24	2	1	561	3	0
57	2	2	84	4	1	76	2	3	67	4	3	84	0	2
16	3	3	7	0	0	48	2	2	48	2	2	549	3	1
49	2	2	61	2	1	9	2	3	6	3	3	617	1	3

7. Long Measure.

1.			2.			3.			4.		
Ft.	*in.*	*bar.*	*Yd.*	*ft.*	*in.*	*Pol.*	*ft.*	*in.*	*Mil.*	*fur.*	*pol.*
9	11	2	7	2	11	12	11	10	9	7	36
6	9	1	4	1	6	9	10	9	7	3	19
7	0	2	6	0	10	8	12	11	4	1	24
3	10	0	7	2	9	7	15	6	6	5	12
9	6	2	8	1	10	4	14	9	4	6	9
7	10	2	9	2	11	5	11	11	5	1	10

5.						
Deg.	*mi.*	*fur.*	*pol.*	*ft.*	*in.*	*bc*
759	56	6	29	15	10	2
317	39	1	36	11	6	1
497	63	7	24	9	8	1
562	17	0	11	13	11	0
64	48	5	17	9	4	2
764	52	4	19	15	11	1

8. Time.

1.					2.				3.		
W.	*d.*	*h.*	*m.*	*s.*	*Mo.*	*d.*	*h.*	*m.*	*Y.*	*m.*	*d.*
3	6	22	57	42	5	24	19	43	19	10	19
1	5	19	31	28	4	27	21	35	7	9	27
2	3	17	9	15	9	18	0	12	4	8	16
3	0	9	17	58	4	19	23	19	1	11	14
1	1	16	19	10	8	11	12	13	17	6	9
2	2	20	53	48	9	19	8	29	12	5	20

4.						
Y.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
57	11	3	6	23	29	55
4	8	1	1	19	45	38
29	9	2	3	17	18	19
46	10	2	5	11	50	13
19	9	2	1	16	18	17
45	9	3	5	18	17	59

9. Motion.

1			2.			3.			
17°	55′	48″	25°	49′	51″	9s	29°	35′	53″
1	37	51	4	21	36	10	0	18	31
28	19	45	19	47	18	4	17	13	42
19	19	37	25	25	39	6	19	50	0

10. Land *or* Square Measure.

1.			2.			3				
Pol.	*feet.*	*in.*	*Yds.*	*ft.*	*in.*	*Acres.*	*rood.*	*pol.*	*feet.*	*in.*
36	179	137	28	7	119	756	3	37	245	128
19	248	119	9	3	75	29	1	28	93	25
12	96	75	29	6	120	416	3	31	128	119
18	110	122	4	8	12	37	1	19	218	20
9	269	24	9	1	119	61	0	0	92	103
25	221	143	8	3	43	191	1	25	129	136

11. Solid Measure.

1.

Ton.	feet.	in.
29	36	1229
12	19	64
18	11	917
19	8	1001
5	0	523
17	39	1119

2.

Yds.	feet.	in.
75	22	1412
9	26	195
3	19	1091
28	15	1110
49	24	218
18	17	1225

3.

Cord.	feet.	in.
37	119	1015
9	110	159
48	127	1071
8	111	956
21	9	27
9	28	1091

12. Wine Measure.

1.

Tier.	gal.	qts.	pts.
37	39	3	1
9	17	2	1
34	28	0	0
32	19	1	1
9	0	3	1
12	40	1	1

2.

Hhd.	gal.	qts.	pts.
51	53	1	1
27	39	3	0
9	18	0	1
0	9	2	1
16	24	1	1
5	0	3	0

3.

Ton.	hhd.	gal.	qts
37	2	37	2
19	1	59	1
28	2	0	0
19	0	47	1
37	1	17	3
14	2	48	2

15. Ale *and* Beer Measure.

1.

A. B.	fir.	gal.
49	3	7
26	2	2
9	0	4
17	3	0
27	1	6
19	3	7

2.

B. B.	fir.	gal.
29	1	8
19	3	5
16	0	3
9	1	8
14	2	0
17	1	5

3.

Hhd.	gal	qts.
379	53	3
19	0	1
121	37	2
467	19	1
561	16	0
75	0	2

16. Dry Measure.

1.

Qrs.	bu.	p.	qt.
64	7	3	7
9	4	1	5
19	6	2	1
4	0	2	0
17	3	0	6
9	5	3	4

2.

Bus.	p.	qt.	pt.
37	2	5	1
19	3	7	0
16	2	0	1
5	1	6	1
9	0	3	0
19	3	0	1

3.

Ch.	bu.	p.	qts
37	27	3	5
6	29	1	7
15	30	0	0
4	11	3	0
5	0	1	0
2	0	2	1

COMPOUND SUBTRACTION

TEACHES to find the difference, inequality, or excess, between any two sums of divers denominations.

Rule.

Rule.*

Place those numbers under each other, which are of the same denomination, the less being below the greater; begin with the least denomination, and, if it exceed the figure over it, borrow as many units as make one of the next greater; subtract it therefrom; and to the difference add the upper figure, remembering, always, to add one to the next superiour denomination, for that which you borrowed.

Examples.

1. Federal Money.

	D.	c.	m.
From	39	15	5
Take	28	17	2

E.	D.	c.	m.
21	8	1	2
10	7	5	

D.	c.	m.
100		
48	87	5

	D.	c.
Borrowed	100	
Paid	29	18
Remains to pay		

	D.	c.
Lent	200	
Received	145	50
Due to me		

	D.	c.	m.
Borrowed	3000		
Paid at several times.	195		
	1115	49	
	247	37	5
	995	12	5
Paid in all			
Remains to pay			

	D.	c.	m.
Lent	7159	12	8
Received at several times.	245	37	5
	3112	15	7
	2000		
	1092	92	0
Received in all			
Remains due			

2. English Money.

1.

	£.	s.	d.	qu.
Borrowed	349	15	6	1
Paid	195	11	8	1
Rem. to pay	154	3	10	0
Proof				

2.

	£.	s.	d.	qr.
Lent	791	9	8	1
Received	197	16	4	2
Due to me				

3. Troy Weight.

1.

	lb.	oz.	pwt.	dr.
Bought	749	5	13	16
Sold	96	9	19	13
Rem.				

2.

lb.	oz.	pwt.	gr.
189	8	12	10
148	4	16	19

3.

lb.	oz.	pwt.	gr.
543	3	9	13
179	1	15	18

4. Avoirdupois

* The reaſon of this Rule will readily appear, from what was ſaid in Simple Subtraction; for the borrowing depends upon the ſame principle, and is only different, as the numbers to be ſubtracted are of different denominations.

4. Avoirdupois Weight.

1.

	lb.	oz.	dr.
Bought	7	9	12
Sold	3	12	9
Rem.			

2.

C.	qr.	lb.
8	2	13
4	1	15

3.

T.	cwt.	qr.	lb.
5	13	1	12
1	12	2	17

4.

T	cwt.	qr.	lb.	oz.	dr.
9	11	3	17	5	12
3	12	1	19	10	9

5. Apothecaries' Weight.

1.

℔	℥	ʒ	℈	gr.
71	9	3	1	13
37	8	4	1	16

2.

℔	℥	ʒ	℈	gr.
65	10	6	2	10
31	8	4	2	9

3.

℔	℥	ʒ	℈	gr.
84	1	1	1	1
65	9	3	1	17

6. Cloth Measure.

1.

Yds.	qr.	n.
35	1	2
19	1	3

2.

E.E.	qr.	n.
467	3	1
291	3	2

3.

E. Fl.	qr.	n.
765	1	3
149	2	1

4.

E. Fr.	qr.	n.
549	4	2
197	4	3

7. Long Measure.

1.

Yds.	ft.	in.
28	2	10
17	2	11

2.

Pol.	ft.	in.
21	11	9
9	13	8

3.

Mil.	fur.	pol.
76	3	11
27	3	21

4.

Deg.	m.	fur.	p.	yds.	ft.	in.	bar.
38	41	3	29	2	1	7	2
19	35	5	31	3	1	9	1

8. Time.

1.

Mo.	d.	h.	m.	s.
6	17	13	27	19
1	21	16	41	35

2.

Mo.	w.	d.	h.
9	2	5	15
4	3	5	15

3.

Y.	mo.	d.
7	3	13
4	2	19

4.

Y.	mo.	w.	d.	h.	m.	s.
48	9	2	5	19	27	31
19	9	3	4	20	19	49

9. Motion.

79°	21′	31″
41	41	52

6s	11°	12′	48″
3	8	39	29

4s	19°	41′	22″
1	22	19	45

10. Land or Square Measure.

1.

A.	R.	Pol.
29	1	10
24	1	25

2.

A.	R.	Pol.
29	2	17
17	1	36

3.

A.	R.	Pol.	ft.	in.
56	3	19	27	110
29	0	21	210	129

11. Solid Measuee.

1.

Tons.	ft.	in.
49	19	1100
38	36	1296

2.

Yds.	ft.	in.
79	11	917
17	25	1095

3.

Cords.	ft.	in.
349	97	1250
192	127	1349

12. Wine Measure.

1.

Hhd.	gal.	qt.	pt.
79	21	2	1
38	61	3	1

2.

Tier.	gal.	qt.
19	17	1
12	29	2

3.

Hhd.	gal.	qt.
375	41	2
197	36	3

4.

Tun.	hhd.	gal.
532	1	19
197	1	47

13. Ale and Beer Measure.

1.

A.B,	fir.	gal.	qt.
39	1	2	1
24	3	6	2

2.

B. B.	fir.	gal.	qts.	pts.
21	3	5	2	0
19	1	7	2	1

3.

Hhds.	gal.	qts.
769	17	1
391	42	3

14. Dry Measure.

1.

Qu.	bu.	pk.	qt.
56	2	2	1
39	3	1	2

2.

Bu.	pk.	qts.	pts.
91	1	3	2
29	2	1	1

3.

Chal.	bu.	pk.	qts.
39	12	2	1
24	25	3	2

PROBLEMS

RESULTING FROM A COMPARISON OF THE PRECEDING RULES.

Prob. 1. Having the sum of two numbers, and one of them given, to find the other.

Rule. Subtract the given number from the given sum, and the remainder will be the number required.

Let 288 be the sum of two numbers; one of which is 115, the other is required?

From 288 the Sum,
Take 115 the given number.
Rem. 173 the other.

Prob. 2. Having the greater of two numbers, and the difference between that and the less given, to find the less.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other, 198: What is the other?

From 325 the greater,
Take 198 the difference.
Rem. 127 the less.

Prob. 3. Having the least of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given

Given $\begin{cases} 127 \text{ the less number.} \\ 198 \text{ the difference.} \end{cases}$

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule. To half the sum add half the difference, and the sum is the greater, and from half the sum take half the difference, and the remainder is the less. Or, from the sum take the difference, and half the remainder is the least: to the least add the given difference, and the sum is the greatest.

What are those two numbers, whose sum is 48, and difference 14?

2)48 2)14 $24+7=31$ the greater, and $24-7=17$ the less.

$\frac{1}{2}$ sum=24 $\frac{1}{2}$ diff.=7 Or $\overline{48-14}\div 2=17$, & $17+14=31$.

PROB 5. Having the sum of two numbers and the difference of their squares* given, to find those numbers.

Rule. Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference: you will then have their sum and difference to find the numbers by Prob. 4.

What two numbers are those, whose sum is 32, and the difference of whose squares is 256?

	Half sum	16
	Half diff.	4
32)256(8 difference.	Greater	20
256	Less	12

PROB. 6. Having the difference of two numbers and the difference of their squares given, to find those numbers.

Rule. Divide the difference of their squares by the difference of the numbers, and the quotient will be their sum; then proceed by Prob. 4.

What are those two numbers, whose difference is 20, and the difference of whose squares is 2000?

20)2000(100 sum. $50+10=60$, the greater, & $50-10=40$, the less.

For more Questions of this nature, see Miscell. Ques. Problems 46, 47, 48 and 49; but, as the extraction of the square root is there concerned, they could not be admitted here.

PROB. 7. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288 and one of them 8; I demand the other? 8)288 *Answer,* 36

PROB. 8. Having the dividend and quotient, to find the divisor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving *Division.*

Given $\begin{cases} 288 \text{ the Dividend.} \\ 36 \text{ the Quotient.} \end{cases}$ 36)288(8 Divisor.

Required the Divisor. 288

PROB.

* The square of a number is the product of it, multiplied into itself.

Prob. 9. Having the Divisor and Quotient given, to find the Dividend.

Rule. Multiply them together.

Given { 8 the Divisor. 36 the Quotient. — 36 × 8

Required the Dividend. — 288 the Dividend.

By a due consideration and application of these Problems only, many questions (of which kind are some of the following) may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

Application *of the preceding Rules*.

1. The least of two numbers is 19418, and the difference between them is 2384 : What is the greater, and sum of both ?

19418+2384=21802 greater, and 19418+21802=41220 sum.

2. Suppose a man born in the year 1743 ; when will he be 77 years of age ? 1743+77 = 1820 *Answer*.

3. What number is that, which, being added to 19418, will make 21802 ? 21802—19418=2384 *Ans*.

4. Gen. Washington was born in 1732 ; what was his age in 1799 ? 1799—1732=67 *Ans*.

5. America was discovered by Columbus in 1492 and its independence declared in 1776 : How many years elapsed between those two eras ? 1776—1492= 284 *Ans*.

6. The Massacre at Boston, by the British troops, happened March 5th, 1770, and the Battle at Lexington, April 19th, 1775 : How long between ?

April 19th, 1775—March 5th, 1770= 5 y. 1 m. 14 d. *Ans*.

7. Gen. Burgoyne and his army were captured October 17th, 1777, and Earl Cornwallis and his army, October 19th, 1781 : What space of time between ?

Oct. 19th, 1781—Oct. 17th, 1777= 4 years and 2 days, *Ans*.

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783 : How long did the war continue ?

January 20th, 1783—April 19th, 1775=7y. 9m. 1d. *Ans*.

9. A, B, C and D purchased a quantity of goods in partnership ; A paid £.12 10*s*. a dollar* and a crown† piece ; B, 35*s*. C, 29*s*. 10*d*. and D, 79*d*. : What did the goods cost ? *Ans*. £.16 14 1.

10. A man borrowed, at different times, these several sums, viz. £.29 5*s*. £ 18 17*s*. 6*d*. £ 45 12*s*. £.98, 3 dollars, one crown piece and an half : Pray how much was he in debt ? *Ans*. £.193 2 6.

11. There are four numbers ; the first 317, the second 912, the third 1229, and the fourth as much as the other three, abating 97 : What is the sum of them all ? *Ans*. 4819.

12. Bought a quantity of goods for £.125 10*s*. paid for truckage 45*s*. for freight 79*s*. 6*d*. for duties 35*s*. 10*d*. and my expenses were 53*s*. 9*d*. : What did the goods stand me in ? *Ans*. £.136 4 1.

13. A

* 6*s*. † 6*s*. 8*d*.

13. A Gentleman left his son £.1725 more than his daughter, whose fortune was 15 thousand, 15 hundred and 15 pounds: What was the son's portion and what did the whole estate amount to?

Ans. The son's fortune, £.18240, and the whole estate £.34755.

14. A merchant had 6 debtors, who together owed him £.2917 10*s.* 6*d.* A, B, C, D and E, owed him £.1675 13*s.* 9*d.* of it: What was F's debt? *Ans.* 1241 16 9.

15. What is the difference between £.1309 7*s.* 1*d.* and the amount of £.345 13*s.* 4*d.* and £.571 4*s.* 8*d.*? *Ans.* £.392 9 1.

16. A merchant, at his first engaging in trade, owed £.937 15*s.* he had in cash £.1755 3*s.* 6*d.* in goods £. 459 12*s.* 3*d.* in good debts £.197 16*s* and he cleared the first year £.249 19 10: What was the neat balance at the year's end? *Ans.* £.1724 16 7.

17 What sum of money must be divided between 12 men, so as that each may receive £.155? £.155 × 12 = 1860 *Ans.*

18. What number must I multiply by 9, that the product may be 675? 675 ÷ 9 = 75 *Ans.*

19. A privateer of 175 men took a prize, which amounted to £.59 per man, beside the owner's half: What was the value of the prize?

175 × 59 × 2 = £.20650 *Ans.*

20 What is the difference between thrice five and thirty, and thrice thirty five?

$\overline{35 \times 3} - \overline{5 \times 3 + 30} = 60$ *Ans.*

21 The sum of two numbers is 750; the less 248: What is their difference, product, and the square of their difference?

750 — 248 = 502 the greater number, 502 — 248 = 254 difference, 502 × 248 = 124496 product, and 254 × 254 = 64516 square of the difference.

22. What is the difference between six dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the less? *Ans.* 6 × 12 × 12 — 6 × 12 = 792 difference, 6 × 12 × 12 × 6 × 12 = 62208 product, and 6 × 12 × 12 ÷ 6 × 12 = 12 quotient.

23. There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their sum and product are required.

Ans. 78 × 25 = 1950 the greater, $1950 - \overline{15 \times 9} = 1815$ the less. 1950 + 1815 = 3765 the sum, and 1950 × 1815 = 3539250 the prod.

24. A merchant began trade with £.25327; for six years together, he cleared £.1253 per annum; the next 5 years, he cleared £.1729 per annum; but, the last 4 years, had the misfortune to lose £.3019 per annum: What was he worth at the 15 years' end?

Ans. £.29414.

25. If a man spends £.192 in a year: What is that per calendar month? 192 ÷ 12 = £.16 *Ans.*

26. If the Federal Debt, which is 42 million dollars, be equally divided between the 13 States: What will be the share of each?

Ans. $3230769\frac{3}{13}$ dollars.

27. If 9000 men march in a column of 750 deep: How many march abreast? 9000 ÷ 750 = 12 *Ans.*

28. What

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the same ? $\overline{3072 \div 32} - 32 = 64$ *Ans*.

29. What number is that, which, multiplied by 3589, will produce 92050672 ? $92050672 \div 3589 = 25648$ *Ans*.

30. Suppose the quotient arising from the division of two numbers to be 5379, the divisor 37625 ; What is the dividend, if the remainder came out 9357 ? $\overline{37625 \times 5379} + 9357 = 202394232$ *Ans*.

31. There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted, and 30 added to the remainder, the half sum shall make 35 : Can you tell me the number ?

$\overline{35 \times 2 - 30 + 20 \times 5 \times 7} \div 3 = 700$ *Ans*.

32. A sheepfold was robbed three nights successively ; the first night, half the sheep were stolen, and half a sheep more ; the second half the remainder were lost, and half a sheep more ; the last night they took half what were left and half a sheep more ; by which time they were reduced to 30 : How many were there at first ?

Begin with 30, and, reckoning back from the last night to the first, you will find that 31 were stolen the 3d night, 62 the 2d, and 124 the first. *Ans*. 247.

33 Two boys, A and B, had 850 chesnuts between them ; but A had 150 more than B : How many had each ?
$850 \div 2 = 425$ half sum, and $150 \div 2 = 75$ half diff. ; then $425 + 75 = 500$ A's, and $425 - 75 = 350$ B's.

34. A and and B played at marbles, having 14 apiece at the first ; but after playing several games, B, having lost some of his, would play no longer, and it was found that the difference of the squares of the numbers, which each then had, was 336 : Pray, how many did B lose ?

$14 + 14 = 28$ sum, $336 \div 28 = 12$ diff. $28 \div 2 = 14$ half sum, and $12 \div 2 = 6$ half diff.; then $14 + 6 = 20$ A retired with, and $14 - 6 = 8$ B had left, therefore B lost $14 - 8 = 6$.

35. Said Harry to Charles, my father gave me 12 more apples than he gave my brother Jack, and the difference of the squares of our separate parcels was 288 : Now, if you are arithmetician enough to tell how many he gave us, each, you shall have half of mine.

$288 \div 12 = 24$ the whole : $24 \div 2 = 12$ and $12 \div 2 = 6$; then $12 + 6 = 18 =$ Harry's share, and $12 - 6 = 6 =$ Jack's share.

36. What number added to the 27th part of 6615, will make 570 ?
$570 - \overline{6615 \div 27} = 325$ *Ans*.

REDUCTION

TEACHES to bring, or exchange, numbers of one denomination to others of different denominations, retaining the same value.

It is of *two* sorts, viz. Descending and Ascending ; the former of which is performed by multiplication, and the latter by division.

REDUCTION DESCENDING.

RULE.*

Multiply the highest denomination, given, by so many of the next less as make one of that greater, and thus continue until you have brought it down as low as your question requires.

PROOF. Change the order of the question, and divide your last product by the last multiplier, and so on.

Note. From this rule and Case VI. of Simple Multiplication, it appears, that *Federal Money* is reduced from higher to lower denominations by annexing as many cyphers as there are places from the denomination given, to that required ; or, if the given sum be of different denominations, by annexing the several figures of all the denominations in their order, and continuing with cyphers, (if necessary,) to the denomination required ; or, what amounts to the same thing, by reading the whole number from the left to the required denomination, as one number in the required denomination.

EXAMPLES.

1. In 3 eagles 2 dollars, how many mills ? *Ans.* 32000 *m.*
2. In 91 dollars 75 cents, how many cents ? *Ans* 9175 *c.*
3. In 50 eagles, how many dollars ? *Ans.* 500 *D.*
4. In 44 dollars, 1 cent, 4 mills, how many mills ?
5. In 9 dollars, 31 cents, 7 mills, how many mills ?
6. How many cents in 39 dollars 5 cents ?
7. In 28 dollars 17 cents, 5 mills, how many mills ?
8. In £.27 15*s.* 9*d.* 2*qrs.* how many farthings ?

£. *s.* *d.* *qr.*
27 11 9 2
multiplied by 20=*shillings in a pound.*

555=*shillings.*
——*by* 12=*pence in a shilling.*

6669=*pence.*
——*by* 4=*farthings in a penny.*

Ans. = 26678 farthings.

Note. In multiplying by 20, I added in the 15s. by 12, the 9d. and by 4, the 2qrs which must always be done in like cases.

To prove the above question, change the order of it, and it will stand thus : In 26678 farthings how many pounds ?

4)26678

12)6669 2qrs.

2|0)55|5 9d.

Answer, £.27 15 9 2.

9. In

* The reason of this Rule is exceedingly obvious ; for pounds are brought into shillings by multiplying them by 20 ; shillings into pence by multiplying them by 12 ; and pence into farthings by multiplying them by 4 ; and the contrary by division ; and this will be true in the reduction of numbers consisting of any denomination whatever.

9. In £.36 12s. 10d. 1qr. how many farthings? Ans. 35177.
10. In £. 95 11s. 5d 3qrs. how many farthings? Ans. 91751.
11 In £.719 9s. 11d. how many half pence? Ans. 345358.
12. In 29 guineas, at 28s. how many pence? Ans. 9744.
13. In 37 pistoles, at 22s. how many shillings, pence, and farthings? Ans. 814s. 9768d. 39072qrs.
14. In 49 half johannes, at 48s. how many sixpences? Ans. 4704.
15. In 473 French crowns, at 6s. 8d. how many threepences? Ans. 12613$\frac{1}{3}$.
16. In 53 moidores, at 36s. how many shillings, pence and farthings? Ans. 1908s. 22896d. 91584qrs.
17. In £. 29 how many groats, threepences, pence, and farthings? Ans 1740 groats, 2320 threepences, 6960d. 27840qrs.

18. Reduce 47 guineas and one fourth of a guinea into shillings, sixpences, groats, threepences, twopences, pence and farthings.

Ans. 1323 shillings, 2046 sixpences, 3969 groats, 5292 threepences, 7938 twopences, 15876 pence, and 63504 qrs.

REDUCTION ASCENDING.

Rule.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue till you have brought it into that denomination which your question requires.

Note. From this rule and the note under Case II. of Simple Division, it appears, that *Federal Money* is reduced from lower to higher denominations by cutting off as many places as the given denomination stands to the right of that required; the figures cut off belonging to their respective denominations.

Examples.

1. How many eagles in 32000 mills? Ans. 3 E. 2 D.
2. In 9175 cents, how many dollars? Ans. 91 D. 75 c.
3. In 500 dollars how many Eagles? Ans. 50.
4. In 4414 mills, how many dimes?
5. In 9317 mills, how many dollars?
6. How many dollars in 28175 mills?
7. In 547325 farthings, how many pence, shillings, and pounds?

Farthings in a penny = 4)547325

Pence in a shilling = 12)136831 1 qr.

Shillings in a pound = 2|0)1140|2 7d.

£.570 2s. 7d. 1 qr.

Ans. 136831d. 11402s. and £.570

Note. The remainder is always of the same name as the dividend.

8. Bring 35177 farthings into pounds.

9. Bring

9. Bring 91751 farthings into pence, &c.
10. Bring 345358 half pence into pence, shillings, and pounds.
11 Reduce 9744 pence to guineas, at 28s. per guinea.
12 In 39072 farthings, how many pistoles, at 22s. ?
13. In 4704 sixpences, how many half johannes ?
14. In 12613⅓ threepences, how many French crowns, at 6s. 8d. ?
15. In 91584 farthings, how many moidores, at 36s ?
16 In 27840 farthings, how many pence, threepences, groats, shillings and pounds ?
17. In 63504 farthings, how many pence, twopences, threepences, groats, sixpences, shillings and guineas ?

Note. The preceding questions may serve as proofs to those in Reduction descending.

REDUCTION DESCENDING AND ASCENDING.

1. Money.

1. In £.97 how many pence and English or French crowns, at 6s. 8d. ? Ans 23280d. and 291 crowns.
2. In 947 English crowns, at 6s. 8d. how many shillings and English guineas ? Ans. 6313s. 4d. and 225 guineas 13s. 4d.
3. In 519 English half crowns, how many pence and pounds ? Ans. 20760d. and £.86 10s.
4. In 1259 groats, how many farthings, pence, shillings, and guineas ? Ans. 20144qrs. 5036d. 419s. 8d. and 14 guin. 27s. 8d.
5. In 75 pistoles, how many pounds ? Ans. £.82 10s.
6. In 735 French crowns, how many shillings and French guineas, at 26s. 8d. ? Ans. 4900s. and 183 guin. 24s.
7. In 5793 pence, how many farthings, pounds and pistoles ? Ans. 23172qrs. £.24 2s. 9d. and 21 pistoles, 20s 9d.
8. In £.99, how many shillings, and half johannes, at 48s. ? Ans. 1980s. and 41 half joes. 12s.
9. In £.179, how many guineas ? Ans. 127 guin 24s.
10. In £.345 how many moidores ? Ans. 191 moid. 24s.
11. In 59 half joes, 37 moidores, 45 guineas, 63 pistoles, 24 English crowns, and 19 dollars ; how many pounds, half joes, moidores, guineas, pistoles, English crowns, dollars, shillings, pence and farthings ?

Ans. £.354 4s. 147 half joes, 28s 196 moidores, 28s 253 guineas, 322 pistoles, 1062 English crowns, 4s. 1180 dollars, 4s 7084 shillings, 85008d. and 340032qrs.

When it is required to know how many sorts of coin, of different values, and of equal number, are contained in any number of another kind ; reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor ; then reduce the money given, into the same denomination, for a dividend, and the quotient, arising from the division, will be the number required.

Note. Observe the same direction in weights and measures.

1. In

1. In 275 half johannes, how many moidores, guineas, pistoles, dollars, shillings and sixpences, of each the like number ?

A moidore is 36s. that is } 72 sixpences.
A guinea is 28s. that is } 56 ditto.
A pistole is 22s. that is } 44 ditto.
A dollar is 6s. that is } 12 ditto.
One shilling has 2 do.
1 do.

Divisor=187 sixpences.

275 half joes.
48 shil. in a johan.

2200
1100

13200 shillings.
2 sixp. in a shill.

dividend=26400 sixpences.

187)26400(141 of each, and 33 sixp. or 16s. 6d. over, the answer.

2. A Gentleman distributed £.37 10s. between 4 poor persons, in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third, 10s. and the fourth 5s. What did each person receive ? Ans. The first man £.15 second £. 11 15s. third £. 7 10s. fourth £. 3 15s.

2. Troy Weight.

1. How many grs. in a silver bowl, that weighs 3lb. 10oz. 12 pwt.?

```
    lb  oz   pwt.
     3  10   12
    12 ounces in a pound.
    --
    46 ounces.
    20 pennyweights in an ounce.
   ---
   932 pennyweights.
    24 grains in one pwt.
  ----
  3728
 1864
 -----
Proof. 24)22368 grains, answer.
 -----
   2|0)93|2
    12) 46—12 pwt.
    lb. 3—10 oz.
```

2. In 487ozs. how many pwts. and grs. ?

Ans. 9740pwt. and 233760gr.

3. In 13 ingots of gold, each weighing 9oz. 5pwt. how many grains ? Ans. 57720gr.

4. In 97397grs. how many pounds ? Ans. 16lb. 10oz. 18pwt. 5gr.

4. How many rings, each weighing 5pwt. 7gr. may be made of 3lb. 5oz. 16pwt. 2gr. of gold ? Ans. 158.

3. Avoirdupois

3. Avoirdupois Weight.

	Cwt.	qrs.	lb	oz.	
1. In	91	3	17	14	how many ounces ?

4

367 quarters.
28

2943
735

10293 pounds.
16

61762
10294

164702 ounces.

Proof.
16)164702
28)10293 14oz.
4)367 17lb.
Cwt. 91 3qrs.

2. In 12 tons, 15cwt. 1qr. 19lb. 6oz. 12dr. how many drams ?
Ans. 7323500 dr.

3. In 24 lb. 11 oz. 9 dr. how many drams ? Ans. 6329 dr.

4. In 44800 pounds, how many drams and tons ?
Ans. 11468800dr. and 20 tons.

5. In 28lb. Avoirdupois, how many pounds Troy ?
28
7000 grains in 1lb. Avoirdupois.

grs. in 1lb. tr. } =576|0)19600|0(34lb.
1728

2320
2304

160
12

576|0)192|0(0 oz.
20

576|0)3840|0)6 pwt.
3456

3840
24

1536
768

576|0)9216|0(16 gr.
576

3456
3456

	lb.	oz.	pwt.	gr.	
6. In	47	9	13	17	Troy, how many pounds Avoirdupois ?

47 9 13 17
12

573
20

11473
24

45899
22947

7|000)275|369(39lb.
21

65
63

2369
16

14214
2369

37904 carried over.

Brought forward, 7|000)37|904(5 oz.
35

2904
16

17424
2904

7|000)46|464($6\frac{4464}{7000}$ dr.
42

4464

4. Apothecaries' Weight.

1. How many grains are there in 37 lb. 6℥ ?

lb. ℥
37 6
12

450 ounces.
8

3600 drams.
3

10800 scruples.
20

Ans. 216000 grains.

Proof.
2|0)21600|0

3)10800

8)3600

12)450

37 lb. 6℥

2. In 9 lb. 8℥ 1ʒ 2℈ 19 gr. how many grains ? Ans. 55799 gr.
3. In 55799 grains, how many pounds, &c. ?
Ans. 9lb. 8℥ 1ʒ 2℈ 19 gr.

6. Cloth Measure.

In 127 yards, how many quarters and nails ?
4

Ans. 508 qrs.
4

Ans. 2032 nails.

Proof.
4)2032

4)508

127 yards.

2. In 9173 nails, how many yards ? Ans. 573 yds. 1qr. 1n.
3. In 75 ells English, how many quarters and nails ?
Ans. 375qrs. 1500n.
4. In 56 ells Flemish, how many quarters and nails ?
Ans. 168qrs. 672n.
5. In 39 ells French, how many quarters and nails ?
Ans 234qrs. 936n.
6. In 7248 nails, how many yards, ells Flemish, ells English, and ells French ?
Ans. 453yds. 604 ells Flem. 362 ells Eng. 2qrs. 302 ells French.
7. In 19 pieces of cloth, each 15 yards, 2 quarters, how many yards, quarters and nails ? Ans. 294yds. 2qrs. 1178 qrs. and 4712n.

6. Long

6. Long Measure.

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles?

43 miles.		
8	3)8173440 proof.	Here I divide by 11, and multiply the quotient by 2, because twice $5\frac{1}{2}$ is 11; or I might first have multiplied by 2, and, then, have divided the product by 11.
344 furlongs.	12)2724480	
40	3)227040	
13760 rods.	11)75680	
$5\frac{1}{2}$	6880	
68800	2	
6880	4|0)1376|0	
75680 yards.	8)344	
3	43	
227040 feet.		
12		
2724480 inches.		
3		
8173440 Answer.		

2 How many barley corns will reach round the globe, it being 360 degrees? Ans. 4755801600

3. How many inches frow Newburyport to London, it being 2700 miles? Ans. 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles? Ans. 13440 times.

5. In 190080 inches, how many yards and leagues? Ans. 5280 yds. and 1 league.

7 Time.

1. In 20 years how many seconds?

d. h.	Proof.
365 6 in a year.	6|0)63115200|0
24	6|0)1051920|0
1466	2|0)17532|0
730	4×6)8766
8766 hours in 1 year.	4)1461
20	365 d. 6h.
175320 hours in 20 years.	
60	
10519200 minutes in ditto.	
60	
631152000 seconds in ditto.	

2. Suppose

2. Suppose your age to be 15y. 19d. 11h. 37m. 45s. how many seconds are there in it, allowing 365 days and 6 hours to the year?

Ans. 475047465.

3. In 31536000 seconds how many years? Ans. 1 year.

4. How many minutes from the first day of January to the 14th day of August, inclusively? Ans. 325440.

5. How many days since the commencement of the christian Æra?

6. How many minutes since the commencement of the American war, which happened on the 19th day of April, 1775?

7. How many seconds between the commencement of the war, April 19th, 1775, and the independence of the United States of America, which took place the 4th day of July, 1776*? Ans. 38188800.

8. MOTION.

1. In 9 signs, 13° 25′, how many seconds?

```
    9s  13°  25′                6|0)102030|0 Proof.
    30                          ------------
   ----                          6|0)1700|5
   283 degrees.                  ---------
    60                            3|0)28|3—25
   ----                           -------
 17005 minutes.                       9s 13° 25′
    60
1020300 seconds.
```

9. LAND OR SQUARE MEASURE.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

```
   Acres. R. Poles.              Proof.
     29   3   19               4|0)477|9
      4                         -----
    ----                       4)119—19p.
    119 roods.                  ----
     40                         29ac. 3 roods.
Answer 4779 perches.
```

2. In 1997 poles how many acres? Ans. 12a. 1r. 37p.

3. In 89763 square yards how many acres, &c.?

Ans. 18a. 2r. 7p. 101ft. 36in.

4. How many square feet, square yards, and square poles, in a square mile? Ans. 27878400 feet, 3097600 yards, and 102400 poles.

10. SOLID MEASURE.

1. In 15 tons of hewn timber how many solid inches?

```
    15 tons.                        Proof.
    50                                   5|0
    --                     1728)1296000(75|0
   750 feet.                    12096   ----
  1728                          -----   15 tons.
  ----                           8640
  6000                           8640
 1500
5250
750
Ans. 1296000 inches.
```

2. In

* 1776 was a leap year.

2. In 9 tons of round timber how many inches? Ans. 622080.
3. In 25 cords of wood how many inches? Ans. 5529600.

11. Wine Measure.

1. In 9hhds 15galls. 3qts. of wine how many quarts?

```
hhds. gal  qts.                  Proof.
   9   15   3                    4)2331
  63                             ------
 ----                            63)582—3qts.
  32                             ------
  55                                 9hhds.—15gals.
 ----
 582 gallons.
   4
 ----
Ans. 2331 quarts.
```

2. In 12 pipes of wine how many pints? Ans. 12096.
3. In 9758 pints of brandy how many pipes?
Ans. 9p. 1hhd. 22gal. 3qts.
4. In 1008 quarts of cyder how many tons? Ans. 1 ton.

12. Ale or Beer Measure.

1. In 29hhds. of beer how many pints?

```
hhds.                            Proof.
   29                          2)12528
   54                          -------
  ---                           4)6264
  116                            -----
 145                           54(1566
 ----                            -----
 1566 gallons.                    29 hhds.
    4
 ----
 6264 quarts.
    2
 ----
Ans. 12528 pints.
```

2. In 47bar. 18gal. of ale how many pints? Ans. 13680.
3. In 36 puncheons of beer how many butts? Ans. 24.

13. Dry Measure.

1. In 42 chaldrons of coals how many pecks?

```
Chaldrons.                        Proof.
   42                            4)5376
   32                           32)1344(42
  ---                              128
   84                              ---
 126                                64
 ----                               64
 1344 bushels.                      --
    4
 ----
Ans. 5376 pecks.
```

2. In 75 bushels of corn how many pints? Ans. 4800.
3. In 9376 quarts how many bushels? Ans. 293.

VULGAR FRACTIONS.

FRACTIONS, or broken numbers, are expressions for any assignable parts of an unit, or whole number; and are represented by two numbers placed one above another, with a line drawn between them, thus: $\frac{5}{8}$, $\frac{4}{3}$, &c. signifying five eighths, four thirds, that is, one and one third, &c.

The figure above the line is called the *numerator*, and that below it, the *denominator*,

The denominator (which is the divisor in division) shews how many parts the integer is divided into; and the numerator (which is the remainder after division) shews how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed. Any whole number may be made an improper fraction by drawing a line under it, and putting unity, or 1 for a denominator, as 9 may be expressed fractionwise, thus $\frac{9}{1}$, and 12 thus $\frac{12}{1}$, &c.

1. A *single* or *simple* fraction is a fraction expressed in a simple form; as $\frac{1}{2}$, $\frac{5}{9}$, $\frac{7}{16}$, &c.

2. A *compound* fraction is a fraction expressed in a compound form, being a fraction of a fraction; or two or more fractions connected together; as $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{7}$ of $\frac{5}{11}$ of $\frac{19}{20}$ which, are read thus, one half of three fourths, two sevenths of five elevenths of nineteen twentieths, &c.

3. A *proper* fraction is a fraction, whose numerator is less than its denominator; as $\frac{2}{3}$, $\frac{3}{8}$, &c.

4. An *improper* fraction is a fraction, whose numerator exceeds its denominator; as $\frac{5}{2}$, $\frac{8}{3}$, &c.

5. A *mixed number* is composed of a whole number and a fraction, as $7\frac{3}{5}$, $35\frac{4}{13}$, &c. that is, seven and three fifths, &c.

6. A fraction is said to be in its least, or lowest terms, when it is expressed by the least numbers possible.

7. The *common measure* of two, or more numbers, is that number which will divide each of them without a remainder: Thus, 5 is the common measure of 10, 20 and 30; and the *greatest* number, which will do this, is called the *greatest common measure*.

8. A number, which can be measured by two, or more, numbers, is called their *common multiple*: And, if it be the least number, which can be so measured, it is called the *least common multiple*; thus, 40, 60, 80, 100, are multiples of 4 and 5; but their least common multiple is 20.

9. A *prime number* is that, which can only be measured by itself or an unit.

10. That

10. That number, which is produced by multiplying several numbers together, is called a *composite number*.

11. A *perfect number* is equal to the sum of all its aliquot parts.*

PROBLEM I†.

To find the greatest common measure of two, or more, numbers

RULE.

1. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder, till nothing remain, then will the last divisor be the greatest common measure required.

2. When there are more than two numbers, find the greatest common measure of two of them, as before; then, of *that* common measure and one of the other numbers, and so on, through all the numbers, to the last; then will the greatest common measure, last found, be the answer.

3. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.

EXAMPLES.

1. What is the greatest common measure of 1836, 3996, and 1044?

```
1836)3996(2
     3672
     ----
      324)1836(5
          1620
          ----
           216)324(1
               216
               ---
Common meas.=108)216(2
                 216
```

So 108 is the greatest common measure of 3996 and 1836.

```
Hence 108)1044(9
           972
           ---
            72)108(1
                72
                --
Last greatest com. meas.=36)72(2
                            72
                            --
```

Therefore, 36 is the answer required.

2. What

* The following perfect numbers are all which are, at present, known.

6	8589869056
28	137438691328
496	2305843008139952128
8128	2417851639228158837784576
33550336	9903520314282971830448816128

† This and the following problem will be found very useful in the doctrine of fractions, and several other parts of Arithmetick.

The truth of the rule may be shewn from the first example: For, since 108 measures 216, it also measures 216+108, or 324.

Again, since 108 measures 216 and 324, it also measures $\overline{5\times324}+216$, or 1836. In the same manner it will be found to measure $\overline{2\times1836}+324$, or 3996, and so on.

It is also the greatest common measure; for suppose there be a greater, then, since the greater measures 1836 and 3996, it also measures the remainder 324; and since it measures 324 and 1836, it also measures the remainder 216; in the same manner it will be found to measure the remainder 108; that is, the greater measures the less, which is absurd; therefore, 108 is the greatest common measure.

In the same manner, the demonstration may be applied to one or more additional numbers.

2. What is the greatest common measure of 1224 and 1080? Ans 72.

3. What is the greatest common measure of 1440, 672 and 3472? Ans. 16.

PROBLEM II.*

To find the least common multiple of two, or more numbers.

RULE.

1. Divide by any number that will divide two, or more, of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

2 Divide the second line, as before, and so on, till there are no two numbers that can be divided; then, the continued product of the divisors and quotients will give the multiple required.

EXAMPLES.

1. What is the least common multiple of 6, 10, 16 and 20?

*5)6	10	16	20
*2)6	2	16	4
*2)3	1	8	2
*3	1	*4	1

* * * * *
$5\times2\times2\times3\times4=240$ Ans.

I survey my given numbers, and find that five will divide two of them, viz. 10 and 20, which I divide by 5, bringing into a line with the quotients the numbers which 5 will not measure: Again, I view the numbers in the second line, and find 2 will measure them all, and get 3, 1, 8, 2 in the third line, and find that two will measure 8 and 2, and in the fourth line get 3, 1, 4, 1, all prime; I then multiply the prime numbers and the divisors continually into each other, for the number sought, and find it to be 240.

2. What is the least common multiple of 6 and 8? Ans. 24.

3. What is the least number that 3, 5, 8 and 10 will measure? Ans. 120

4. What is the least number which can be divided by the 9 digits, separately without a remainder? Ans. 2520.

REDUCTION OF VULGAR FRACTIONS.

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE

* The reafon of this rule may alfo be fhewn from the firft example: Thus, it is evident that $6\times10\times16\times20$ ($=19200$) may be divided by 6, 10, 16 and 20, without a remainder; but 20 is a multiple of 5; therefore. $6\times10\times16\times4$, or 3840, is alfo divifible by 6, 10, 16 and 20. Alfo, 16 is a multiple of 4: therefore $6\times10\times4\times4=960$, is alfo divifible by 6, 10, 16 and 20. Alfo, 10 is a multiple of 2; therefore, $6\times5\times4\times4=480$, is alfo divifible by 6, 10, 16 and 20. Alfo, 6 is a multiple of 2; therefore, $3\times5\times4\times4=240$, is alfo divifible by 6, 10, 16 and 20; and is evidently the leaft number that can be fo divided.

CASE I.*

To abbreviate, or reduce fractions to their lowest terms.

RULE

Divide the terms of the given fraction by any number, which will divide them without a remainder, and the quotients, again, in the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms. Or,

Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{288}{480}$ to its lowest terms.

$$8\left\{\frac{288}{480}=\frac{36}{60}\overset{(4)}{=}\frac{9}{15}\overset{(3)}{=}\frac{3}{5}\right.$$ the answer.

Or thus:

```
288)480(1
    288
    ---
    192)288(1
        192
        ---
Com. meas. 96)192(2
              192
```

Therefore 96 is the greatest common measure.

and $96\left\{\frac{288}{480}=\frac{3}{5}\right.$ the same as before.

2. Reduce $\frac{96}{352}$ to its lowest terms. Ans. $\frac{3}{11}$.
3. Reduce $\frac{384}{1152}$ to its lowest terms. Ans. $\frac{1}{3}$.
4. Reduce $\frac{57}{456}$ to its lowest terms. Ans. $\frac{1}{8}$.
5. Reduce $\frac{46}{184}$ to its lowest terms. Ans. $\frac{1}{4}$.
6. Reduce $\frac{1429}{2858}$ to its lowest terms. Ans. $\frac{1}{2}$.

CASE

* That dividing both the terms, that is, both numerator and denominator of the fraction, equally by any number whatever, will give another fraction, equal to the former, is evident: And if those divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. Any number, ending with an even number or cypher, is divisible by 2.

2. Any number, ending with 5 or 0, is divisible by 5.

3. If the right hand place of any number be 0, the whole is divisible by 10.

4. If the two right hand figures of any number be divisible by 4, the whole is divisible by 4.

5. If the three right hand figures of any number be divisible by 8, the whole is divisible by 8.

6. If the sum of the digits, constituting any number, be divisible by 3 or 9, the whole is divisible by 3 or 9.

7. If a number cannot be divided by some number less than the square root thereof, that number is a *prime*.

8. All *prime* numbers, except 2 and 5, have 1, 3, 7, or 9 in the place of units: and all other numbers are *compossite*.

9. When numbers, with the sign of Addition or Subtraction between them, are to be divided by any numbers, each of the numbers must be divided: Thus, $\frac{6+9+12}{3}=2+3+4=9$.

10. But if the numbers have the sign of Multiplication between them; then only one of them must be divided: Thus, $\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction, and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ \times 8 + 5 \\ \hline \text{Ans. } 293 \\ \hline 8 \end{array}$$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum 293 is the numerator of the fraction sought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

Or, $\frac{\overline{36\times8}+5}{8}=\frac{293}{8}$ Answer as before.

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction. Ans. $\frac{2163}{17}$.
3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction. Ans. $\frac{12410}{19}$.

CASE III.†

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8.
$6\times8=48$, and $\frac{48}{8}$ the Ans.—Proof $\frac{48}{8}=48\div8=6$.
2. Reduce 15 to a fraction, whose denominator shall be 12. Ans. $\frac{180}{12}$.
3. Reduce 100 to a fraction, whose denominator shall be 70. Ans. $\frac{7000}{70}=\frac{100}{1}=100$.

CASE IV.‡

To reduce an improper fraction to its equivalent whole, or mixed number.

RULE.

Divide the numerator by the denominator: the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

EXAMPLES.

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$; from whence the rule is manifest; for if any number is multiplied and divided by the same number, it is evident the quotient must be the same as the quantity first given.

† Multiplication and Division are here equally used, and consequently the result is the same as the quantity first proposed.

‡ This case is, evidently, the reverse of case 2d, and has its reason in the nature of common division.

EXAMPLES.

1. Reduce $\frac{293}{8}$ to its equivalent whole, or mixed number.

$$\begin{array}{l} 8)293(36\frac{5}{8}\ \text{Ans.} \\ \ \ 24 \\ \ \ \overline{\ \ 53} \\ \ \ \ \ 48 \\ \ \ \overline{\ \ \ \ \ 5} \end{array}$$

Or, $\frac{293}{8}=293\div8=36\frac{5}{8}$ as before.

2. Reduce $\frac{2163}{17}$ to its equivalent whole, or mixed number. Ans. $127\frac{4}{17}$.

3. Reduce $\frac{12410}{19}$ to its equivalent whole, or mixed number. Ans. $653\frac{3}{19}$.

4. Reduce $\frac{45}{5}$ to its equivalent whole number. Ans. 9.

CASE V.*

To reduce a compound fraction to an equivalent simple one.

RULE.

Multiply all the numerators continually together for a new numerator, and all the denominators, for a new denominator, and they will form the simple fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction, by case 2d, or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged, and the other members continually multiplied, as by the rule, will produce the fractions required in lower terms.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a a simple fraction.

$$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5}\ \text{the Answer.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction.

$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24}$ Ans. Or, by expunging the equal numerators and denominators, it will be $\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24}$ as before.

3. Reduce $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{15}{19}$ to a simple fraction. Ans. $\frac{225}{532}$.

4. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a simple fraction.

Ans. $\frac{624}{306}=2\frac{2}{51}$.

2. Reduce

* That a compound fraction may be represented by a simple one is very evident; since a part of a part must be equal to some part of the whole. The truth of the rule for this reduction may be shown as follows.

Let the compound fraction to be reduced, be $\frac{3}{4}$ of $\frac{6}{10}$. Then $\frac{1}{4}$ of $\frac{6}{10}=\frac{6}{10}\div4=\frac{6}{40}$, and consequently $\frac{3}{4}$ of $\frac{6}{10}=\frac{6}{40}\times3=\frac{18}{40}$ the same as by the rule.

If the compound fraction consists of more numbers than two, the two first may be reduced to one, and that one and the third will be the same as a fraction of two numbers, and so on.

5. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple fraction. Ans. $\frac{75}{64}=1\frac{11}{64}$.

CASE VI.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

RULE I*.

Multiply each numerator into all the denominators except its own, for a new numerator, and all the denominators into each other, continually, for a common denominator.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{5}{8}$ to equivalent fractions, having a common denominator. $1\times5\times8=40$ the new numerator for $\frac{1}{4}$.
$2\times4\times8=64$ the new numerator for $\frac{2}{5}$.
$5\times4\times5=100$ ditto for $\frac{5}{8}$.
$4\times5\times8=160$ the common denominator.
Therefore the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$ and $\frac{100}{160}$, the Answer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$ to fractions having a common denominator. Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$, and $\frac{3}{13}$, to a common denominator.
Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$.

4. Reduce $\frac{11}{15}$, $\frac{3}{4}$ of $2\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$, to a common denominator.
Ans. $\frac{8448}{11520}$, $\frac{21600}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

RULE II.

To reduce any given fractions to others, which shall have the least common denominator.

1. By Problem 2, Page 61, find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{8}$ to fractions, having the least common denominator possible. 4)

* By placing the numbers multiplied properly under one another, it will be seen that the numerator and denominator of every fraction are multiplied by the very same number, and consequently their values are not altered. Thus, in the first example.

$$\frac{1\times5\times8}{4\times5\times8} \quad \frac{2\times4\times8}{5\times4\times8} \quad \frac{5\times4\times5}{8\times4\times5}$$

In the second rule, the common denominator is a multiple of all the denominators, and consequently will divide by any of them: Therefore, proper parts may be taken for all the numerators as required.

$$\begin{array}{r} 4)3\ \ 4\ \ 8 \\ \hline 3\ \ 1\ \ 2 \\ \hline \end{array}$$

$4\times3\times2 = 24 =$ least common denominator.

$\overline{24\div3}\times1=8$ the first numerator; $\overline{24\div4}\times3=18$ the second numerator; $\overline{24\div8}\times7=21$ the third numerator.

Whence, the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions having the least common denominator. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$ and $\frac{48}{60}$.

CASE VII.

To reduce a fraction of one denomination to the fraction of another, but greater, retaining the same value.

Rule.*

Reduce the given fraction to a compound one by comparing it with all the denominations between *it* and *that* denomination you would reduce it to; lastly, reduce this compound fraction to a single one, by case 5th, and you will have a fraction of the required denomination, equal in value to the given fraction.

Examples.

1. Reduce $\frac{4}{7}$ of a cent to the fraction of a dollar.

By comparing it, it becomes $\frac{4}{7}$ of $\frac{1}{10}$ of $\frac{1}{10}$, which, reduced by case 5, will be

$$\frac{4\times1\times1 = 4}{\text{and } 7\times10\times10 = 700} = \frac{1}{175} \text{ D. Ans.}$$

2. Reduce $\frac{3}{5}$ of a mill to the fraction of an eagle. Ans. $\frac{3}{50000}$ E.
3. Reduce $\frac{11}{15}$ of a mill to the fraction of a dollar. Ans. $\frac{11}{1500}$ D.
4. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound. Ans. £. $\frac{1}{400}$.
5. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound. Ans. $\frac{1}{1280}$.
6. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea. Ans. $\frac{5}{2688}$ guinea.
7. Reduce $\frac{12}{19}$ of a shilling to the fraction of a moidore. Ans. $\frac{1}{57}$ moidore.
8. Reduce $\frac{4}{7}$ of an ounce to the fraction of a lb. Avoirdupois. Ans. $\frac{1}{28}$ lb.
†9. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea. Ans. $\frac{4}{7}$ guin.
10. Reduce $\frac{7}{8}$ of a pwt. to the fraction of a pound Troy. Ans. $\frac{7}{1920}$ lb.
11. Reduce $\frac{8}{9}$ of a lb. Avoirdupois to the fraction of 1 Cwt. Ans. $\frac{1}{126}$ Cwt.
12. Express $5\frac{1}{2}$ furlongs in the fraction of a mile. Ans. $\frac{11}{16}$ mile.

CASE

* The reason of this and the next rule is explained in the rule reducing compound fractions to simple ones.

† $\frac{4}{5}$£. $= \frac{4}{5}$ of $\frac{20}{1} = \frac{4\times20}{5\times1} = \frac{80}{5}$s. and $\frac{80}{5}$ of $\frac{1}{28} = \frac{80\times1}{5\times28} = \frac{80}{140} = \frac{4}{7}$ guinea.

CASE VIII.

To reduce a fraction of one denomination to the fraction of another, but less, retaining the same value.

Rule.

Multiply the given numerator by the parts in the denominations between *it* and *that* denomination you would reduce it to, for a new numerator, which place over the given denominator: *Or*, only invert the parts contained in the integer, and make of them a compound fraction as before, then, reduce it to a simple one.

Examples.

1. Reduce $\frac{1}{175}$ of a dollar to the fraction of a cent.

By comparing the fraction it will be $\frac{1}{175}$ of $\frac{10}{1}$ of $\frac{10}{1}$; then $\frac{1}{175} \times \frac{10}{1} \times \frac{10}{1} = \frac{100}{175} = \frac{4}{7}$ c. Answer.

2 Reduce $\frac{3}{50000}$ of an eagle to the fraction of a mill. Ans. $\frac{3}{5}$m.

3. Reduce $\frac{11}{15000}$ of a dollar to the fraction of a mill. Ans. $\frac{11}{15}$m.

4. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny. Ans. $\frac{3}{5}$d.

5. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing. Ans. $\frac{3}{4}$qr.

6. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny. Ans. $\frac{5}{8}$d.

7. Reduce $\frac{1}{57}$ of a moidore to the fraction of a shilling. Ans. $\frac{12}{13}$s.

8. Reduce $\frac{1}{28}$ of a lb. Avoirdupois to the fraction of an ounce. Ans. $\frac{4}{7}$oz.

*9. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound. Ans. $\frac{4}{5}$£.

10. Reduce $\frac{7}{1920}$ of a lb. Troy to the fraction of a pwt. Ans. $\frac{7}{8}$pwt.

11. Reduce $\frac{1}{126}$ of Cwt. to the fraction of a lb. Avoirdupois. Ans. $\frac{8}{9}$lb.

CASE IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, measure, &c.

Rule.†

Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator; and if any thing remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on, as far as necessary; and the quotients placed after one another, in their order, will be the answer required; *or*, reduce the numerator, as if it were a whole number, to the lowest denomination, and divide the result by the denominator; the quotient will be the number of the lowest denomination, (which must be brought into higher denominations as far as it will go,) and the

* $\frac{4}{7}$ Guin. $= \frac{4}{7}$ of $\frac{28}{1} = \frac{4 \times 28}{7 \times 1} = \frac{112}{7}$s. & $\frac{112}{7}$ of $\frac{1}{20} = \frac{112}{140} = \frac{4}{5}$£.

† As the numerator of a fraction may be considered as a remainder, and the denominator as a divisor: This rule therefore has its reason in the nature of division.

the remainder will be a numerator to be placed over the given denominator for a fraction of the lowest denomination.

Note. From this rule, in connexion with what has been said of *Reduction of Federal Money*, it appears, that, annexing to the given numerator as many cyphers, as will fill all the places to the lowest denomination, and dividing the number so formed by the denominator, the quotient will be the answer in the several denominations, and the remainder a numerator to be placed over the given denominator, forming a fraction of the lowest denomination.

EXAMPLES.

1. What is the value of $\frac{5}{8}$ of a dollar?

By the general rule.

```
     5
    10
  8)50( 2
  d. 6  10      Ans. 6d. 2c. 5m.
      8)20( 4        or 62c. 5m.
        ——
      c. 2  10
            ——
          8)40
            ——
          m. 5
```

By the note.

```
     D.  d.  c.  m.
   8) 5 . 0 . 0   0
   ——————————————————
         6   2   5
```

Or thus.

5 D.=5000m. and $\frac{5000}{8}$m.=625m.=62c. 5m. Ans. as before.

2. What is the value of $\frac{17}{64}$ of a dollar?

```
      D   d.  c.  m.
   64)17  0   0   0        (2d. 6c. 5 5/8 m.
      128
      ———                  or 26c. 5 5/8 m. Ans.
       420
       384                 Or, 17D.=17000m. And
       ———
        360                17000/64 m.=265 5/8 m.=
        320
        ———
         40                26c. 5 5/8 m.        Ans. as before.
         —— = 5/8
         64
```

3. What is the value of $\frac{9}{48}$ of an eagle? Ans. 1D. 87c. 5m.
4. What is the value of $\frac{7}{16}$ of a dollar? Ans. 43c. $7\frac{1}{2}$m.
5. What is the value of $\frac{5}{7}$ of a pound? Ans. 14s. 3d. $1\frac{5}{7}$qr.
6. What is the value of $\frac{9}{24}$ of a shilling? Ans $4\frac{1}{2}$d.
7. What is the value of $\frac{7}{40}$ of a £? Ans. 3s. 6d.
8. What is the value of $\frac{27}{44}$ of a pistole? Ans. 13s. 6d.
9. What is the value of $\frac{17}{29}$ of a Cwt.?
Ans. 2 qrs. 9lb. 10oz. $7\frac{21}{29}$dr.
10. What is the value of $\frac{4}{5}$ of a lb. Avoirdupois?
Ans. 12oz. $12\frac{4}{5}$dr.

11. What

11. What is the value of $\frac{3}{5}$ of a lb. Troy? Ans. 7oz. 4pwt.
12. What is the value of $\frac{3}{13}$ of a ton?
Ans. 4cwt. 2qrs. 12lb. 14oz. $12\frac{4}{13}$dr.
13. What is the value of $\frac{6}{9}$ of a yard? Ans. 2qrs. $2\frac{2}{3}$n.
14. What is the value of $\frac{7}{8}$ of an ell English? Ans. 4qrs. $1\frac{1}{2}$n.
15. What is the value of $\frac{5}{6}$ of a mile? Ans. 6fur. 26p. 11ft.
16. What is the value of $\frac{9}{13}$ of a day? Ans 16h. 36m. $55\frac{5}{13}$s.
17. The value of $\frac{12}{17}$ of a Julian year is required?
Ans. 257d. 19h. 45m. $52\frac{16}{17}$s.
18. The value of $\frac{9}{14}$ of a guinea is demanded? Ans. 18s.
19 What is the value of $\frac{15}{16}$ of a dollar? Ans. 5s. $7\frac{1}{2}$d.
20. What is the value of $\frac{3}{5}$ of a moidore? Ans. 21s. $7\frac{1}{3}$d.
21. What is the value of $\frac{6}{7}$ of an acre? Ans. 3r. $17\frac{1}{7}$p.

CASE X.

To reduce any given quantity to the fraction of any greater denomination of the same kind.

Rule.*

Reduce the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term for a denominator; which will be the fraction required.

Note. It appears from this rule and what has been said before, that, in *Federal Money*, where the given quantity contains no fraction of its lowest denomination, the annexing of as many cyphers to 1 of the required denomination, as will extend to the lowest denomination in the given quantity, will form a denominator, which placed under the given quantity used as one number for a numerator, will make the answer, which may be reduced to its lowest terms. Or, if there be a fraction of the lowest denomination, multiply the given whole numbers by its denominator, adding its nnmerator, for a numerator; and let the denominator itself at the left of as many cyphers as were mentioned above be a denominator; the fraction so formed will be the answer; which may be reduced to its lowest terms.

Examples.

1. Reduce 6d 2c. 5m. to the fraction of a dollar.

By the general rule.

6d.	10d. int. pt.
×10+2	10
62	100
×10+5	10
625	1000

And, $\frac{625}{1000}=\frac{5}{8}$ D. Ans.

By the note.

D.	d.	c.	m.
	6	2	5
1	0	0	0

$=\frac{5}{8}$D.

Ans. as before.

2. Reduce.

* This case is the reverse of the former, therefore proves it.

Note. If there be a fraction given with the said quantity, it must be farther reduced to the denominative parts thereof, adding thereto the numerator.

2. Reduce 26c. $5\frac{5}{8}$ m. to the fraction of a dollar.

By the general rule.

26c.	100c. int. pt.
×10+5	10
265	1000
×8+5	8
2125	8000

By the note.

D d. c. m.
265×8+5=2 ·1 2 5 $=\frac{17}{64}$D.
And 1D.×8=8 0 0 0

Ans. as before.

And, $\frac{2125}{8000}=\frac{17}{64}$D. Ans.

3. Reduce 1D. 87c. 5m. to the fraction of an eagle. Ans. $\frac{9}{48}$E.
4. Reduce 43c $7\frac{1}{2}$m. to the fraction of a dollar. Ans. $\frac{7}{16}$D.
5. Reduce 14s. $3\frac{1}{4}$d. $\frac{5}{7}$ to the fraction of a pound. Ans. $\frac{4800}{6720}=\frac{5}{7}$£.
6. Reduce $4\frac{1}{2}$d. to the fraction of a shilling. Ans. $\frac{3}{8}$s.
7. Reduce 3s. 6d. to the fraction of a pound. Ans $\frac{7}{40}$l.
8. Reduce 13s. 6d. to the fraction of a pistole. Ans $\frac{27}{44}$pistole.
9. Reduce 2qrs. 9lb. 10oz. $7\frac{21}{29}$ dr. to the fraction of a cwt. Ans. $\frac{17}{29}$cwt.
10. Reduce 12oz. $12\frac{4}{5}$dr. to the fraction of a lb. Avoirdupois. Ans. $\frac{4}{5}$lb.
11. Reduce 7oz. 4pwt to the fraction of a lb. Troy. Ans. $\frac{3}{5}$lb.
12. Reduce 4cwt. 2qrs. 12lb. 14oz. $12\frac{4}{13}$dr. to the fraction of a ton. Ans. $\frac{3}{13}$ton.
13. Reduce 2qrs. $2\frac{2}{3}$n. to the fraction of a yard. Ans. $\frac{2}{3}$yd.
14. Reduce 4qrs. $1\frac{1}{2}$n. to the fraction of an ell English. Ans. $\frac{7}{8}$E. E.
15. Reduce 6fur. 26po. 11ft. to the fraction of a mile. Ans. $\frac{5}{8}$m.
16. Reduce 16h. 36m. $55\frac{5}{13}$s. to the fraction of a day. Ans. $\frac{9}{13}$ day.
17. Reduce 257d. 19h. 45m. $52\frac{16}{17}$s. to the fraction of a Julian year. Ans. $\frac{12}{17}$ J. Y.
18. Reduce 18s. to the fraction of a guinea. Ans. $\frac{9}{14}$G.
19. Reduce 5s. $7\frac{1}{2}$d. to the fraction of a dollar. Ans. $\frac{15}{16}$dol.
20. Reduce 21s. $7\frac{1}{5}$d. to the fraction of a moidore. Answer $\frac{3}{5}$moidore.
21. Reduce 3r. $17\frac{1}{7}$p. to the fraction of an acre. Ans. $\frac{6}{7}$acre.

ADDITION OF VULGAR FRACTIONS.

Rule.*

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and

* Fractions, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts of the same thing; their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum or difference of their individuals; whence the reason of the rules, both for Addition and Subtraction, is manifest.

and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLES.

1. Add $7\frac{4}{5}$, $\frac{5}{7}$ of $\frac{3}{8}$, and 7 together.
First. $7\frac{4}{5}=\frac{39}{5}$, $\frac{5}{7}$ of $\frac{3}{8}=\frac{15}{56}$, and $7=\frac{7}{1}$.
Then the fractions are $\frac{39}{5}$, $\frac{15}{56}$, and $\frac{7}{1}$; therefore,

$$\begin{array}{r} 39\times56\times\ 1=2184 \\ 15\times\ 5\times\ 1=\ \ \ 75 \\ 7\times\ 5\times56=1960 \\ \hline \end{array}$$

$$\frac{4219}{5\times56\times1=280}=15\tfrac{19}{280}$$

Or thus, $\dfrac{2184+75+1960}{280}=15\frac{19}{280}$.

2. Add $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. Ans. $9\frac{101}{105}$.
3. What is the sum of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{4}$, and $8\frac{4}{13}$? Ans. $9\frac{2319}{12480}$.
4. What is the sum of $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $9\frac{1}{4}$? Ans. $12\frac{59}{80}$.
5. Add together $\frac{5}{7}$E. $\frac{3}{8}$D. and $1\frac{1}{2}$c. Ans. 7D. 53c. $2\frac{6}{7}$m.
6. Add together $\frac{1}{5}$D. $\frac{5}{8}$c. $\frac{3}{16}$c. and $\frac{7}{8}$m. Ans. 20c. 9m.
7. Add $\frac{1}{9}$l. $\frac{3}{7}$s. and $\frac{4}{5}$d. together. Ans. 2s. $8\frac{64}{105}$d.
8. What is the sum of $\frac{2}{5}$ of 17£. $9\frac{5}{8}$£. and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{4}{7}$£.? Ans. £. 16 12s. $3\frac{5}{7}$d.
9. Add $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{5}{8}$ of a mile together. Ans. 1100yds. 2ft. 7inches.
10. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. Ans. 2 days, 2 hours, 30 minutes, 45 seconds.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

EXAMPLES.

* In ſubtracting mixed numbers, when the fractions have a common denominator, and the numerator in the ſubtrahend is leſs than that in the minuend, the difference of the whole numbers will be a whole number, and the difference of the numerators a numerator to be placed over the given denominator: this whole number and the fraction thus formed will be the remainder: but, when the numerator in the ſubtrahend is greater than that in the minuend, ſubtract the numerator in the ſubtrahend from the common denominator, adding the numerator in the minuend, and carrying 1 to the integer of the ſubtrahend.

Hence, A fraction is ſubtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

EXAMPLES.

From	$12\frac{3}{5}$	$12\frac{2}{5}$	12
Take	$7\frac{2}{5}$	$7\frac{3}{5}$	$\frac{2}{5}$
Rem.	$5\frac{1}{5}$	$4\frac{4}{5}$	$11\frac{3}{5}$

EXAMPLES.

1. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$.

$\frac{2}{7}$ of $\frac{5}{8}=\frac{10}{56}=\frac{5}{28}$. Then the fractions are $\frac{3}{4}$ and $\frac{5}{28}$.

$3\times28 = 84$
$5\times 4 = 20$
$4\times28 = 112$ com. den.

$\frac{3}{4}=\frac{84}{112}$, and $\frac{5}{28}=\frac{20}{112}$, therefore, $\frac{84}{112}-\frac{20}{112}=\frac{64}{112}=\frac{4}{7}$ remainder.

2. From $\frac{49}{50}$ take $\frac{5}{9}$. Ans. $\frac{191}{450}$.
3. From $37\frac{1}{4}$ take $19\frac{4}{7}$. Ans. $17\frac{19}{28}$.
4. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. Ans. $2\frac{1}{12}$.
5. From $\frac{1}{2}$D take $\frac{7}{8}$c. Ans. 49c. $1\frac{1}{4}$m.
6. Take $3\frac{1}{3}$c. from $\frac{1}{5}$ of $2\frac{1}{3}$D. Ans. $43\frac{1}{3}$c.
7. From $\frac{7}{8}$ of $\frac{4}{9}$ of 5D. take $\frac{5}{8}$ of 96c. added to $\frac{1}{3}$ of $1\frac{1}{8}$D. Ans 96c. $9\frac{4}{9}$m.
8. From $\frac{1}{4}$£. take $\frac{9}{10}$s. Ans. 4s. $1\frac{1}{5}$d.
9. From $\frac{5}{7}$oz. take $\frac{3}{4}$pwt. Ans. 13pwt. $12\frac{6}{7}$gr.
10. From $\frac{1}{2}$ of a league take $\frac{3}{8}$ of a mile. Ans. 1mi. 1fur.
11. From 5 weeks take $19\frac{4}{5}$ days. Ans. 15da. 4ho. 48min.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.*

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.—*Note*, where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

EXAMPLES.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$$4\tfrac{1}{3}=\tfrac{13}{3},\ \tfrac{1}{4}\text{ of }\tfrac{7}{8}=\frac{1\times7}{4\times8}=\tfrac{7}{32},\text{ and }6=\tfrac{6}{1}.$$

$$\text{Then }\tfrac{13}{3}\times\tfrac{1}{5}\times\tfrac{7}{32}\times\tfrac{6}{1}=\frac{13\times1\times7\times6}{3\times5\times32\times1}=\tfrac{546}{480}=1\tfrac{11}{80}\text{ the Answer.}$$

2. Multiply $\frac{4}{17}$ by $\frac{5}{27}$. Ans. $\frac{20}{459}$.
3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans $\frac{7}{8}$.
4. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. Ans. $\frac{5}{14}$.
5. Multiply $\frac{3}{7}$ of $\frac{5}{9}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{7}$. Ans. $\frac{64}{147}$.
6. Multiply $9\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{5}$, and $12\frac{4}{7}$ continually together. Ans. $24\frac{18}{35}$.
7. What is the continual product of $\frac{3}{4}$ of $\frac{2}{3}$, $5\frac{1}{2}$, 7 and $\frac{1}{3}$ of $\frac{5}{8}$? Ans. $4\frac{1}{96}$.
8. What is the continual product of 7, $\frac{1}{2}$, $\frac{5}{7}$ of $\frac{3}{8}$, and $3\frac{1}{9}$? Ans. $1\frac{11}{24}$.

Another

* Multiplication of a fraction implies the taking of ſome part or parts of the multiplicand, and therefore may truly be expreſſed by a compound fraction. Thus $\frac{4}{5}$ multiplied by $\frac{3}{8}$ is the ſame as $\frac{4}{5}$ of $\frac{3}{8}$; and as the directions of the rule agree with the method already given, to reduce theſe fractions to ſimple ones, it is ſhown to be right.

Another method for the Multiplication of mixed Quantities.

Case 1. To multiply a *whole number* by a *fraction*, or a *fraction* by a *whole number*.

Rule. Multiply the whole number by the numerator of the fraction and divide the product by the denominator: But if the numerator be 1, divide by the denominator only.

	1.	2.	3.	4.	5.	6.	7.
Mult.	8	15	28	36	48	325	259
By	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{12}$
	2	$7\frac{1}{2}$	$9\frac{1}{3}$	3)72	4)144	8)1625	12)1813
				Prod. 24	36	$203\frac{1}{8}$	$151\frac{1}{12}$

Case 2. To multiply a *whole number* by a *mixed one*.

Rule. Multiply by the fraction as in Case 1st; then multiply by the whole number, and add the two products, as in the examples—or, to multiply a mixed number by a whole one, change the place of the factors, and proceed as the rule directs.—See example 6.

	1.	2.	3.	4.	5.	6.
Mult.	15	35	68	42	129	$1\frac{7}{15}$
By	$3\frac{1}{2}$	$5\frac{1}{3}$	$7\frac{11}{12}$	$9\frac{3}{7}$	$8\frac{5}{8}$	24
	$7\frac{1}{2}$	$11\frac{2}{3}$	748	126	645	Mult. 24
	45	175	$62\frac{4}{12}$	18	$80\frac{5}{8}$	By $1\frac{7}{15}$
	$52\frac{1}{2}$	$186\frac{2}{3}$	476	378	1032	15)168
			$538\frac{4}{12}$	396	$1112\frac{5}{8}$	$11\frac{3}{15}$
						24
						Prod. $35\frac{3}{15}=\frac{1}{5}$

Case 3. To multiply a *mixed number* by a *mixed number*.

Rule. Multiply the integral part of the multiplicand by the denominator of its fractional part, and add thereto its numerator: Then multiply by the mixed multiplier, by Case 2d, and divide the product by the denominator of the fractional part of the multiplicand, as in the following example.

Mult. $42\frac{3}{5}$ } 1st. $42\frac{3}{5} = 213$
By $8\frac{2}{3}$ } which mult. by $8\frac{2}{3}$

3)426

142

1704

5)1846

Product $= 369\frac{1}{5}$

After this manner may feet and inches be multiplied, calling 1 inch $\frac{1}{12}$ of a foot, 2 inches $\frac{1}{6}$, 3 inches $\frac{1}{4}$, 4 inches $\frac{1}{3}$, 5 inches $\frac{5}{12}$, 6 inches $\frac{1}{2}$, 7 inches $\frac{7}{12}$, 8 inches $\frac{2}{3}$, 9 inches $\frac{3}{4}$, 10 inches $\frac{5}{6}$, 11 inches $\frac{11}{12}$ of a foot.

DIVISION

K

DIVISION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as before: then, invert the divisor and proceed exactly as in Multiplication: The products will be the quotient required.

EXAMPLES.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$

$$\tfrac{1}{3} \text{ of } 17 = \tfrac{1}{3} \text{ of } \tfrac{17}{1} = \frac{1 \times 17}{3 \times 1} = \tfrac{17}{3} \text{ and } \tfrac{2}{3} \text{ of } \tfrac{6}{8} = \tfrac{12}{24} = \tfrac{1}{2}\text{; therefore,}$$

$$\tfrac{17}{3} \div \tfrac{1}{2} = \frac{17 \times 2}{3 \times 1} = \tfrac{34}{3} = 11\tfrac{1}{3} \text{ the quotient required.}$$

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$. Ans. $1\frac{4}{21}$.
3. Divide $12\frac{1}{5}$ by $\frac{1}{3}$ of 7. Ans. $5\frac{8}{35}$.
4. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. Ans. $\frac{41}{62}$.
5. Divide $\frac{3}{7}$ by 9: Ans. $\frac{1}{21}$.
6. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$. Ans. $\frac{8}{9}$.
7. Divide 7 by $\frac{3}{8}$. Ans. $18\frac{2}{3}$.
8. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112. $42\frac{529}{588}$.

DECIMAL FRACTIONS.

DECIMAL Fractions are of such a nature, that they vary in the same proportion, and are managed by the same method of operation, as whole numbers are.

On this account, every proper Fraction is supposed to be reducible to another, whose denominator shall be 10, 100, 1000, &c. viz. Unity, with a number of cyphers annexed; and Fractions with such denominators are called *Decimal Fractions:* Such are $\frac{5}{10}$, $\frac{55}{100}$, $\frac{675}{1000}$, &c.

As the denominator of a decimal fraction is always 10, or 100, or 1000, &c. the denominators need not be expressed: For the numerator only may be made to express the true value: For this purpose it is only required to write the numerator with a point before it at the left hand, to distinguish it from a whole number, when it consists of so many figures as the denominator hath cyphers annexed to unity, or 1: So $\frac{5}{10}$ is written ·5; $\frac{33}{100}$ ·33; $\frac{735}{1000}$ ·735, &c.

Note.

* The reaſon of the rule may be ſhewn thus. Suppoſe it were required to divide $\frac{4}{5}$ by $\frac{2}{7}$. Now $\frac{4}{5} \div 2$ is manifeſtly $\frac{1}{2}$ of $\frac{4}{5}$ or $\frac{4}{2 \times 5}$; but $\frac{2}{7} = \frac{1}{7}$ of 2; therefore, $\frac{1}{7}$ of 2, or $\frac{2}{7}$, muſt be contained 7 times as often in $\frac{4}{5}$ as 2 that is $\frac{4 \times 7}{5 \times 2}$ = the anſwer, which is according to the rule.

Note. To multiply a fraction by an integer, divide the denominator, or multiply the numerator by it; and to divide by an integer, divide the numerator, or multiply the denominator by it.

Note. The point prefixed is called a Separatrix.

But if the numerator has not so many places as the denominator has cyphers, put so many cyphers before it, viz. at the left hand, as will make up the defect; so write $\frac{5}{100}$ thus, ·05; and $\frac{6}{1000}$ thus ·006, &c. And thus do these fractions receive the form of whole numbers.

The 1st, 2d, 3d, 4th, &c. places of decimals, counting from the left hand toward the right, are called primes, seconds, thirds, fourths, &c.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand to 0, as in the following

TABLE.*

C Millions	X Millions	Millions	C Thousands	X Thousands	Thousands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thousandth Parts	X Thousandth Parts	C Thousandth Parts	Millionth Parts	X Millionth Parts	C Millionth parts
9	8	7	6	5	4	3	2	1	·2	3	4	5	6	7	8	9

From this table it is evident that, in decimals, as well as in whole numbers, each figure takes its value by its distance from unit's place: If it be in the first place after units (or the separating point) it signifies

* It will be very apparent to the learner from the nature of decimals, and what has been ſaid of *Federal Money*, that this money is purely decimal; and, the dollar being the money unit, the lower denominations are plainly ſo many decimal parts of a dollar; thus 9 dollars and 8 dimes are expreſſed $9{\cdot}8=9\frac{8}{10}$ doll.—12 dollars, 4 dimes, and 7 cents thus, $12{\cdot}47=12\frac{47}{100}$ doll.—20 dollars, 3 dimes, 4 cents and 5 mills, thus $20{\cdot}345=20\frac{345}{1000}$ doll.—100 dollars and 9 mills, thus $100{\cdot}009=100\frac{9}{1000}$ doll. and 50 dollars, 5 cents, thus $50{\cdot}05=50\frac{5}{100}$ doll. wherefore, it is, in all reſpects, added ſubtracted, multiplied and divided, the ſame as decimals; and, of all coins, it is the moſt ſimple.

It may alſo be obſerved that the ſum exhibits the particular number of each different piece of money contained in it, viz. 455997 mills $=45599\frac{7}{10}$ cents $=4559\frac{97}{100}$ dimes $=455\frac{997}{1000}$ dollars $=45\frac{5997}{10000}$ eagles

E. D. d. c. m.
= 4 5 5· 9 9 7.

Alſo, the names of the coins, leſs than a dollar, are ſignificant of their values. For the *mill*, which ſtands in the 3d place at the right hand of the ſeparatrix or place of thouſandths, is contracted from *mille*, the Latin for *thouſand*: *Cent*, which occupies the ſecond place, or place of hundredths, is an abbreviation of *centum*, the Latin for *hundred*: And *dime*, which is in the firſt place, or place of tenths, is derived from, *diſme*, the French for *tenths*.

Such being the nature of *Federal Money*, its operations can in no other way be ſo well underſtood as in obtaining a good knowledge of decimals, and applying their ſeveral rules to the various caſes of money matters.

fies tenths; if in the second, hundredths, &c. decreasing in each place in a tenfold proportion.

Consequently, every single figure expressing a decimal, has for its denominator an unit or 1, with so many cyphers as its place is distant from unit's place: Thus 2 in the decimal part of the table $=\frac{2}{10}$; 3 $=\frac{3}{100}$; 4 $=\frac{4}{1000}$, &c. And if a decimal be expressed by several figures, the denominator is 1, with so many cyphers as the lowest figure is distant from unit's place. So ·357 signifies $\frac{357}{1000}$, and ·0053 $=\frac{53}{10000}$, &c.

Cyphers, placed at the right hand of a decimal fraction, do not alter its value, since every significant figure continues to possess the same place: So ·5, ·50, and ·500, are all of the same value, and each equal to $\frac{1}{2}$.

But cyphers, placed at the left hand of a decimal, do alter its value, every cypher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place further from the place of units. So ·5, ·05, ·005, all express different decimals, viz. ·5, $\frac{5}{10}$; ·05, $\frac{5}{100}$; ·005, $\frac{5}{1000}$.

Hence may be observed the contrary effects of cyphers being annexed to whole numbers, and decimals.

It is likewise evident from the table, that since the places of decimals decrease in a tenfold proportion from units downwards, so they consequently increase in a tenfold proportion from the right hand toward the left, as the places of whole numbers do: For, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $=\frac{10}{100}=\frac{1}{10}$, $\frac{10}{10}=1$, and $1\times10=10$, which proves that decimals are subject to the same law of Notation, and consequently of operation, as whole numbers are.

Decimal fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of those, which have fewer places, so many cyphers, as make them equal in places with that which has the most. So these decimals, ·5, ·06, ·455, may be reduced to the decimals, ·500, ·060, and ·455, which have, all, 1000 for their denominator.

Of Decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places: Thus, ·5 is greater than, ·459, for if it be reduced to the same denominator with ·459, it will be ·500.

A mixed number, viz. a whole number, with a decimal annexed, is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number, and the denominator, that of the decimal part. So 45·309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction:

Thus, $45\times1000+\cdot309=\frac{45309}{1000}$ as above.

ADDITION

ADDITION OF DECIMALS.

Rule.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2 Find their sum as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Examples.

1. Find the sum of 19·073+2·3597+223+·0197581+3478·1+12·358.

```
  19·073
   2·3597
 223·
    ·0197581
3478·1
  12·358
-----------
3734·9104581 the sum.
```

2. Required the sum of 429+21·37+355·003+1·07+1·7 ?
Ans. 808·143.

3. Required the sum of 5·3+11·973+49+·9+1·7314+34·3 ?
Ans. 103.2044.

4. Required the sum of 973+19+1·75+93·7164+·9501 ?
Ans. 1088·4165.

SUBTRACTION OF DECIMALS.

Rule.

Place the numbers according to their value ; then subtract as in whole numbers, and point off the decimals as in Addition.

Examples.

1. Find the difference of 1793·13 and 817·05693 ?

```
From 1793·13
Take  817·05693
      ---------
Remainder 976·07307
```

2. From 171·195 take 125·9176. Ans. 45·2774.
3. From 219·1384 take 195·91. Ans. 23 2284.
4. From 480 take 245·0075. Ans. 234·9925.

MULTIPLICATION OF DECIMALS.

CASE 1.

Rule.

1. Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. Point off so many figures from the product as there are decimal places in both the factors ; and if there be not so many places in the product, supply the defect by prefixing cyphers.

Examples.

EXAMPLES.

1. Multiply ·02345
by ·00163

7035
14070
2345
·0000382235 the product.

2. Multiply 25·238 by 12·17. Ans. 307·14646.
3. Multiply ·3759 by ·945. Ans. ·3552255.
4. Multiply ·84179 by ·0385. Ans. ·032408915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has cyphers.

$$\text{So } \cdot345 \text{ Multiplied by} \left\{\begin{array}{l}10\\100\\1000\end{array}\right\} \text{makes} \left\{\begin{array}{l}3\cdot45\\34\cdot5\\345\end{array}\right.$$

For ·345×10 is 3·450, &c.

CASE II.

To contract the operation, so as to retain so many decimal places in the Product as may be thought necessary.

RULE

1. Write the unit's place of the multiplier under that figure of the multiplicand, whose place you would reserve in the product; and dispose of the rest of the figures in a contrary order to what they are usually placed in.

2. In multiplying, reject all the figures which are to the right hand of the multiplying digit, and set down the products, so that their right hand figures may fall in a straight line below each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the preceding figures, when you begin to multiply, and the sum will be the product required.

EXAMPLES.

1. It is required to multiply 56·7534916 by 5·376928, and to retain only five places of decimals in the product.

Contracted way.	Common way.
56·7534916	56·7534916
829673·5	5·376928
28376746 . . .	45|40279328
1702605 . . .	113|5069832
·397274	5107|814244
34052	34052|09496
5108	397274|4412
113	1702604|748
45	28376745|80
305·15943	305·15943|8081804 8

By

By the operation in the *common way*, it is evident that all the figures which are cut off at the right hand, by the perpendicular line, are wholly omitted in the *contracted way*, and the last product here is the first there ; consequently the reason of placing the multiplier in a reverse order, must appear very plainly.

DIVISION OF DECIMALS.

Rule.*

1. The places of decimal parts in the divisor and quotient counted together, must always be equal to those in the dividend, therefore divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness.

Examples.

1.

```
219)·117841075(·000538087, &c.
     1095
     ----
      834
      657
     ----
     1771
     1752
     -----
       1907
       1752
       ----
        1555
        1533
        ----
          22
```

2.

```
·3719)38·0000(102·178, &c.
      3719
      ----
       8100
       7438
       ----
        6620
        3719
        -----
        29010
        26033
        -----
         29770
         29752
         -----
            18
```

In Example 1ſt, the diviſor having no decimals, the quotient muſt have ſo many as there are in the dividend. In Example 2, the dividend being an integer muſt have at leaſt ſo many cyphers annexed as there are decimals in the diviſor, and ſo far the quotient will be whole numbers, then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the Rule.

3d. 133)5737(43·1353+ (4th.) 23·7)65321(2756·16+

5th. †72)918·217(12753+ (6th.) 25·17)315·6293(1253+

7th. ·317)29·417(92+ (8th) 37·9)·0059374(156+

9th. ·375)·173948375(463862+

Having a multiplier, to find a divisor which shall give a quotient equal to the product by that multiplier.

Rule.

Divide unity by the given multiplier, and the quotient will be the divisor sought.

What

* The reaſon of pointing off ſo many decimal places in the quotient, as thoſe in the dividend exceed thoſe in the diviſor, will eaſily appear, for, ſince the number of decimal places in the dividend is equal to thoſe in the diviſor and quotient taken together, by the nature of multiplication : It therefore follows that the quotient contains ſo many as the dividend exceeds the diviſor.

† The following queſtions are left unpointed in the quotient to exerciſe the learner.

What divisor is that, by which dividing 5357, shall give a quotient equal to the product of the same number multiplied by 250 ?

250)1·000(·004 the Answer. And ·004)5357·000(1339250.
Proof. 5357×250=1339250.

Having a divisor, to find a multiplier which shall give a product equal to the quotient by that divisor.

RULE.

Divide unity by the given divisor, and the quotient will be the multiplier sought.

What multiplier is that, by which multiplying 5357, shall give a product equal to the quotient of the same number divided by ·004 ?

·004)1·000(250 the Answer : Therefore, 5357×250=5357÷·004 =1339250.

CASE II.

To contract Division, when there are many decimals in the dividend, and the divisor is large.

RULE.

1. Whatever place of the dividend corresponds with the unit's place of the divisor, at the first step of the division, the same place must the first figure of the quotient have.

2. In dividing, reject the last right hand figure of the divisor, at every step, (instead of bringing down a figure, as is common,) and make the last remainder the dividend for the new divisor at every step : Thus continue the division until the divisor shall be exhausted.

```
99·5678)4·6789837568(·0469931 Quotient.
        3 982712
        --------
99·567)696271
       597402
       ------
 99·56) 98869
        89604
        -----
  99·5)  9265
         8955
         ----
    99)   310
          297
          ---
     9)    13
            9
          ---
Remainder   4
```

Here, the unit's place of the divisor in the first step falls under 7 in the place of hundredths in the dividend, therefore, I put 4, the first quotient figure, in the place of hundredths, by prefixing a cypher.

I have set down every divisor, to explain the work ; but you need only put a dash over every figure rejected, as you proceed, to show it is omitted.

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers) it is performed by removing the separatrix, in the dividend, so many places toward the left hand as there are cyphers in the divisor.

EXAMPLES.

10	Dividing	7654	The Quot. is	765·4
100				76·54
1000				76·55
10000				·7654

REDUCTION

REDUCTION OF DECIMALS.

CASE I.

To reduce a Vulgar Fraction to its equivalent Decimal.

RULE.*

Divide the numerator by the denominator, as in division of decimals, and the quotient will be the decimal required :—Or, so many cyphers as you annex to the given numerator, so many places must be pointed off in the quotient, and if there be not so many places of figures in the quotient, the deficiency must be supplied by prefixing so many cyphers before the quotient figures.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal. 8)1·000

·125 Ans.

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{2}{3}$ to decimals. Answers, ·375, ·625, ·666+.
3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$ to decimals.
   Answers, ·25, ·5, ·75, ·333+, ·8, ·833+, ·875.
4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$, and $\frac{9}{36}$ to decimals.
   Answers, ·263+, ·692+, ·025, ·25.
5. Reduce $\frac{7}{375}$, $\frac{9}{1129}$, and $\frac{5}{1875}$ to decimals.
   Answers, ·0186+, ·00797+, ·00266+.

CASE II.

To reduce numbers of different denominations, as of Money, Weight and Measure, to their equivalent decimal values.

RULE.†

1. Write the given numbers perpendicularly, under each other, for dividends ; proceeding orderly from the least to the greatest.

2. Opposite to each dividend, on the left hand, place such a number, for a divisor, as will bring it to the next superiour denomination, and draw a line perpendicularly between them.

3. Begin with the highest, and write the quotient of each division as decimal parts on the right hand of the dividend next below it, and so on, until they are all used, and the last quotient will be the decimal sought.

EXAMPLES.

* Let the given vulgar fraction, whofe decimal expreffion is required, be $\frac{9}{15}$. Now, fince every decimal fraction has 10, 100, 1000, &c. for its denominator ; and if two fractions be equal, it will be, as the denominator of 1 is to its numerator ; fo is the denominator of the other to its numerator ; therefore, as 15 : 9 :: 10 &c. :$\frac{9\times10}{15}$ $=\frac{90}{15}=$·6 the numerator of the decimal required ; and is the fame as by the rule.

† The reafon of the rule may be explained from the firft example : Thus, three farthings are $\frac{3}{4}$ of a penny, which, reduced to a decimal, is, ·75 ; confequently, $8\frac{3}{4}$d. may be expreffed, 8·75d. but 8·75 is $\frac{875}{100}$ of a penny $=\frac{875}{1200}$ of a fhilling, which, reduced to a decimal, is, ·729166s.+ In like manner, 17·729166s.+ are $\frac{17729166}{1000000}$s. $=\frac{17729166}{20000000}$£.= ·886458+ as by the rule.

EXAMPLES.

1. Reduce 17s. 8¾d. to the decimal of a pound.

4	3·
12	8·75
20	17·729166, &c.
	·886458, &c. the decimal required.

Here, in dividing 3 by 4, I suppose 2 cyphers to be annexed to the 3, which make it 3·00, and ·75 is the quotient, which I write against 8 in the next line; this quotient, viz. 8·75 being pence and decimal parts of a penny, I divide them by 12, which brings them to shillings and decimal parts, I therefore divide by 20, and, there being no whole number, the quotient is decimal parts of a pound.

2. Reduce 1, 2, 3, 4, and so on to 19 shillings, to decimals.

Shillings.	1	2	3	4	5	6	7	8	9	10
Answers.	·05,	·1,	·15,	·2,	·25,	·3,	·35,	·4,	·45,	·5,
Shillings.	11	12	13	14	15	16	17	18	19	
Answers.	·55,	·6,	·65,	·7,	·75,	·8,	·85,	·9,	·95.	

Here, when the shillings are even, half the number, with a point prefixed, is their decimal expression; but if the number be odd, annex a cypher to the shillings, and then halving them, you will have their decimal expression.

3. *Reduce 1, 2, 3, and so on to 11 pence, to the decimals of a shilling.

Pence.	1	2	3	4	5	6
Answers.	·083+,	·166	·25,	·333+,	·416+,	·5,
Pence.	7	8	9	10	11	
Answers.	·583+,	·666+,	·75,	·833+,	·916+.	

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

Pence.	1	2	3	4	5	
Answers.	·00416+,	·0083+,	·0125,	·01666+,	·0208+,	
Pence.	6	7	8	9	10	11
Answers.	·025,	·02916+,	·0333+,	·0375,	·0416+,	·04583+.

5. Reduce 1, 2 and 3 farthings to the decimals of a penny.
1qr.=·25d. 2qr.=·5d. and 3qr.=·75d. Answers.

6. Reduce 1, 2 and 3 farthings to the decimals of a shilling.
Answers. 1qr.=·02083+s. 2qrs.=·04166+s. 3qrs.=·0625s.

7. Reduce 1, 2 and 3 farthings to the decimals of a pound.
Ans. 1qr.=·0010416+£. 2qrs.=·002083+£. 3qrs.=·003125£.

8. Reduce 15s. 5½d. to the decimal of a pound. Ans. ·6729+.

9. Reduce 7Cwt. 3qrs. 17lb. 10oz. 12dr. to the decimal of a ton.
Ans. ·39538+.

10. Reduce 10oz. 13pwt. 9gr. to the decimal of a pound Troy.
Ans. ·8890625.

11. Reduce 3qrs. 3n. to the decimal of a yard. Ans. ·9375.

12. Reduce 5fur. 12po. to the decimal of a mile. Ans. ·6625.

13. Reduce 55m. 37sec. to the decimal of a day. Ans. ·03862+.

* The anſwers to this queſtion are the ſame as the decimal parts of a foot.

CASE

CASE III.

To find the decimal of any number of shillings, pence and farthings, by inspection.

RULE.*

1. Write *half* the greatest even number of shillings for the first decimal figure.

2. Let the farthings in the given pence and farthings possess the second and third places: observing to increase the *second* place or place of hundredths, by 5 if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

EXAMPLES.

1. Find the decimal of 13s. $9\frac{3}{4}$d. by inspection.

	·6 . .	$=\frac{1}{2}$ of 12s.
	5	for the odd shilling.
	39	=the farthings in $9\frac{3}{4}$d.
Add	2	for the excess of 36.
	·691	= decimal required.

2. Find, by inspection, the decimal expressions of 18s. $3\frac{1}{4}$d. and 17s. $8\frac{1}{2}$d. Ans. £·914 and £·885.

4. Value the following sums, by inspection, and find their total, viz. 15s. 3d.+8s. $11\frac{1}{2}$d.+10s. $6\frac{1}{4}$d.+1s. $8\frac{1}{2}$d.+$\frac{1}{2}$d.+$2\frac{3}{4}$d.

Ans. £.1·834 the total.

CASE IV.

To find the value of any given decimal in the terms of the integer.

RULE.

1. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

2. Multiply the remainder by the next inferiour denomination, and cut off a remainder as before.

3. Proceed in this manner through all the parts of the integer, and the several denominations, standing on the left hand, make the answer.

EXAMPLES.

* The invention of the rule is as follows: As shillings are so many 20ths of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimals; but when they are odd, their half will always consist of two figures, the first of which will be half the even number, next less, and the second a 5: Again, farthings are so many 960ths of a pound, and had it happened that 1000, instead of 960, had made a pound, it is plain any number of farthings would have made so many thousandths, and might have taken their place in the decimal accordingly. But 960 increased by $\frac{1}{24}$ part of itself, is=1000, consequently any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal expression for them: Whence, if the number of farthings be more than 12, $\frac{1}{24}$ part is greater than $\frac{1}{2}$qr. and, therefore, 1 must be added; and when the number of farthings is more than 36, $\frac{1}{24}$ part is greater than $1\frac{1}{2}$qr. for which 2 must be added.

EXAMPLES.

1. Find the value of ·73968 of a pound.

```
   20
-------
14·79360
      12
-------
 9·52320
       4
-------
 2·09280   Ans. 14s. 9½d.
```

2. What is the value of ·679 of a shilling? Ans. 8·148d.
3. What is the value of ·9999£.? Ans. 19s. 11¾d. $\frac{9}{10}$—or £1.
4. What is the value of ·617 of a Cwt.?
Ans. 2qrs. 13lb. 1oz. 10$\frac{6}{10}$dr.
5. What is the value of ·8593 of a lb Troy?
Ans. 10oz. 6pwt. 5gr.
6. What is the value of ·397 of a yard? Ans. 1qr. 2·352n.
7. What is the value of ·8469 of a degree?
Ans. 58m. 6fur. 35po. 0ft. 11in.
8. What is the value of ·569 of a year?
Ans. 207da. 16ho. 26m. 24sec.
9. What is the value of ·713 of a day? Ans. 17h. 6m. 43sec.

CASE V.

To find the value of any decimal of a pound by inspection.

RULE.

Double the first figure, or place of tenths, for shillings, and if the second figure be 5, or more than 5, reckon another shilling; then, after the 5 is deducted, call the figures in the second and third places so many farthings, abating 1 when they are above 12, and 2 when above 36, and the result will be the answer.

Note. When the Decimal has but 2 figures, if any thing remain after the shillings are taken out, a cypher must be annexed to the right hand, or supposed to be so.

EXAMPLES.

1. Find the value of ·876£. by inspection.

```
        16s.  = double of 8.
         1s.  for the 5 in the second place, which is to be taken out
And          6½d.=26 farthings remain to be added.          [of 7.
Deduct       ¼d. for the excess of 12.
       ------
        17s. 6¼d. the Ans.
```

2. Find, by inspection, the value of ·49£.

```
      8s. - - = double of 4.
      1s. - - for the 5 in the place of hundredths.
         10d. = 40 farthings, a 0 being annexed to the remaining 4.
Ded.      ½d. for the excess of 36.
     ------
      9s. 9½d. the Answer.
```

3. Find

3. Find the value of ·097£. by Inspection. Ans. 1s. 11¼d.

4. Value the following decimals, by Inspection, and find their sum, viz. ·785£. + ·537£. + ·916£. + ·74£. + ·5£. + ·25£. + ·09£. + ·008£. Ans. £3 16s. 6d.

DECIMAL TABLES OF COIN, WEIGHT *and* MEASURE.

TABLE I. COIN.
£. 1 the Integer.

Shil.	dec.	Shil.	dec.
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

Pence.	Decimals.
11	·045833
10	·041666
9	·0375
8	·033383
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Farthings.	Decimals.
3	·003125
2	·0020833
1	·0010416

TABLE II.
COIN & Long Meas.
1 Shil. & 1 Foot the Integer.

Pence & Inches.	Decimals.
11	·916666
10	·833333
9	·75
8	·666666
7	·583333
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Farthings.	Decimals.
3	·0625
2	·041666
1	·020383

TABLE III.
TROY WEIGHT.
1lb. the Integer.
Ounces the same as TABLE II.

Pwts.	Decimals.
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Grains.	Decimals.
12	·002083
11	·00191
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 Oz. the Integer. Pennyweights the same as Shillings in in the first Table.

Grains.	Decimals.
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125
5	·010416

Grains.	Decimals.
4	·008333
3	·00625
2	·004166
1	·002083

TABLE IV.
AVOIRDUPOIS Wt.
112lb. the Integer.

Qrs.	Decimals.
3	·75
2	·5
1	·25

Pounds.	Decimals.
27	·241071
26	·232143
25	·223214
24	·214286
23	·205357
22	·196428
21	·1875
20	·178571
19	·169643
18	·160714
17	·151786
16	·142857
15	·133928
14	·125
13	·110671
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Ounces.	Decimals.
15	·008370
14	·007812
13	·007254
12	·006696
11	·006138

Ounces	Decimals.
10	·00558
9	·005022
3	·004464
7	·003906
6	·003348
5	·00279
4	·002232
3	·001674
2	·001116
1	·000558

qrs. of ozs.	Decimals.
3	·000418
2	·000279
1	·000139

TABLE V.
AVOIRDUPOIS Wt.
1lb. the Integer.

Ounces.	Decimals,
15	·9375
14	·875
13	·8125
12	·75
11	·6875
10	·625
9	·5625
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

Drams.	Decimals.
15	·058593
14	·054687
13	·050781
12	·046875
11	·042968
10	·039062
9	·035156
8	·03215
7	·027343
6	·023437
5	·019531

Drams.	Decimals.
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.

CLOTH MEASURE.

1 Yard the Integer.

Quarters.	Decimals.
3	·75
2	·5
1	·25

Nails.	Decimals.
3	·1875
2	·125
1	·0625

TABLE VII.

LONG MEASURE.

1 Mile the Integer.

Yards.	Decimals.
1000	·568182
900	·511364
800	·454545
700	·397727
600	·34
500	·284091
400	·227272
300	·170454
200	·113636
100	·056818
90	·051136
80	·045454
70	·039773
60	·034091

Yards.	Decimals.
50	·028409
40	·022727
30	·017045
20	·011364
10	·005682
9	·005114
8	·004545
7	·003977
6	·003409
5	·002841
4	·002273
3	·001704
2	·001136
1	·000568

Feet.	Decimals.
2	·0003787
1	·0001892

Inches.	Decimals.
6	·0000947
5	·000079
4	·0000632
3	·0000474
2	·0000316
1	·0000158

TABLE VIII.

LIQUID MEASURE.

1 Gallon the Integer.

Quarts the same as qrs. in TABLE VI.

1 Pint.	·125
3 Gills.	·09375
2	·0625
1	·03125

TABLE IX.

TIME.

1 Year the Integer.

Months the same as Pence in TABLE II.

Days.	Decimals.
365	1·000000
300	·821928
200	·547945
100	·273973
90	·246575
80	·219178
70	·191781
60	·164383
50	·136986
40	·109589
30	·082192
20	·054794
10	·027397
9	·024657
8	·021918
7	·019178
6	·016438
5	·013698
4	·010959
3	·008219
2	·005479
1	·002739

1 Day the Integer.

Hours.	Decimals.
23	·958333
22	·916666
21	·875
20	·833333
19	·791666
18	·75
17	·708333
16	·666666
15	·625
14	·583333
13	·541666
12	·5
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041666

Minutes.	Decimals.
30	·020833
20	·013888
10	·006944
9	·00625
8	·005555
7	.004861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001388
1	·000694

COMPOUND MULTIPLICATION*

IS extremely useful in finding the value of Goods, &c. And as in Compound Addition, we carry from the lowest denomination to the next higher, so we begin and carry in Compound Multiplication: One general rule being to multiply the price by the quantity.

CASE I.

When the quantity does not exceed 12 *yards, pounds, &c:* Set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and, in multiplying by it, observe the same rules for

* The product of a number, consisting of several parts or denominations, by any simple number whatever, will be expressed by taking the product of that simple number, and each part by itself, as so many distinct questions: Thus £.33 15s 9d. multiplied by 5, will be £.165 75s. 45d.=(by taking the shillings from the pence, and the pounds from the shillings, and placing them in the shillings and pounds respectively,) £.168 18s. 9d. and this will be true when the multiplicand is any compound number whatever.

for carrying from one denomination to another as in Compound Addition.

INTRODUCTORY EXAMPLES.

	1.			2.			3.				4.		
	£.	s.	d.	£.	s.	d.	D.	d.	c.	m.	£.	s.	d.
Multiply	15	17	1	25	12	8	8	5	1	7	67	18	$6\frac{1}{2}$
by			2			3				4			5

5.		6.			7.			8.				
D.	c.	£.	s.	d.	£.	s.	d.	E.	D.	d.	c.	m.
4	75	13	12	11	31	16	$8\frac{1}{4}$	2	7	8	9	1
	6			7			8					9

9.			10.			11.			12.		
£.	s.	d.	£.	s.	d.	D.	c.	m.	£.	s.	d.
4	13	$4\frac{3}{4}$	8	15	$11\frac{3}{4}$	35	87	5	14	17	$18\frac{3}{4}$
		10			11			12			9
									133	19	$2\frac{1}{4}$

In the last example, I say, 9 times 1 is 9 farthings$=2\frac{1}{4}$d. I set down $\frac{1}{4}$ and carry 2, saying, 9 times 8 is 72, and 2 I carry makes 74 pence, =6s. 2d. I set down 2 in the pence and carry 6; then, 9 times 7 (the unit figure in the shillings) is 63, and 6 I carry is 69, I set down 9 under the unit figure of the shillings, and carry 6, saying, 9 times 1 is 9, and 6 I carry is 15, then I halve 15, which is 7 and 1 over, which I set in the ten's place in the shillings, and that, with the 9, makes 19 shillings; then I carry the 7 as pounds: Lastly, 9 times 4 is 36, and 7 I carry, are 43 pounds: I set down 3 and carry 4, saying, 9 times 1 is 9, and 4 I carry makes 13, which I set down, and the product is £.133 19s. $2\frac{1}{4}$d.

PRACTICAL QUESTIONS.

Note. The facility of reckoning in the *Federal money*, compared with *pounds*, *shillings*, &c. may be seen from the examples in this and the following cases; where the same questions are given in both the currencies, as near as can be, avoiding small fractions. It may be observed, that the variety of cases here given is applicable only to the *old* currency, while the same questions in the Federal are solved by plain decimals.

1. What will 9 yards of cloth at $\left\{\begin{matrix}\text{5s. 4d.}\\ \text{88c. 9m.}\end{matrix}\right\}$ per yard, come to?

	£.0	5s.	4d.	price of one yard,	·88c.	9m.
Multiplied by			9	yards.		9
Ans.	£.2	8	0	price of 9 yds.	D. 8 ·00	1

2. 3 yards at $\left\{\begin{matrix}\text{15s. 4d.}\\ \text{2D. 55c. 6m.}\end{matrix}\right\}$ per yard$=\left\{\begin{matrix}\text{£.2 6s.}\\ \text{D.7 66c. 8m.}\end{matrix}\right\}$

3. 6 - - - $\left\{\begin{matrix}\text{9s. 10d.}\\ \text{D.1 63c. 9m.}\end{matrix}\right\}$ - - - $=\left\{\begin{matrix}\text{£.2 19s}\\ \text{D.9 83c. 4m.}\end{matrix}\right\}$

4. 9

4. 9 - - - { 13s. 7½d. / D.2 27c. 1m. } - - = { £.6 2s. 7½d. / D.20 43c. 9m. }

CASE II.

When the multiplier, that is, the quantity, is above 12: You must multiply by two such numbers, as, when multiplied together, will produce the given quantity.

Examples.

1. What will 144 yards of cloth cost, at { 3s. 5¼d / 57c. 6⅔m. } per yard?

	£. s. d.		c. m.
	0 3 5¼	price of 1 yard	·5764
Multiply by	12		144
Produces	2 1 6	price of 12 yards.	23056
Multiplied by	12		23056
			5764
Answer	£.24 18 0	price of 144 yards.	
			Ans. D.83·0016

Questions.			Answers.
2. 24 yards, at	{ 6s. 3¾d. / D.1 5c. 2m. }	per yard=	{ £.7 11s. 6 / D.25 24c. 8m. }
3. 27 ——	{ 9s. 10d. / D.1 63c. 9m. }	——	{ £.13 5s. 6d. / D.44 21c. 3m. }
4. 44 ——	{ 12s. 4½d. / D.2 6c. 2¼m. }	——	{ £.27 4s. 6d. / D.90 75c. }

CASE III.

When the quantity is such a number, as that no two numbers in the table will produce it, exactly: Then multiply by two such numbers as come the nearest to it; and for the number wanting, multiply the given price of one yard by the said number of yards wanting, and add the products together for the answer; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

Examples.

1. What will 47 yards of cloth, at { 17s. 9d. / D.2 95c. 8m. } per yard, come to?

	£. s. d.		
	0 17 9	price of 1 yard.	D.2·958
Multiplied by	5		47
Produces	4 8 9	price of 5 yards.	20706
Multiplied by	9		11832
Produces	39 18 9	price of 45 yards.	Ans. D.139·026
Add	1 15 6	price of 2 yards.	
Ans.	£.41 14 3	price of 47 yards.	

Note.

Note. This may be performed by first finding the value of 48 yards, from which if you subtract the price of 1, the remainder will be the answer as above.

Questions.				Answers.
2. 75 yards, at	5s. 7½d. / 93c. 7½m.	per yard=		£.21 1s. 10½d. / D.70 31c. 2½m.
3. 67½	—	16s. 3¼d. / 2D. 71c. 1⅘m	— =	£.54 18s. 3¾d. / D.183 4c. 6½m
4. 59	—	9s. 7d. / D.1 59c. 7m.	— =	£.28 5s. 5d. / D.94 22c. 3m

CASE IV.

When the quantity is any number above the Multiplication Table: Multiply the price of 1 yard by 10, which will produce the price of 10 yards: *This* product, multiplied by 10, will give the price of 100 yards; then, you must multiply the price of one hundred by the number of hundreds in your question; the price of ten, by the number of tens; and the price of unity, or 1, by the number of units: Lastly, add these several products together, and the sum will be the answer.

Examples.

1. What will 359 yards of cloth, at {4s. 7½d. / 77c. 1m.} per yard, amount to?

	£.	s.	d.		c. m.
	0	4	7½	price of 1 yard.	·771
					359
			10		
					6939
	2	6	3	price of 10 yards.	3855
			10		2313
	23	2	6	price 100 yds.	Ans. D.276·789
			3		
	69	7	6	price of 300 yards.	
5 times the price of 10 yds.=	11	11	3	price of 50 yards.	
9 times the price of 1 yd.=	2	1	7½	price of 9 yards.	
Answer	£.83	0	4½	price of 359 yards.	

2. 297 yards at	17s. 3½d. / D 2 88c. 2m.	per yard=		£.256 15s. 7½d. / D.855 95c. 4m.
3. 473	—	9s. 11¼d. / D.1 65c. 6¼m.	— =	£.235 0s. 5¼d. / D.783 40c. 6¼m.
4. 512	—	15s. 10d. / D 2 63c. 9m.	— =	£.405 6s. 8d. / D.1351 16c. 8m.
5. 765	—	19s. 9½d. / D.3 29c. 9m.	— =	£.757 0s. 7½d. / D.2523 73c 5m.

CASE V.

To find the value of one hundred weight: As 112 is the gross hundred, so 112 farthings are = 2s 4d and 112 pence = 9s. 4d. ; therefore, if the price be farthings, or not more than 3d multiply 2s. 4d. by the farthings in the price of 1 lb or, if the price be pence, multiply 9s. 4d. by the pence in the price of 1 lb. and in either case, the product will be the answer.

EXAMPLES.

1. What will 1 Cwt. of chalk come to at { $1\frac{1}{2}$d. / 2c. 1m. } per pound?

112 farthings = 0 2 4 price of 1 Cwt. at $\frac{1}{4}$ per lb.
$1\frac{1}{2}$d. = 6 farthings in the price.

Answer £.0 14 0 price of 1 Cwt. at $1\frac{1}{2}$ per lb.

·021
112
42
21
21
Ans. 2·352

2. 1Cwt. of tin at $2\frac{1}{4}$d. per lb.?

	s.	d.	
	2	4	price of 1 Cwt. at $\frac{1}{4}$d. per lb.
		9	farthings in the price of 1 lb.
Ans. £.1	1	0	price of 1 Cwt. at $2\frac{1}{4}$ per lb.

·03125
112
6250
3125
3125
Ans. D.3·50000

3. 1 Cwt. of lead at { 7d. / 9c. 8m. } lb.?

	s.	d.	
	9	4	price of 1 Cwt. at 1d. per lb.
		7	pence in the price of 1 lb.
£.3	5	4	price of 1 Cwt. at 7d. per lb.

·098
112
196
98
98
Ans. D.10·976

Questions.			Answers.
4. 1 Cwt. of copper at	{ $0\frac{3}{4}$d. / 1c. 0·4m. }	per lb. =	{ £.0 7s. / D.1 16c. 5m. }
5. 1 ——	{ $2\frac{1}{2}$d. $\frac{1}{2}$q. / 3c. $6\frac{1}{2}$m. }	—— =	{ £.1 4s. 6d. / D.4 8c. 8m. }
6. 1 ——	{ $4\frac{1}{2}$d. / $6\frac{1}{4}$c. }	—— =	{ £.2 2s. / D.7 }

CASE VI.

To find the value of two, or more hundreds, by having the price of one pound: First, find the price of 1 Cwt. by the last Case, and then proceed to find the value of the whole by Case 1st. or 2d. as the question may require.

EXAMPLES.

EXAMPLES.

1. What is the value of $5\frac{1}{4}$cwt. of sugar at $\left\{\begin{array}{c}6d.\\ 8c.\ 3\frac{1}{3}m.\end{array}\right\}$ per lb.

	£.	s.	d.		Cwt.	qr.	D.
	0	9	4	price of 1 cwt. at 1d. per lb.	5	1	·083$\frac{1}{3}$.
			6		4		588
	2	16	0	price of ditto at 6d. per lb.	21		664
			5		28		664
							415
	14	0	0	price of 5cwt.	168		196
1 qr. =	0	14	0	price of $\frac{1}{4}$cwt.	42		
							D.49·000 Ans.
Ans. £.	14	14	0	price of $5\frac{1}{4}$cwt.	588 lb.		

Questions.. Answers.

2. 4 cwt. of sugar at $\left\{\begin{array}{c}2\frac{1}{2}d.\\ 3\frac{1}{2}c.\end{array}\right\}$ per pound = $\left\{\begin{array}{l}£4\ 13s.\ 4d.\\ D\ 15\ 68c.\end{array}\right\}$

3. $8\frac{1}{2}$ —— $\left\{\begin{array}{c}5d.\\ 7c.\end{array}\right\}$ —— $\left\{\begin{array}{l}£19\ 16s.\ 8d.\\ D\ 66\ 64c.\end{array}\right\}$

4. 7 —— $\left\{\begin{array}{c}4\frac{3}{4}d.\\ 6c.\ 6m.\end{array}\right\}$ —— $\left\{\begin{array}{l}15\ \ 10s.\ 4d\\ D.51\ 74c.\ 4m.\end{array}\right\}$

CASE VI.

To find the value of a hundred weight, when the price of 1 lb. *is any number of pounds and shillings ; or shillings, pence and farthings* : Multiply the price of 1 lb. by 7, its product by 8, and *this* product by 2 ; which last product will be the answer required.

EXAMPLES.

1. What will 1 cwt. of tobacco cost at $\left\{\begin{array}{c}5s.\ 7\frac{1}{2}d.\\ 93c.\ 7\frac{1}{2}m.\end{array}\right\}$ per lb.

	£.	s.	d.		D.
	0	5	$7\frac{1}{2}$	price of 1 lb.	·9375
			7		112
	1	19	$4\frac{1}{2}$	price of 7 lb.	18750
			8		9375
					9375
	15	15	0	price of 56 lb. or $\frac{1}{2}$ cwt.	
			2		D.105· Ans.
Ans. £.	31	10	0	price of 112 lb. or 1 cwt.	

Questions. Answers.

2. 1 Cwt. at $\left\{\begin{array}{c}3s.\ \ 10\frac{1}{2}d.\\ 64c.\ \ 6m.\end{array}\right\}$ per lb. = $\left\{\begin{array}{c}£.21\ \ 14s.\\ D.72\ \ 35c.\ \ 2m.\end{array}\right\}$

3. 1 — $\left\{\begin{array}{c}9s.\ \ 6d.\\ D.1\ \ 58\frac{1}{3}c.\end{array}\right\}$ — = $\left\{\begin{array}{c}£.53\ \ 4s.\\ D.177\ \ 33\frac{1}{3}c.\end{array}\right\}$

4. 1 cwt.

4. 1 Cwt. at	{ 16s. 11½d. / D.2 82c. 6m. }	— =	{ £.94 19s. 4d. / D.316 51c. 2m. }
5. 1 —	{ 19s. 8¼d. / D.3 28c. 1¼m. }	— =	{ £.110 5s. / D.367 50c. }
6. 1 —	{ £.1 7s. 10d. / D.4 63c. 9m. }	— =	{ £.155 17s. 4d / D.519 56c 8m }

PRACTICAL QUESTIONS IN WEIGHTS AND MEASURES.

1. What is the weight of 4 hogsheads of sugar, each weighing 7cwt. 3qrs. 19lb. ? Ans. 31cwt. 2qrs. 20lb.

2. What is the weight of 6 chests of tea, each weighing 3cwt. 2qrs. 9lb. ? Ans. 21cwt. 1qr 26lb.

3. If I am possessed of 1½ dozen of silver spoons, each weighing 3oz. 5pwt.—2 dozen of tea spoons, each weighing 15pwt. 14gr.—3 silver cans, each 9oz. 7pwt —2 silver tankards, each 21oz. 15pwt. and 6 silver porringers, each 11oz. 18pwt. Pray, what is the weight of the whole ? Ans. 18 lb. 4oz. 3pwt.

4. In 35 pieces of cloth, each measuring 27¾ yards, how many yards ? Ans. 971¼ yards.

5. How much brandy in 9 casks, each containing 45gal. 3qts 1pt ? Ans. 412gal. 3qts. 1pt.

6. If I have 9 fields, each of which contains 12 acres, 2 roods and 25 poles ; how many acres are there in the whole ? Ans. 113A. 3r. 25p.

COMPOUND DIVISION*

IS the dividing of numbers of different denominations : In doing which, always begin at the highest, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and so on, till you have divided the whole, taking care to set down your quotient figures under their respective denominations.

INTRODUCTORY EXAMPLES.

1.	2.	3.
£. s. d.	D. d. c. m.	£. s. d.
Divide 549 17 9 by 5	3)14 1 9 6	4)731 5 10½
Quot.£. 109 19 6½		

4.

* To divide a number consisting of several denominations by any simple number whatever, is the same as dividing all the parts or members of which that number is composed by the same number. And this will be true when any of the parts are not an exact multiple of the divisor ; for, by conceiving the number, by which it exceeds that multiple, to have its proper value by being placed in the next lower denomination, the dividend will still be divided into parts, and the true quotient found as before : Thus £.41 17s. 6d. divided by 6, will be the same as £.36 117s. 42d. divided by 6, which is equal to £.6 19s. 7d. as by the rule.

4.	5.	6.	7.
£. s. d.	£. s. d.	D. c. m.	£. s. d.
2)97 19 $10\frac{1}{2}$	6)37 11 $4\frac{3}{4}$	7)25 49 4	8)739 12 $1\frac{1}{2}$

8	9	10	11.
D. c.	£. s. d.	£. s. d.	E. D. d. c. m.
10)37 50	10)79 13 $9\frac{1}{2}$	11)58 19 $11\frac{1}{2}$	12)3 9 8 7 5

In the first example, having divided the pounds, the 4, which remains, is 4 pounds, which are equal to 80 shillings, and 17 in the shillings make 97 ; I then find 5 is contained 19 times in 97, and 2 over : I set down 19 under the shillings, and reduce the 2 shillings, which remain, into pence, and they make 24, and the 9 pence, in the question, added, make 33 : Then, how often 5 in 33 ; I find it 6 times, and 3 over : I set down 6 under the pence, and reduce the 3 pence, which remain, to farthings, and they make 12 ; then, how often 5 in 12 ; I find it to be twice : I therefore set down $\frac{1}{2}$d. and the 2 which remains, is $\frac{2}{5}$ of a farthing, which I make no account of.

12.	13.	14.	15.
T. cwt. qr. ℔ oz. dr.	T. cwt. qr. ℔	cwt. qr. ℔	℔ oz. dr.
3)29 13 2 25 12 13	4)6 11 3 19	5)14 1 12	6)10 13 9

16.	17.	18.	19.
℔ oz.	℔ oz. pwt. gr.	℔ oz. pwt gr.	lb. oz pwt. gr.
7)20 13	8)7 10 15 2	9)56 9 13 19	10)849 11 12 14

20.	21.	22.
M. w. d. h. m.	M. d. h. m.	
6)6 3 5 10 29	7)9 21 12 45	8)3s. 25° 55′ 25″

23.	24.
9)19° 45′ 38″	12)189° 37′ 29″

25. Suppose that two places lie east and west of each other, and it is found by observation that it is noon at the former 2 hours, 6′ 30″ sooner than at the latter ; how many degrees are they asunder ?

4′)2h. 6′ 30″

31° 37′ 30″ Ans.

Reduce the hours and minutes to minutes, then, minutes divided by 4′ give degrees in the quotient.

26. The longitude of Cambridge is 4h 44′ 17″, and that of Philadelphia, 5h 0′ 43″ ; how many degrees difference ?

5h 0′ 43″
4 44 17

4′)0 16 26

4° 6′ 30″ Ans.

PRACTICAL

PRACTICAL QUESTIONS.

CASE I.

The same note, which was given under Practical Questions in Compound Multiplication, is applicable in this place.

Having the price of any number of yards, &c. within the pence table, to find the price of unity, or 1 *yard* : If the quantity do not exceed 12, proceed by setting down the price, and dividing it by the quantity ; which quotient will be the price of one yard, required ; but if the quantity exceed 12, then divide by 2 such numbers, as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

Note. This case proves the first and second cases in Compound Multiplication.

EXAMPLES.

1. If 9 yards of cloth cost $\left\{\begin{matrix}£.4\ \ 3s.\ \ 7\frac{1}{2}d. \\ D.\ 13.\ \ 93\frac{3}{4}c.\end{matrix}\right\}$ what is it per yard ?

£.	s.	d.		D.
9)4	3	$7\frac{1}{2}$		9)13 9375
0	9	$3\frac{1}{2}$ Ans.		D.1·5486+ Ans.

2. If 7 ells cost $\left\{\begin{matrix}£.5\ \ 17s.\ \ 5d. \\ D.19\ \ 57c.\end{matrix}\right\}$ what cost 1 ell? Ans. $\left\{\begin{matrix}16s.\ \ 9\frac{1}{4}d. \\ D.2.796\end{matrix}\right\}$

3. If 11 sheep cost $\left\{\begin{matrix}£.6\ \ 5s.\ \ 9d. \\ D.20\ \ 95c.\ \ 8m.\end{matrix}\right\}$ what did each cost ?

Ans. $\left\{\begin{matrix}11s.\ \ 5\frac{1}{4}d. \\ D.1.905+\end{matrix}\right\}$

4. If 12 gallons of rum cost $\left\{\begin{matrix}£.8\ \ 11s.\ \ 9\frac{1}{2}d. \\ D.28\ \ 63c.\ \ 2m.\end{matrix}\right\}$ what is it per gallon ?

Ans. $\left\{\begin{matrix}14s.\ \ 3\frac{3}{4}d. \\ D.2\ \ 38c.\ \ 6m.\end{matrix}\right\}$

5. If 84 cows cost $\left\{\begin{matrix}£.253\ \ 13s. \\ D.845\ \ 50c.\end{matrix}\right\}$ what is the price of each ?

Ans. $\left\{\begin{matrix}£.3\ \ 0s.\ \ 4\frac{2}{3}d. \\ D.10\ \ 6\frac{1}{2}c.\end{matrix}\right\}$

6. If 132 bushels of corn cost $\left\{\begin{matrix}£.20\ \ 12s.\ \ 6d. \\ D.68\ \ 75c.\end{matrix}\right\}$ what is that per per bushel ?

Ans. $\left\{\begin{matrix}3s.\ \ 1\frac{1}{2}d \\ 52c.\ \ 1m\end{matrix}\right\}$

Note. When the given quantity, or divisor, is large, and not a composite number, the operation may be performed by Long Division.

CASE II.

Having the price of a hundred weight, to find the price of 1 lb : Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1 lb. required.

EXAMPLES.

EXAMPLES.

1. If 1 cwt. of flax cost $\left\{\begin{array}{l}£\,2\ \ 7s.\ \ 8d.\\ D.7\ \ 94c.\ \ 4m\end{array}\right\}$ what is that per lb. ?

8)2l. 7s. 8d.
———
7)0 5 11$\frac{1}{2}$
———
2)0 0 10d.0$\frac{6}{7}$qr.
———
0 0 5$\frac{3}{28}$d price of 1 pound.

112)7·944(·071—
784
———
104
Ans. 7c. 1m. (nearly.)

2. At $\left\{\begin{array}{l}£\,3\ \ \ 10s\\ D\ 11\ 67c.\end{array}\right\}$ per cwt. what cost 1 lb ? Ans $\left\{\begin{array}{r}7\frac{1}{2}d.\\ 10c.\ \ 4m.\end{array}\right\}$

3. At $\left\{\begin{array}{l}£.6\ \ 6s.\\ D\ 21\end{array}\right\}$ per cwt. what cost 1 lb. ? Ans $\left\{\begin{array}{l}1s.\ \ 1\frac{1}{2}d.\\ 18c\ \ 7\frac{1}{2}m.\end{array}\right\}$

4. At $\left\{\begin{array}{l}£\,42\ \ 11s.\ \ 8d.\\ D.141\ \ 94c.\ \ 4m.\end{array}\right\}$ per cwt. what cost 1 lb. ?

Ans. $\left\{\begin{array}{l}7s.\ \ \ 7\frac{1}{4}d.\\ D.1\ \ 26c.\ \ 7m.\end{array}\right\}$

CASE III.

Having the price of several hundred weight, to find the price per lb : Divide the whole price by the number of hundreds, which will give the price per cwt. and then proceed as in the last Case.

EXAMPLES.

1. If 5cwt. of sugar cost $\left\{\begin{array}{l}£.13\ \ 8s.\ \ 4d.\\ D.44\ \ 72c.\ \ 2m.\end{array}\right\}$ what is that per lb. ?

5)13l. 8s. 4d.
———
8) 2 13 8 price of 1 cwt.
———
7) 0 6 8$\frac{1}{2}$d. price of 14 lb. or $\frac{1}{8}$cwt.
———
2) 0 0 11$\frac{1}{2}$d. price of 2 lb. or $\frac{1}{56}$ cwt.
———
0 0 5$\frac{3}{4}$ price of 1 lb.
———

112lb. in a cwt.
5cwt. c.m.
56|0)44·722(·079$\frac{7}{8}$—,or 8c.
392 [nearly, Ans.
———
552
504
———
482
——=$\frac{7}{8}$ nearly.
560

2. If 8 cwt. of cocoa cost $\left\{\begin{array}{l}£.15\ \ 17s.\ \ 4d.\\ D.52\ \ 88c.\ \ 9m\end{array}\right\}$ what is that per lb. ?

Ans. $\left\{\begin{array}{r}4\frac{1}{4}d.\\ 5c.\ \ 9m.+\end{array}\right\}$

3. If 3$\frac{1}{4}$ cwt. of sugar cost $\left\{\begin{array}{l}£\,9\ \ 17s.\ \ 2d.\\ D.32\ \ \ 86c.\end{array}\right\}$ what is that per lb ?

Ans. $\left\{\begin{array}{l}6\frac{1}{2}d.\\ 9c.+\end{array}\right\}$

4. If 1$\frac{3}{4}$ cwt. of cotton wool cost $\left\{\begin{array}{l}£.6\ \ 10s.\ \ 8d.\\ D.21\ \ 77c.\ \ 8m.\end{array}\right\}$ what is that per. lb ?

Ans. $\left\{\begin{array}{r}8d.\\ 11c.\ \ 1m+\end{array}\right\}$

Note. This Case proves the 6th in Compound Multiplication.

CASE

CASE IV.

Having the price of any number of yards, &c. to find the price of 1 *yard:* Divide the price by the quantity, beginning at the highest denomination, and, if any thing remain, reduce it into the next, and every inferiour denomination, and, at each reduction, divide as before, remembering each time, to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed; or, in federal money, work by decimals.

Examples.

1. If $95\frac{1}{2}$lb. of figs cost $\left\{\begin{array}{l}\text{£.16 13s. } 6\frac{3}{4}\text{d.}\\ \text{D.55 59c. } 3\frac{3}{4}\text{m.}\end{array}\right\}$ what are they per lb.?

	lb		£.	s.	d.
Quantity =	$95\frac{1}{2}$	Price =	16	13	$6\frac{3}{4}$
Mult by	4				4
Produces	382 for a divisor.	Product	£.66	14	3 for a dividend.

382)66 14 3 (0 3 $5\frac{3}{4}$ $\frac{250}{382}$ per lb.
20
———
382)1334(3
1146
———
188
12
———
382)2259(5
1910
———
349
4
———
382)1396(3
1146
———
250

D. c.m.dec. c. m.
$95\frac{1}{2}$=95·5)55.59375(·58 2+Ans.
4775
———
7843
7640
———
2037
1910
———
1275

2. If 147 bushels of rye cost $\left\{\begin{array}{l}\text{£.47 12s. 6d.}\\ \text{D.158 76c.}\end{array}\right\}$ what is it per bushel?

Ans. $\left\{\begin{array}{l}\text{6s. } 5\frac{3}{4}\text{d}\\ \text{D.1 8c.}\end{array}\right\}$

3. If $33\frac{1}{4}$ yards of baize cost $\left\{\begin{array}{l}\text{£.25 13s. } 9\frac{1}{2}\text{d.}\\ \text{D.85 63c. 2m.}\end{array}\right\}$ what is it per yd.?

Ans. $\left\{\begin{array}{l}\text{15s. } 5\frac{1}{4}\text{d. } \frac{95}{133}\\ \text{D.2. 57c. 5m.}\end{array}\right\}$

Note. This proves the 3d and 4th Cases in Multiplication.

PRACTICAL

PRACTICAL QUESTIONS IN MONEY.

1. Divide { £. 273 9s 4d / D. 911 55c 5m. } among 5 men and 4 women, and give the men twice as much as the women.

Men. Women.
5 and 4
Mult. by 2
—
10 ſhares.
Add 4 women's ſhares.
—
14 the number of equal ſhares in the whole=Diviſor.

£. s. d. £. s. d.
Divide by 14)273 9 4(19 10 8=1 woman's ſhare.
14 — 4 women.
—
133 — 78 2 8=women's ſhare.
126
— £.19 10 8
7 — 2
20
— £.39 1 4=1 man's ſhare.
14)149(10 — 5 men.
14
— £.195 6 8=men's ſhare.
9 — 78 2 8=women's ſhare.
12
— £.273 9 4 Proof.
14)112(8
112

D.
14)911·555(65·111+ =1 woman's ſhare.
84 — 4 women.
—
71 — 260·444=women's ſhare.
70
— 65·111+
15 — 2
14
— 130·222+ =1 man's ſhare.
15 — 5
14
15 — 651·111+ =men's ſhare.
14 — 260·444+ =women's ſhare.
—
1 — 911·555+ Proof.

2. Divide { £. 120 17s. 4d. / D. 402 88c. 9m. } among 7 men and 7 women, and give the women 3 times so much as the men.

Answer, { { £. 4 6s 4d. / D. 14 38c 9m. } = a man's share. { £ 12 19s. / D. 43 16c 7m. } = a woman's share. }

3. Divide { £. 39 12s. 5d. / D. 132 7c. } among 4 men, 6 women, and 9 boys: Give each man double to a woman, and each woman double to a boy.

Answer, { { £.1 1s. 5d. / D.3 57c.— } = a boy's share. { £.2 2s. 10d. / D.7 14c.— } = a woman's ditto. { £.4 5s. 8d. / D.14 28c.— } = a man's ditto. }

4. Divide 5 guineas among 8 men :—Give A 8d. more than B, and B 8d. more than C, &c. Ans. H's share =15s. 2d.

REDUCTION OF COINS.

RULES for reducing the Federal Coin, and the currencies of the several United States ; also English, Irish, Canada, Nova-Scotia, Livres Tournois and Spanish milled Dollars, each to the *par* of all the others.

I. *To reduce New-Hampshire, Massachusetts, Rhode-Island, Connecticut, and Virginia currency* :

1. *To Federal Money.*

Rule.—Reduce the shillings, pence and farthings, to decimals, by Inspection (Case 3d, Dec. Frac.) divide the whole by 3, putting the comma one figure further to the right hand in the quotient, than in the pounds of the dividend, and the quotient will be the answer in dollars, cents and mills.

1. Reduce £.349 19s. 1d. to dollars.

·9 = $\frac{1}{2}$ the shillings.
·05 = odd shillings.
·004 = qrs. in 1d.

·954 = decimal.

3)349·954 D. c. m.
1166·513=1166 51 3 Ans.

2. Reduce 19s. $1\frac{3}{4}$d. to dollars.

·9
·05
·007

·957 = decimal.

3)·957 D. c.
3·19 = 3 19

3. Reduce 1s. to cents.

1s. = ·05 then
3)0 5 c. m.
0· $166\frac{2}{3}$ = 16 $6\frac{2}{3}$

4. Reduce 1d.

1d. = 4qrs.
3)·004 c. m.
0·01 $3\frac{1}{3}$ = 1 $3\frac{1}{3}$

5. Reduce 1 qr.

1qr. = ·001041 and
3)·0 01 041
0·00 347 = $3\frac{47}{100}$ mills.

3. *To New-York and North-Carolina currency.*

Rule.—Add one third to the New-Hampshire, &c. sum, and the sum total will be the New-York, &c. currency.

Reduce £.100 New-Hampshire, &c. to New-York, &c.

£.
3)100
+ 33 6 8

£.133 6 8 Ans.

3. *To Pennsylvania, New-Jersey, Delaware and Maryland currency.*

Rule.—Add one fourth to the New-Hampshire, &c. sum.

Reduce £.100 New-Hampshire, &c. to Pennsylvania, &c.

4)100
+ 25

£.125 Ans.

4. *To South-Carolina and Georgia currency.*

Rule.—Multiply the New-Hampshire, &c. sum by 7, and divide the product by 9, and the quotient is the answer.

Reduce £.100 New-Hampshire, &c. to South-Carolina, &c.

100
7

9)700
£.77 15 $6\frac{2}{3}$ Ans.

5. To

5. *To English Money.*

Rule.—Deduct one 4th from the New-Hamshire, &c. sum.

Reduce £.100 New-Hampshire, &c. to English Money.

```
4)100
 - 25
 ----
£.75 Answer.
```

6. *To Irish Money.*

Rule.—Multiply the New-Hampshire, &c. sum by 13, and divide the product by 16.

Reduce £.100 New-Hampshire, &c. to Irish Money.

```
       100
       ---
       4×3+ the given sum.
       ---
       400
         3
      ----
      1200
      +100
      ----
16 = 4×4)1300
      ----
      4)325
      ----
     £.81 5 Ans.
```

7. *To Canada and Nova-Scotia currency.*

Rule. Multiply the New-Hampshire, &c. sum by 5, and divide the product by 6

Reduce £.100 New-Hampshire, &c. to Canada, &c.

```
  100
    5
  ---
6)500
  ---
£ 83  6  8 Answer.
```

8. *To livres Tournois.*

Note. 12 deniers, or pence, make 1 sol, or shilling, 20 sols, or sous, 1 livre, or pound.

Rule. Multiply the New-Hampshire, &c. pounds, by $17\frac{1}{2}$, and the product will be livres : Or, multiply the sum in shillings by 7 : Divide the product by 8, and the quotient will be livres, sous, &c.

Reduce £ 100 New-Hampshire &c. to Livres Tournois.

```
     100        Or, 100
     17½             20
    ----          -----
     700           2000
     100              7
      50          -----
    ----        8)14000
Ans. 1750 liv.    -----
    ----    Ans. 1750 livres.
```

1d.=1sou. $5\frac{1}{3}$den. 1s.=$17\frac{1}{2}$sous. 1£.=$17\frac{1}{2}$ livres.

II. *To reduce Federal Money to New-England and Virginia currency.*

Rule. Multiply the Federal money by 3, and if it consist of dollars only, cut off 1 figure, if of cents also, cut off 3, and if of mills, 4 figures at the right hand ; then reduce the figures so cut off to farthings each time cutting off as at first and the left hand figures are pounds, shillings, &c Or, reduce them by inspection.

1. Reduce 1166dolls. 51c. 3m. to New England currency.

```
      D   c. m.
   1166·51 3
           3
   ---------
 £.349·953 9
          20
   ---------
   s.19·0780
          12
        ----
      ·936 =1d. nearly.
Or, 18s.   = double of 9.
    1s.    = 5 in the 2d place.
     1d    = 3·9 or 4qrs that
                        [remain.
   ------
 19s 1d. Ans.
```

2. Reduce 45 dollars.

```
     45
      3
   ----
 £.13·5
     20
   ----
 s.10·0
```

3. Reduce

3. Reduce 12D. 7c. to lawful money.

```
  D  c.
  12·07
      3
  -----
£.3·621
     20
  -----
s.12·420
     12
  -----
d 5·040
```

III. *To reduce New-Jersey, Pennsylvania, Delaware and Maryland currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut, and Virginia currency.*

Rule. Deduct one fifth from the New-Jersey, &c. sum, and the remainder will be New-Hampshire, &c. currency.

Reduce £.100 New-Jersey, &c. to New-Hampshire, &c.

```
5)100
— 20
  --
£ 80 Answer.
```

2. *To New-York and North-Carolina currency.*

Rule. Add one fifteenth to the New-Jersey, &c. sum.

Reduce £ 100 New-Jersey, &c. to New-York, &c.

```
15=3×5)100
       ---
      3)20
       ---
     + 6 13 4+giv. sum.
     ----------
£.106 13 4 the Answer.
```

3. *To South-Carolina and Georgia currency.*

Rule. Multiply the New-Jersey, &c. sum by 28, and divide the product by 45, and the quotient is South-Carolina, &c.

Reduce £ 100 New-Jersey, &c. to South-Carolina, &c.

```
          100
          ---
          4×7=28
          ---
          400
            7
          ---
45=5×9)2800
          ---
        5)311 2 2⅔
          --------
      £.62 4 5⅓ Ans.
```

4. *To English Money.*

Rule. Multiply the New-Jersey, &c by 3, and divide the product by 5

Reduce £.100 New-Jersey, &c. to English money.

```
   100
     3
   ---
 5)300
   ---
£.60 Answer.
```

5 *To Irish Money.*

Rule. Multiply the New-Jersey, &c. by 13, and divide the product by 20

Reduce 100l. New-Jersey, &c. to Irish.

```
          100
          ---
          4×3+the giv. sum.
          ---
          400
            3
          ---
         1200
         +100
          ---
20=4×5)1300
          ---
        4)260
          ---
         65l. Answer.
```

6. *To Canada and Nova-Scotia currency.*

Rule. Deduct one third from the New-Jersey, &c.

Reduce

Reduce 100l. New-Jersey, &c. to Canada, &c.

```
3)100
— 33  6 8
  ———————
£ 66 13 4 Ans.
```

7. *To Livres Tournois.*

Rule Multiply the New-Jersey, &c. pounds by 14, and the product will be Livres Tournois—or multiply the sum in shillings by 7; divide the product by 10, and the quotient will be livres, sous, &c.

Reduce 100l New-Jersey, &c. to Livres Tournois

```
 100       Or, 100 ⎫ 1d.=1 1/6 sous.
  14            20 ⎬ 1s.=14 sous.
 ———          ———— ⎭ 1£.=14liv.
 400          2000
 100             7
 ———          ————
Ans. 1400 liv. 10)14000
              —————
               1400
```

8. *To Spanish milled dollars.*

Rule. Multiply the New-Jersey, &c. pounds by $2\frac{2}{3}$ and the product will be dollars.—Or, multiply them by 8: Divide the product by 3, and the quotient will be dollars.—If there be shillings in the given sum, for every 7s. 6d. add 1 dollar to the quotient.

Reduce 100l. 10s. New-Jersey, &c. to dollars.

```
  100            Or 100
    8                 2
  ———               ———
3)800               200
  ———     100×2/3=  66 2/3
  266 2/3   10s.=    1 1/3
10s.=1 1/3          ———
  ———               268 as be-
Ans. 268 dol.            [fore.
```

IV. *To reduce New-York and North-Carolina currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule. Deduct one fourth from the New-York, &c.

Reduce 100l. New-York, &c. to New-Hampshire, &c.

```
4)100
 —25
 ———
£.75 Answer.
```

2. *To New-Jersey, Pennsylvania, Delaware, and Maryland currency.*

Rule. Deduct one sixteenth from the New-York &c sum.

Reduce 100l. New-York, &c. to New-Jersey, &c.

```
16=4×4)100
        ——
      4)25
        ——
     —£.6  5
      —————
   £. 93 15 Answer.
```

8 *To South-Carolina and Georgia currency.*

Rule. Multiply the New-York, &c sum by 7, and divide the product by 12: The quotient is South-Carolina, &c.

Reduce £ 100 New-York, &c. to South-Carolina, &c.

```
    100
      7
    ———
 12)700
    ————————
    £.58 6 8 Answer.
```

4. *To English Money.*

Rule. Multiply the New-York, &c sum by 9: Divide the product by 16, and the quotient is English.

Reduce £.100 New-York, &c. to English money.

```
        100
          9
        ———
16=4×4)900
        ———
      4)225
        ———
     £.56 5 Answer.
```

5. *To Irish Money.*

Rule. Multiply the New-York, &c.

&c. sum by 39 : Divide the product by 64, and the quotient is Irish.

Reduce 100l. New-York, &c. to Irish money.

100
6×6+thrice the giv. sum
———
600
6
———
3600
+300=100×3
———
64=8×8)3900
———
8)487 10
———
£.60 18 9 Ans.

6. *To Canada and Nova-Scotia currency.*

Rule. Multiply the New-York, &c. sum by 5, and divide the product by 8

Reduce 100l. New-York, &c. to Canada, &c.

100
5
———
8)500
———
£.62 10 Ans.

7. *To Livres Tournois.*

Rule. Multiply the New-York, &c sum in shillings by 21 : Divide the product by 32, and the quotient will be livres, sous, &c.

Reduce 100l. New-York, &c. to Livres Tournois.

100 Note. 1d.=$1\frac{3}{32}$sou.
20 1s.=$13\frac{1}{8}$sou.
——— 1l.=$13\frac{1}{8}$liv.
2000
21
———
2000
4000
———
32=4×8)42000
———
4)5250
Ans. $1312\frac{1}{2}$ livres.

8. *To Spanish milled Dollars.*

Rule. If the New-York sum be pounds only, annex a cypher to them, then divide by 4, and the quotient is dollars : But if it be pounds and shillings, annex half the shillings to the pounds, and divide as before, and the quotient is dollars

Reduce 100l. New-York, &c. to Dollars.

4)1000
———
250 Dolls. Ans.

Reduce 100l. 8s. to Dollars.

4)1004
———
251 Dolls. Ans.

V. *To reduce South-Carolina and Georgia currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule. Multiply the South Carolina, &c. sum by 9, and divide the product by 7.

Reduce 100l South-Carolina, &c. to New-Hampshire, &c.

100
9
———
7)900
———
£.128 11 $5\frac{1}{7}$ Ans.

2. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the South-Carolina, &c. sum by 45, and divide the product by 28.

Reduce 100l. South-Carolina, &c. to New-Jersey, &c.

100
9×5=45
———
900
5
———
28=4×7(4500
4)642 17 $1\frac{5}{7}$
———
£.160 14 $3\frac{3}{7}$ Ans.

3. *To*

3. *To New-York and North-Carolina currency.*

Rule. Multiply the South-Carolina, &c. sum by 12, and divide the product by 7.

Reduce 100l. South-Carolina, &c. to New-York, &c.

$$\begin{array}{r} 100 \\ 12 \\ \hline 7)1200 \\ \hline \end{array}$$

£.171 8 $6\frac{6}{7}$ Ans.

4. *To English Money.*

Rule. From the South-Carolina, &c. sum, deduct one twenty-eighth.

Reduce 100l. South-Carolina, &c. to English Money.

$$\begin{array}{l} 28=4\times7)100 \\ \quad 4)14 \quad 5 \quad 8\frac{4}{7} \\ \hline \quad - 3 \quad 11 \quad 5\frac{1}{7} \text{ from } 100. \\ \hline \end{array}$$

£.96 8 $6\frac{6}{7}$ Ans.

5. *To Irish Money.*

Rule. Multiply the South-Carolina, &c. sum by 117, and divide the product by 112.

Reduce 100l. South-Carolina, &c. to Irish.

$$\begin{array}{r} 100 \\ \hline 12\times9+9 \text{ times} \\ \text{[the giv. sum} \\ 1200 \\ 9 \\ \hline 10800 \\ +100\times9=900 \\ \hline 112=4\times4\times7)11700 \\ \hline 4)1671 \quad 8 \quad 6\frac{6}{7} \\ \hline 4)417 \quad 17 \quad 1\frac{5}{7} \\ \hline \end{array}$$

£.104 9 $3\frac{3}{7}$ Ans.

6. *To Canada and Nova-Scotia currency.*

Rule. Multiply the South-Carolina, &c. sum by 15, and divide the product by 14.

Reduce 100l. South-Carolina, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ \hline 5\times3 \\ \hline 500 \\ 3 \\ \hline 14=2\times7)1500 \\ \hline 2)214 \quad 5 \quad 8\frac{4}{7} \\ \hline \end{array}$$

£.107 2 $10\frac{2}{7}$ Answer.

7. *To Livres Tournois.*

Rule. Multiply the South-Carolina, &c pounds by $22\frac{1}{2}$, and the product will be livres.

Reduce 100l. South-Carolina, &c. to Livres.

$$\begin{array}{r} 100 \\ 22\frac{1}{2} \\ \hline 200 \\ 200 \\ 50 \\ \hline \end{array}$$

Note. 1d.=$1\frac{7}{8}$ sous.
1s =$1\frac{1}{8}$ livre.
1l.=$22\frac{1}{2}$ livres.

Ans. 2250 livres.

8. *To Spanish milled Dollars.*

Rule. Multiply the South-Carolina, &c. pounds by 30, and divide the product by 7, and if there be shillings, turn them into dollars, and add them.

Reduce 100l. South-Carolina, &c. to Dollars.

$$\begin{array}{r} 100 \\ \hline 10\times3=30 \\ \hline 1000 \\ 3 \\ \hline 7)3000 \\ \hline \end{array}$$

Dollars $428\frac{4}{7}$. Note. $\frac{1}{7}$=8d.

VI. *To reduce English Money.*

1. *To New-Hampshire, Massachusetts,*

chusetts, Rhode-Island, Connecticut, and Virginia currency.

Rule. To the English, sum add one third.

Reduce 100l English to New-Hampshire, &c.

3)100
+ 33 6 8

£ 133 6 8 Answer.

2. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the English money by 5, and divide the product by 3

Reduce 100l English, to New-Jersey, &c.

100
5

3)500

£.166 13 4

3. *To New-York and North-Carolina currency.*

Rule. Multiply the English money by 16, and divide the product by 9.

Reduce 100l English, to New-York, &c.

100

4×4

400
4

9)1600

£.177 15 $6\frac{2}{3}$ Answer.

4. *To South-Carolina and Georgia currency.*

Rule. To the English money add one twenty-seventh

Reduce 100l. English, to South-Carolina, &c.

27=3×9)100

3)11 2 $2\frac{2}{3}$

+ 3 14 $0\frac{8}{9}$

£ 103 14 $0\frac{8}{9}$ Ans.

5 *To Irish Money.*

Rule To the English sum add one twelfth.

Reduce 100l. English money to Irish money.

12)100
+ 8 6 8

£.108 6 8 Answer.

6 *To Canada and Nova-Scotia currency*

Rule. To the English sum add one ninth

Reduce 100l. English, to Canada, &c.

9)100
+ 11 2 $2\frac{2}{3}$

£ 111 2 $2\frac{2}{3}$ Answer.

7. *To Livres Tournois.*

Rule. Multiply the English pounds by $23\frac{1}{3}$, and the product will be livres.

Reduce 100l. English, to Livres Tournois.

100 Note. 1d.=$1\frac{7}{8}$sous.
$23\frac{1}{3}$ 1s=$1\frac{1}{6}$livre.
1l.=$23\frac{1}{3}$livres.

300
200
$33\frac{1}{3}$

Liv. sou. den.
Ans $2333\frac{1}{3}$ Liv =2333 6 8

VII *To reduce Irish Money.*

1 *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule Multiply the Irish sum by 16, and divide the product by 13.

Reduce 100l. Irish, to New-Hampshire, &c.

100

100
4×4
—
400
4
13)1600
£ 123 1 $6\frac{6}{13}$ Answer.

2 *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule Multiply the Irish sum by 20, and divide the product by 13.

Reduce 100l. Irish to New-Jersey, &c.

100
4×5=20
—
400
5
13)2000(153 16 $11\frac{1}{13}$ Answer.
13 11 12
— 20 12
70 — —
65 13)220(16 13)144(11
— 13 13
50 — —
39 90 14
— 78 13
11 — —

3. *To New-York and North-Carolina currency.*

Rule. Multiply the Irish sum by 64, and divide the product by 39.

Reduce 100l. Irish to New-York, &c.

100
8×8=64
800
8
— £. s.
39)6400(164 $2\frac{2}{39}$ Answer.
39
250
234
160
156
—
4

4. *To South-Carolina and Georgia currency.*

Rule. Multiply the Irish sum by 112, and divide the product by 117.

Reduce 100l. Irish to South-Carolina, &c.

100
—
7×4×4=112
—
700
4
—
2800
4
— £. s. d.
117)11200(95 14 $6\frac{42}{117}$ Ans.
1053
—
670
585
—
85

5. *To English Money.*

Rule. From the Irish sum deduct one thirteenth.

Reduce 100l. Irish to English money.

13)100(7
91
—
9
20 £. s. d.
— 100 0 0
13)180(13 —7 13 $10\frac{2}{13}$
13
— £.92 6 $1\frac{11}{13}$ Ans.
50
39
—
11
12
—
13)132(10
13
—
2

6 *To Canada and Nova-Scotia currency.*

Rule.

Rule. To the Irish sum add one thirty-ninth.

Reduce 100l. Irish to Canada, &c.

```
39)100(2
    78
    —
    22
    20
    ——
39)440(11
   39        100
   ——        +2 11 3 5/13
   50        ————————
   39   £.102 11 3 5/13 Ans.
   —
   11
   12
   ——
39)132(3
   117
   ——
   15  5
   —=—
   39 13
```

7. *To Livres Tournois.*

Rule. Multiply the Irish sum, in pence, by 70; divide that product by 39, and the quotient will be sous, which, divided by 20, will be livres.

Reduce £.100 Irish to Livres Tournois.

$100 \times 20 \times 12 = 24000$d.

```
              70
        ——————— 2·0
    39)1680000(4307|6
              ———sou.
```

Ans. Livres. 2153 $16\frac{12}{13}$

1d.$=1\frac{31}{39}$ sous 1s $=21\frac{7}{13}$ sous.

1£.$=21$ liv. $10\frac{10}{13}$ sous.

VIII. *To reduce Canada and Nova-Scotia currency.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule. To the Canada, &c. sum add one fifth.

Reduce £.100 Canada, &c. to New-Hampshire, &c.

```
  5)100
  + 20
  ——
£.120 Answer.
```

2. *To New-York and North-Carolina currency.*

Rule. Multiply the Canada, &c. sum by 8, and divide the product by 5.

Reduce £.100 Canada, &c. to New York, &c.

```
   100
     8
   ——
 5)800
   ——
£ 160 Answer.
```

3. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. To the Canada, &c. sum add one half.

Reduce £.100 Canada, &c. to New-Jersey, &c.

```
  2)100
  + 50
  ——
£.150 Answer.
```

4. *To South-Carolina and Georgia currency.*

Rule. From the Canada, &c. sum deduct one fifteenth.

Reduce 100l Canada, &c. to South-Carolina, &c.

```
15=3×5)100
       ——
       3)20
       —
     — 6 13 4
     ————
£.93  6 8 Answer.
```

5. *To English Money.*

Rule. From the Canada, &c. deduct one tenth

Reduce 100l. Canada, &c. to English money.

```
 10)100
   — 10
     —
£.90 Answer.
```

6. To

6. *To Irish Money.*

Rule. From the Canada, &c. deduct one fortieth.

Reduce 100l. Canada, &c. to Irish money.

```
40)100
 — 2 10
 ——
£.97 10 Answer.
```

7. *To Livres Tournois.*

Rule. Multiply the Canada, &c. pounds by 21, and the product will be livres.

Reduce 100l. Canada, &c. to livres Tournois.

```
   100
   ——
   7×3=21
   ——
   700          1d.=1¾sous.
     3          1s.=21sous.
   ——           1l.=21livres.
Ans. 2100
```

8. *To Spanish Milled Dollars.*

Rule. Reduce the Canada, &c. sum to shillings: Divide them by 5, and the quotient is dollars. Or, Multiply the pounds by 4, and the product is dollars: And if there be shillings turn them into dollars, and add them to the product.

Reduce 100l. Canada, &c. to dollars.

```
   100         155 15
    20           4
   ——          ——
 5)2000        620
   ——          + 3=15s.
Doll 400 Ans.  ——
           Doll. 623 Ans.
```

IX *To reduce Livres Tournois.*

1. *To New Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule. Multiply the livres by 2: Divide the product by 35, and the quotient will be pounds. Or, Multiply the livres by 8: Divide the product by 7, and the quotient will be shillings.

Reduce 1750 livres to New-Hampshire, &c. currency.

```
     1750          Or,  1750
        2                  8
     —— £.              ——
35)3500(100 Ans.   7)14000
   35                 ——
   ——               2|0)200|0
   00                 ——
                    £ 100asbef.
```

2. *To New-York and North-Carolina currency.*

Rule. Multiply the livres by 32: Divide the product by 21, and the quotient will be shillings.

Reduce 1312½ livres to New-York, &c. currency.

```
      1312·5
         3 2
      ——
      26250
      39375
      ———2|0
21)42000(200|0
      £ 100 Answer.
```

3. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Divide the livres by 14, and the quotient will be pounds. Or, multiply the livres by 10: Divide the product by 7, and the quotient will be shillings.

Reduce 1400 livres to New-Jersey, &c. currency.

```
      1400
        10
      ——               Or,
   7)14000        14)1400(£.100.
      ——              14
  20)200|0            ——
      ——              00
  £.100 Ans.
```

4. *To South-Carolina and Georgia currency.*

Rule.—Multiply the livres by 2, divide the product by 45, and the quotient will be pounds. Or, deduct one ninth, and the remainder will be shillings.

Reduce

Reduce 2250 livres to South-Carolina, &c. currency.

```
 2250              Or,
    2            9)2250
 ---- £.          — 250
45)4500(100 Ans.  -----
   45           2|0)200 0
   --             -----
   00           £.100 as bef.
```

5. *To English Money.*

Rule.—Multiply the livres by 6: Divide the product by 7, and the quotient is shillings: Or, deduct one seventh from the livres, and the remainder will be shillings.

Reduce 2333⅓ livres to English money.

```
 2333⅓          Or,
     6        7)2333⅓
 -----         — 333⅓
7)14000        ------
  -----       2|0)200|0
2|0)200.0       ----
    ---       £.100 as bef.
Ans. £.100      ---
```

6. *To Irish Money.*

Rule.—Reduce the livres to sous, then multiply them by 39: divide this product by 70, and the quotient will be pence.

Reduce 2153 liv. $16\frac{12}{13}$so. to Irish money.

```
        20
    ------
    43076 12/13
        39
    ------
    387720
   129228
   -------
7|0)168000|0
   -------
 12)24000
    -----
2|0)200|0
    ----
  £.100 Answer.
```

7. *To Spanish milled Dollars, or to Federal Dollars.*

Rule.—Multiply the livres by 4: Divide the product by 21, and the quotient will be Spanish, or Federal Dollars.

Reduce 1000 livres to dollars.

```
 1000                          Or, 1000
    4                                 4
 ----                              ----
21)4000(190 { Span. Dol.       21)4000(190 { Fed. Dol.
   21                             21
   --                             --
   190=190 10/21                  190
   189 Dollars.                   189
   ---                            ---
    10                             10
     6                             10
   — s. d. q.                     —d. c. m.
21)60(2 10 1                  21)100(4 7 6 4/21
```

X. *To reduce Spanish milled Dollars.*

1. *To New-Hampshire, Massachusetts, Rhode-Island, Connecticut and Virginia currency.*

Rule.—Multiply the Dollars by 3, and double the right hand figure of the product, for shillings; the left hand figures are pounds.

Reduce 529 dollars to New-Hampshire, &c.

```
   529
     3
   ---
£.158 14 Answer.
```

2. *To New-York and North-Carolina currency.*

Rule.—Multiply the number of dollars by 4: Double the right hand figure of the product for shillings, and the left hand figures are pounds.

Reduce 529 dollars to New-York, &c.

```
   529
     4
   ---
£.211 12 Answer.
```

3. *To New-Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule.—Multiply the number of dollars by 3, and divide by 8.

Reduce 529 dollars to New-Jersey, &c.

```
529
```

529
3
——
8)1587(198 7 6 Answer.
8
—
78
72
——
67
64
—
3

Or,
8)1587
——
£·198$\frac{3}{8}$ Ans.

4. *To South-Carolina and Georgia currency.*

Rule.—Multiply the number of dollars by 7, and divide by 30.

Reduce 529 dollars to South-Carolina, &c.

529
7
——
3|0)370|3
——
£.123$\frac{13}{30}$ Answer.

*5. *To English Money, at* 4s. 6d. *per dollar.*

* Note, that in England dollars are Bullion, that is, they are bought and fold by weight, and their value varies as other articles of merchandize.

Rule.—Multiply the dollars by 9, and divide by 40.

Reduce 529 dollars to English money.

529
9
——
4|0)476|1
——
£.119$\frac{1}{40}$ Answer.

6. *To Canada and Nova-Scotia currency.*

Rule.—Divide the dollars by 4.

Reduce 529 dollars to Canada, &c.

4)529
——
£.132$\frac{1}{4}$ Answer.

7. *To Livres Tournois.*

Rule.—Multiply the dollars by 5$\frac{1}{4}$, and the product will be livres. Or, multiply them by 21 : divide by 4, and the quotient will be livres.

Reduce 100 Spanish dollars to livres.

100
5$\frac{1}{4}$
——
500
100×$\frac{1}{4}$=25
——
Ans. 525 livres.

Or,
100
21
——
4)2100
——
525 *as bef.*

Note. { 1 Cent = 1$\frac{1}{20}$ Sous. 1 Dime = 10$\frac{1}{2}$ Sous. 1 Dollar = 5$\frac{1}{4}$ Livres. }

The Pound	contains	Grains of fine silver.	Grains of Stand silv.
Sterling,		1718$\frac{2}{3}$	1858
New-Hamp.&c.		1289	1393$\frac{1}{2}$
New-York, &c.		966$\frac{3}{4}$	1045
New-Jerfey,&c.		1031$\frac{1}{4}$	1114$\frac{3}{7}$
South-Carol.&c.		1657·366	1791·819

The proportion of alloy being $\frac{3}{37}$ of the fine filver.

The Federal	contains	Grains of fine		
Dollar		375·64	Silver	409·78 grs. of Stand.filv.
Eagle		246·268	Gold	268·659 grs. of Stand.gold.

The alloy being $\frac{1}{11}$ of the fine { Silver. Gold. } The Subdivisions are in the same proportion.

DUODECIMALS,

DUODECIMALS,

OR CROSS MULTIPLICATION,

IS a Rule, made use of by workmen and artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus; inches or primes (′) seconds (″), thirds (‴), fourths (⁗), &c.

This method of multiplying is not confined to *twelves*; but may be greatly extended: For any number, whether its inferiour denominations decrease from the integer in the same ratio, or not, may be multiplied crosswise; and, for the better understanding of it, the learner must observe, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand; but if he multiplies by any number of an inferiour denomination, the value of an unit in the product will be so much inferiour to the value of an unit in the multiplicand as an unit of the multiplier is less than an integer.

Thus, pounds, multiplied by pounds, are pounds; pounds, multiplied by shillings, are shillings, &c. shillings, multiplied by shillings are twentieths of a shilling; shillings, multiplied by pence, are twentieths of a penny; pence, multiplied by pence, are 240ths of a penny, &c.

Rule.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term, observing, in duodecimals, to carry an unit for every 12 from each lower denomination to its next superiour, and for other numbers accordingly.

3. In the same manner multiply all the multiplicand by the primes or second denomination in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand.

4. Do the same with the seconds in the multiplier, setting the result of each term two places to the right hand of those in the multiplicand.

5. Proceed in like manner with all the rest of the denominations, and their sum will be the answer required.

Examples.

EXAMPLES.

1. Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

```
 F.  '
 2   6
 2   6
 -----
 5   0
 1   3   0
 ---------
Ans.6  3
```

Or thus.

$2\frac{1}{2}$
$2\frac{1}{2}$
5
$1\frac{1}{4}$

Ans. $6\frac{1}{4}$ square feet = 6ft. 36in.
So that the 3 is not 3 inches, but 36 inches, or $\frac{1}{4}$ of a square foot.

Or thus.

```
 2·5
 2·5
 ---
 125
 50
 ----
```

Ans. 6.25 square feet.

2. Multiply 9f. 8′ 6″ by 7f. 9′ 3″

```
 F.  '   "
  9  8   6
  7  9   3
 ----------
 67 11   6            = Product by the feet in the multiplier.
  7  3   4   6'''     = ditto by the primes.
     2   5   1   6''''  = ditto by the seconds.
 --------------------
 75  5   3   7   6 Answer.
```

3. How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide ? Ans. 24ft. 10′ 11″

4. How many cubick feet in a stick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick ?
Ans. 35ft. 6′ 8″ 6‴

5. How many cubick feet of wood in a load 6 feet 7 inches long, 3 feet 5 inches high, and 3 feet 8 inches wide ? Ans. 82ft. 5′ 8″ 4‴

6. There is a house with 4 tiers of windows, and 4 windows in a tier ; the height of the first tier is 6ft. 8′ ; of the second, 5ft. 9′ ; of the third, 4ft. 6′ ; and of the fourth, 3ft. 10′ ; and the breadth of each is 3ft. 5′ ; how many square feet do they contain in the whole ?
Ans. 283ft. 7′

The two following questions are Sexcessimals.

7. If 2 places differ in longitude 2° 12′ ; what is their difference of time ?

```
Mult. 2°  12'  00"  00'''
by         3'  59"  20''' the time in which the sun passes through 1°
      ---------------------
           8'  46"  32''' Answer.
```

8. Two places differ in longitude 31° 27′ 30″ ; What is the difference, in time, of the sun's coming to the meridian of those places, the sun passing through 15° in an hour ?

```
31°  37'  30"
      4'  00" In 4' of a solar day, or day of 24 hours, the sun passes 1°
-------------
 2°   6'  30"  00''' Answer.
```

9. Multiply

9. Multiply £.3 6 8 by £.2 5 7.

	£.	s.	d.
	3	6	8
	2	5	7
£3×£2=£6 =	6	0	0
6s.×£.2=12s. =	0	12	0
8d.×£.2=16d. =	0	1	4
£.3×5s. =15s. =	0	15	0
6s. ×5s. =$\frac{30}{20}$s. =	0	1	6
8d.×5s. =$\frac{40}{20}$d. =	0	0	2
£.3×7d.=21d. =	0	1	9
6s. ×7d.=$\frac{42}{20}$d. =	0	0	$2\frac{1}{10}$
8d.×7d.=$\frac{56}{240}$d.=	0	0	$0\frac{7}{30}$
Ans.	7	11	$11\frac{1}{3}$

10. A, B and C bought a drove of sheep in company; A paid £.14 5s. B, £.13 10s. and C, £.11 5s. They agreed to dispose of them at the market; that each man should take 18s. as pay for his time, &c. and that the remainder should be divided in proportion to their several stocks: At the close of the sale, they found themselves possessed of £.46 5s. what was each man's gain, exclusive of the pay for his time, &c.?

£.14 5+£.13 10+£.11 5=£.39, and £.46 5—£.39=£.7 5, and £.7 5—18s.×3=£.4 11s. whole gain, and £.4 11÷39=2s. 4d. gain in the pound.

A		B		C
£14 5 0		£.13 10 0		£.11 5 0
× 2 4		× 2 4		× 2 4
1 8 6		1 7 0		1 2 6
4 9		4 6		3 9
A. £.1 13 3		B. £.1 11 6		C. £.1 6 3

	£.	s.	d.
Proof.	1	13	3
	1	11	6
	1	6	3
	£.4	11	0

SINGLE RULE OF THREE DIRECT.

THE Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

If *more* require *more*, or *less* require *less*, the question belongs to the Rule of Three Direct.

But if *more* require *less* or *less* require *more*, it belongs to the Rule of Three Inverse.*

Rule.

* *More* requiring *more*, is when the third term is greater than the firſt, and requires the fourth term to be greater than the ſecond. And *leſs* requiring *leſs*, is when the third term is leſs than the firſt, and requires the fourth term to be leſs than the ſecond.

Alſo, *more* requiring *leſs*, is when the third term is greater than the firſt, and requires the fourth term to be leſs than the ſecond. And *leſs* requiring *more*, is when the third term is leſs than the firſt, and requires the fourth term to be greater than the ſecond.

Rule.*

1. State the question by making that number, which asks† the question, the third term, or putting it in the third place; that, which is of the same name or quality as the demand, the first term; and that, which is of the same name or quality with the answer required, the second term.

2. Multiply the second and third numbers together; divide the product by the first, and the quotient will be the answer to the question, which (as also the remainder) will be in the same denomination you left the second term in, and which may be brought into any other denomination required.

Two, or more statings are sometimes necessary, which may always be known from the nature of the question.

The method of proof is by inverting the question.

But,

* This Rule, on account of its great and extenfive ufefulnefs, is fometimes called the *Golden Rule of Proportion:* For, on a proper application of it and the preceding rules, the whole bufinefs of Arithmetick, as well as every mathematical inquiry depends. The rule itfelf is founded on this obvious principle, that the magnitude or quantity of any effect varies conftantly in proportion to the varying part of the caufe: Thus, the quantity of goods bought, is in proportion to the money laid out; the fpace, gone over by an uniform motion, is in proportion to the time, &c.

As the idea, annexed to the term, *proportion*, is eafily conceived, the truth of the rule, as applied to ordinary inquiries, may be made evident by attending to principles, already explained.

It has been fhewn, in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole; and in Divifion, that the price of the whole, divided by the quantity, is the price of one: Now, in all cafes of valuing goods, &c. where one is the firft term of the proportion, it is plain that the anfwer found by this rule, will be the fame as that, found by Multiplication of Money; and, where one is the laft term of the proportion, it will be the fame as that found by Divifion of Money.

In like manner, if the firft term be any number whatever, it is plain, that the product of the fecond and third terms will be greater than the true anfwer, required, by as much as the price in the fecond term exceeds the price of one, or as the firft term exceeds an unit; confequently, this product, divided by the firft term, will give the true anfwer required.

Direct and Inverfe proportion are properly only parts of the fame general rule; but I have preferved the common diftinction, and given fome loofe definitions, which, to young perfons in general, are more intelligible.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and Compound Divifion.

2. If the firft term, and either the fecond or third can be divided by any number without a remainder, let them be divided and the quotient ufed inftead of them.

The following methods of operation, when they can be ufed, perform the work in a much fhorter manner than the general rule.

1. Divide the fecond term by the firft: Multiply the quotient into the third, and the product will be the anfwer.

2. Divide the third term by the firft; multiply the quotient into the fecond, and the product will be the anfwer.

3. Divide the firft term by the fecond, and the third by that quotient, and the laft quotient will be the anfwer.

4. Divide the firft term by the third, and the fecond by that quotient, and the laft quotient will be the anfwer.

† Note. The term which afks or moves the queftion, has generally fome words like thefe before it, viz. What will? What coft? How many? How far? How long? or, How much? &c.

But, that I may make the method of working this excellent Rule as intelligible as possible to the learner, I shall divide it into the several cases following:

1. The fourth number is always found in the same name in which the second is given, or reduced to; which, if it be not the highest denomination of its kind, reduce to the highest when it can be done.

2. When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name to which the second is reduced, which reduce back to the highest possible.

3. If the first and third be of different names, or one or both of divers denominations, reduce them both to the lowest denomination mentioned in either.

4. When the product of the second and third is divided by the first; if there be a remainder after the division, and the quotient be not the least denomination of its kind; then multiply the remainder by that number, which one of the same denomination with the quotient contains of the next less, and divide this product again by the first number; and thus proceed till the least denomination be found, or till nothing remain.

5. If the first number be greater than the product of the second and third; then bring the second to a lower denomination.

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the lowest name, and then multiplied by the given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal contents, (as it most generally happens) put the separate content of each properly under one another, then add them together, and you will have the whole quantity.

Examples.

1. If 6lb. of sugar cost $\left\{\begin{array}{c}\text{9s.}\\ \text{D.1 50c.}\end{array}\right\}$ what will 30lb. cost at the same rate?

```
     lb.   s.   lb.
As 6 : 9 :: 30 : the Answer.
             9
            ---
         6)270
            ---
          45s.=£.2 5s. Ans.
```

Here the first clause (if 6lb. of sugar cost 9s. or D.1 50c.) supposes the rate; then follows the question: What will 30lb. cost?

30lb. which moves the question, is the 3d term. 6lb. the same kind, is the 1st, and 9s. (or D.1 50c.) the 2d.

```
  lb.  D. c.   lb.
As 6 : 1 50 :: 30 : the Answer.
             30
           -----
        6)45·00
           -----
        D.7·50 Ans.
```

Again, By inverting the order of the question, it will be,

2. If

2. If $\left\{ \begin{array}{c} 9s. \\ D.1 \ 50c. \end{array} \right\}$ buy 6lb. of sugar, how much will $\left\{ \begin{array}{c} £.2 \ 5s. \\ D.7 \ 50c. \end{array} \right\}$ buy at that rate ?

```
   s. lb.   s.                      D. c.  lb.  D. c.
As 9 : 6 : : 45 : the Ans.       As 1 50 : 6 : : 7 50 : the Ans.
             6                                   6
            ---                                 ---
        9)270                          1·5)45·00(30lb. Ans.
         ---                                45
        30lb. answer.                      ---
         ---                                 0
```

Again, 3. If 30lb. of sugar be worth $\left\{ \begin{array}{c} £.2 \ 5s. \\ D.7 \ 50c. \end{array} \right\}$ how much may I buy for $\left\{ \begin{array}{c} 9s. \\ D.1 \ 50c. \end{array} \right\}$?

```
  s.  lb.   s.                     D. c.   lb.   D. c.
As 45 : 30 : : 9 : the Ans.      As 7·50 : 30 : : 1·50 : the Ans.
        9                                         30
       ---                                        ---
   45)270(6lb. the Ans.                  7·5)45·00(6lb. Ans.
      270                                    45 0
      ---                                    ---
```

Again, 4. Suppose $\left\{ \begin{array}{c} £.2 \ 5s. \\ D.7 \ 50c. \end{array} \right\}$ will buy 30lb. of sugar : What will 6lb. of the same sugar cost ?

```
  lb.  s.  lb.                       lb.  D. c.  lb.
As 30 : 45 : : 6 : the Ans.       As 30 : 7·50 : : 6 : the Answer.
        6                                  6
       ---                                ---
  3|0)27|0                          3|0)45·0|0
      9s. Ans.                        D.1·50c. Ans.
```

N. B. The three last questions are only the first *varied*, being put merely to show how any question, in this Rule, may be inverted.

5 If 5yds. cloth cost $\left\{ \begin{array}{c} £.1 \ 10s. \\ D.5 \end{array} \right\}$ what will 20yds. ditto come to?

```
        yds.  s.   yds.
      As 5 : 30 : : 20
        30 ÷ 5 =  6
                 ---
          2|0)12|0s.=£.6 Ans.
```

Here I divide the 2d term by the 1st, and multiply the quotient into the 3d for the answer.

```
              yds.  s.  yds.
Again, 6.    As 5 : 30 :: 20
             20÷5 = 4
                   ---
                  120s. = £.6
```

Here I divide the 3d term by the 1st, and multiply the quotient into the 2d, for the answer.

These

These operations would be, perhaps, still more apparent, if performed in Federal money. Thus:

yds. D. yds.	yds. D. yds.
As 5 : 5 : : 20	As 5 : 5 : : 20
5÷5 = 1	20÷5 =4
D.20 Ans.	D.20 Ans.

7. If 20 yds. cost D.120, how many yards may I have for D.30?

D. yds D.
As 120 : 20 : : 30
120÷20=6 quot. & 30÷6=5 yards, Answer.

Here I divide the 1st term by the 2d, and then, the 3d term by the quotient for the answer.

D. yds. D.
Again, 8. As 120 : 20 :: 30
120÷30=4 quot. and, 20÷4=5 yards, Ans.

Here I divide the 1st term by the 3d, and then, the 2d term by that quotient for the answer.

9. If 1cwt. of tobacco cost £.5 12 9½; what will 8cwt. ditto cost?

cwt. £. s. d. cwt.
As 1 : 5 12 9½ :: 8
8

Ans. £.45 2 4

Here there is no need of reducing the middle term, because it can be performed by compound multiplication, the first term being an unit.

10. If 8cwt of tobacco cost £.45 2 4; what is that per cwt.?

£. s. d.
8)45 2 4

Ans. 5 12 9½

Here there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

11. If 9cwt. 3qrs. sugar cost { £.27 17s. 6d / D.92 91c. 7m. } what will 2cwt. 1qr. 1lb. cost?

	£. s. d.	2 C. 1qr. 1lb.
4	27 17 6	4
	20	
39		9
28	557	28
	12	
312		73
78	6690	19
1092		263
		lbs.

```
     lb.     d.     lb.
As 1092 ; 6690 :: 263 : the answer.
               263
              ----
              2007
             4014
            1338
            -------12
1092)1759470(1611
     1092     ----
     ------2|0)13|4 3d.
      6674  £.6 14s. 3d. Answer.
      6552
      ----
       1227
       1092
       ----
        1350
        1092
        ----
         258
           4
        ----
1092)1032)0qr.
```

Note 1. If you look at the stating, you will see that the first and third terms are of the same kind, but of different denominations, and therefore are reduced to the same name or denomination, and that the demand of the question lies on the 3d term.

2. That the middle term, being given in pounds, shillings and pence, is reduced to pence. But,

3. If the second term were in federal money, it would be sufficient to proceed according to decimals. Thus:

```
          lb.  D. c. m.   lb.
     As 1092 : 92·917 :: 263 : the Ans.
                  263
               ------
               278751
              557502
             185834
     --------------D. c. m.
1092)24437·171(22·378+, Ans.
     2184
     ----
      2597
      2184
      ----
       4131
       3276
       ----
        8557
        7644
        ----
         9131
         8736
         ----
          395
```

12. If

12. If 57yds. cost $\left\{\begin{matrix}£.69\\D.230\end{matrix}\right\}$ what will 9yds. cost at that rate?

yds. £. yds.
As 57 : 69 :: 9
9

57)621(10£. Ans.
57
51
20

57)1020(17s.
57
450
399
51
12

57)612(10d.
57
42
4

57)168($2\frac{54}{57}$qrs.
114

Here, all the terms being whole numbers, there is no need of reducing the middle one until after stating.

The same in Federal money would stand thus :

yds. D. yds.
As 57 : 230 :: 9 : the answer.
9

57)2070(36 D. 31c. $5\frac{15}{19}$m. Ans.
171
360
342
180
171
90
57
330
285
45

$\frac{45}{57} = \frac{15}{19}$

18. If

13. If my income be 109 guineas per annum, I desire to know what I may spend per day, so that I may lay up 45l. at the year's end ?

Ans. £.0 5 $10\frac{3}{4}$ $\frac{1}{365}$ per day.

Note 1. You must subtract 45l. from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus :

D. Guin. £. D. s. d qr.
As 365 : 109 — 45 :: 1 : 5 10 $3\frac{1}{365}$ the Ans.

14. If my salary be 43l. 12s. 5d. per annum, what does it amount to per week ?

Ans. £ 0 16s. $9\frac{17}{52}$d.

The Stating.
W. £. s. d. W.
As 52 : 43 12 5 :: 1 : the Ans.

Note. As there are 52 weeks and 1 day in a year, you will get the true answer to the above question by the following ratio.

D. £. s. d. D.
As 365 : 43 12 5 :: 7 : 16s. $8\frac{283}{365}$d.

15. Suppose my income to be 16s. $8\frac{283}{365}$d. per week, what is it per annum ?

Ans. 43l. 13s. $7\frac{1}{4}$ $\frac{51}{365}$d.

The Stating. D. s. d. D.
As 7 : 16 $8\frac{283}{365}$ :: 365 : £.43 12s. 5d. Ans.

Note. 1. You must first reduce the middle term to pence.

2. You must multiply by 365 (the denominator of the fraction) and add to the product the 283 which remains ; and remember always to do so in similar cases.

3. You must divide by 7, the first term and the quotient will be the answer in 365ths of a penny, which (in all similar cases) must be first divided by the denominator, and then brought into pounds.

16. If I am to pay 1s. 7d. per week for pasturing a cow ; what must I give per week for 37 cows ?

C. s. d. C. £. s. d.
As. 1 : 1 7 :: 37 : 2 18 7 Ans.

17. How many yards of cloth may be bought for 195dol. 75c. of which $9\frac{1}{2}$ yds. cost 11dol. 2c. ?

Dol. c. yds. Dol. c. yds. qrs.
As 11 02 : $9\frac{1}{2}$ :: 195 75 : 168 3 Ans.

18. If I buy 57 yards of cloth for 49 guineas ; what did it cost per ell English ?

yds. guin. yds.
As 57 : 49 :: $1\frac{1}{4}$: £.1 10s. $1\frac{13}{228}$d. Ans.

19. A merchant, failing in trade, owes in all £.3475, and has in money and effects but £.2316 13 4 : Now, supposing his effects are delivered up, pray, what will each creditor receive on the pound ?

£. £. s. d. £.
As 3475 : 2316 13 4 :: 1 : £0 13s. 4d. Ans.

20. A owes B 3475l. but B compounds with him for 13s. 4d. on the pound ; pray, what must he receive for his debt ?

£. s. d. £. £. s. d.
As 1 : 13 4 :: 3475 : 2316 13 4

21. If

21. If the distance from Newburyport to York be 31 miles ; I demand how many times a wheel, whose circumference is $15\frac{1}{2}$ feet will turn round in performing the journey ?

Feet. Cir. M. Cir.
As $15\frac{1}{2}$: 1 :: 31 : 10560 times, Answer.

22. Bought 9 chests of tea, each weighing 3cwt. 2qrs. 21lb. at 4l. 9s. per cwt. what came they to ?

Cwt. £. s. C. qr. lb. £. s. d.
As 1 : 4 9 :: 3 2 21 × 9 : 147 13 $8\frac{1}{4}$.

23. What will $37\frac{1}{2}$ gross of buttons come to at 13 cents per dozen ?

Doz. c. Gross. D. c.
As 1 : 13 :: $37\frac{1}{2}$: 58 50 Ans.

24. A farm, containing 125 A. 3r. 27p. is rented at D.11 50c. per acre ; what is the yearly rent of that farm ?

A. D. c. A. R. P. D. c. m.
As 1 : 11 50 :: 125 3 27 : 1447 6 $5\frac{5}{8}$ Ans.

25. If a ship cost 537l. what are $\frac{3}{8}$ of her worth ?

Eigh. £. Eigh. £. s. d.
As 8 : 537 :: 3 : 201 7 6 Ans.

26. If $\frac{7}{16}$ of a ship cost 1163D. what is the whole worth ?

Sixt. D. Sixt. D. c. m.
As 7 : 1163 :: 16 : 2658 28 5 Ans.

27. Bought a cask of wine at 76c. 5m. per gallon, for 125 dollars : How much did it contain ;

Ans. 163 gal., 1qt. $1\frac{3}{17}$pt.

28. What come the insurance of 537l. 15s. to at $4\frac{1}{2}$l. per centum ?

£. £. £. s. £. s. d
As 100 : $4\frac{1}{2}$ :: 537 15 : 24 3 $11\frac{1}{2}\ \frac{8}{10}$ Ans.

29. What come the commissions of 785l. to at $3\frac{1}{2}$ guineas per cent. ?

Ans. 38l. 9s $3\frac{1}{2}\ \frac{4}{10}$d.

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards : each package contained 8 parcels, each parcel, 12 pieces, and each piece, 20 yards ; how many dollars came the whole to, and how many per yard ?

Yds. guin. pack. D.
As 7 : 3 :: 9 : 34560 Ans. for the whole cost.
Yds. guin. yd. D.
As 7 : 3 :: 1 : 2 Ans. per yard.

31. A merchant bought 49 tuns of wine for D.910 ; freight cost D.90 ; duties D.40 ; cellar D.31 67c. ; other charges D.50 and he would gain D.185 by the bargain ; what must I give him for 23 tuns ?

Tuns. D. D. D D. c. D. D. Tuns. D.
As 49 : 910 + 90 + 40 + 31 67 + 50 + 185 :: 23 : 613 33c. Ans.

32. If D.100 gain D.6 in a year, what will D.475 gain in that time ?

Ans. D.28 50c.

33. The

33. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles?

H. D. M. M. M.
As 24 : 360 × 51 :: 1 : $12\frac{3}{4}$ Ans.

34. Shipt for the West Indies 225 quintals of fish, at 15s. 6d. per quintal; 37000 feet of boards, at $8\frac{1}{3}$ dolls. per 1000; 12000 shingles, at $\frac{1}{2}$ guin. per 1000; 19000 hoops at $1\frac{1}{2}$ doll. per 1000, and 53 half joes; and in return, I have had 3000 galls. of rum, at 1s. 3d. per gallon; 2700 gallons of molasses, at $5\frac{1}{2}$d. per gallon; 1500lb. of coffee, at $8\frac{1}{2}$d. per lb.; and 19Cwt. of sugar, at 12s. 3d. per cwt. and my charges on the voyage were 37l. 12s. pray, did I gain or lose, and how much by the voyage?

Ans. lost 134l. 9s. 9d.

35. If a staff, 4 feet long, cast a shade (on level ground) 7 feet; what is the height of that steeple, whose shade, at the same time, measures 198 feet?

F. sh. F. hei. F sh. F. hei.
As 7 : 4 :: 198 : $113\frac{1}{7}$ Ans.

*36. Suppose a tax of D.755 be laid on a town, and the inventory of all the estates in the town amounts to D.9345, what must A pay whose estate is D.149?

D. D. D. D. c. m.
As 9345 : 755 :: 149 : 12 12 7 Ans.

37. If

* It may not be amiss to show the general method of assessing town or parish taxes. First, then, an inventory of the value of all the estates, both real and personal, and the number of polls, for which each person is rateable, must be taken in separate columns: The most concise way is then to make the total value of the inventory the first term, the tax to be assessed, the second, and D.1 the third, and the quotient will show the value on the dollar: 2dly, Make a table, by multiplying the value on the dollar by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, or otherwise, before stating to find the value on the dollar, you must deduct the sum of the average tax from the whole sum to be assessed: for which average, you must have a separate column, as well as for the real and personal estates.

EXAMPLE.

Suppose the General Court should grant a tax of D.500000, of which the town of Newburyport is to pay D.5312 50c. and, of which the polls, being 1550, are to pay D.1 25c. each:—The town's inventory amounts to D.450000, what will it be on the dollar, and what is A's tax, whose estate (as by the inventory) is as follows, viz. real D.1376, personal D.1149, and he has 3 polls.?

Pol. D. c. Pol. D. c.
First, As 1 : 1 25 :: 1550 : 1937 50 the average part of the tax to be deducted from D.5312 50c. and there will remain D.3375.

D. D. D.
Secondly, As 450000 : 3375 :: 1 : $7\frac{1}{2}$m. on the dollar.

TABLE.

37. If 50 gallons of water, in one hour, fall into a cistern, containing 230 gallons, and by a pipe in the cistern 35 gallons run out in an hour; in what time will it be filled ?

Gal. gal. h. gal. h.
As 50—35 : 1 :: 230 : $15\frac{1}{3}$ Ans.

38. A butcher went with £.416, to buy cattle : Oxen, at £.22 each, cows at £.4, steers at £.3 10s. and calves at £.2 10s. and of each a like number ; how many of each could he purchase with that sum ?

£. £. £ s. £. s. each £. each.
As 22+4+3 10+2 10 : 1 :: 416 : 13 Ans.

39. Said Harry to Dick, my purse and money are worth $3\frac{1}{4}$ guineas but the money is worth eleven times as much as the purse ; pray, how much money is there in it ?

Guin s. d.
As 12 : 1 :: $3\frac{1}{4}$: 7 7 then £.4 11s.—7s. 7d.=£.4 3s. 5d. Ans.

40. How many dozen pair of gloves, at 13 groats per pair, may I have for 125 dollars ?

Gr pr. dol doz. pr.
As 13 : 1 :: 125 : 14 $5\frac{4}{52}$ Ans.

41. There is a cistern, having four cocks ; the first will empty it in ten minutes ; the second in 20 minutes ; the third in 40, and the fourth in 80 minutes ; in what time will all four, running together, empty it ?

$$\text{As}\left\{\begin{matrix}10\\20\\40\\80\end{matrix}\right\}\begin{matrix}\text{Cist. Min.}\\ :1\ ::\ 60\ :\end{matrix}\left\{\begin{matrix}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{matrix}\right\}\begin{matrix}\text{Cist. Min. Cist. Min.}\\ \text{As } 11\frac{1}{4} : 60 :: 1 : 5\frac{1}{3} \text{ Ans.}\end{matrix}$$

$11\frac{1}{4}$ Cist.

42. A

TABLE.

D.	D.	c.	m.	D.	D.	c.	m.	D.	D.	c.
1 is	0	0	$7\frac{1}{2}$	20 is	0	15	0	200 is	1	50
2 —	0	1	5	30 —	0	22	5	300 —	2	25
3 —	0	2	$2\frac{1}{2}$	40 —	0	30	0	400 —	3	00
4 —	0	3	0	50 —	0	37	5	500 —	3	75
5 —	0	3	$7\frac{1}{2}$	60 —	0	45	0	600 —	4	50
6 —	0	4	5	70 —	0	52	5	700 —	5	25
7 —	0	5	$2\frac{1}{2}$	80 —	0	60	0	800 —	6	00
8 —	0	6	0	90 —	0	67	5	900 —	6	75
9 —	0	6	$7\frac{1}{2}$	100 —	0	75		1000 —	7	50
10 —	0	7	5							

Now, to find what A's rate will be,
His real estate being D.1376, I find,
by the table, that D.1000 is D.7 50c.
that D.300 is 2 25
that D.70 is 52 5m.
and that D.6 is 4 5
for his real estate D. 10 32

Real. D. c. m.	Personal. D. c. m.	Polls. D. c. m.	Total. D. c. m.
10 32 0	8 61 $7\frac{1}{2}$	3 75	22 68 $7\frac{1}{2}$

In like manner I find his tax for personal estate to be } D.8 61 $7\frac{1}{2}$
His 3 polls, at D.1 25c. each are 3 75 } +D.10 32=D. 22 68c. $7\frac{1}{2}$m.

or, D.22 69c. Ans.

42. A and B depart from the same place, and travel the same road; but A goes 5 days before B, at the rate of 20 miles per day; B follows at the rate of 25 miles per day: In what time and distance will he overtake A?

M. M. D. M. D. D. D. M. D. M.
As 25—20 : 1 :: 20 × 5 : 20. And, As 1 : 25 :: 20 : 500

43. If the earth revolves 366 times in 365 days, in what time does it perform one revolution?

Revol. days. Revol.
As 366 : 365 :: 1 : 23h. 56′ 3″ 56‴ + = 1 Sidereal day.*

44. If the earth makes one complete revolution in 23h. 56′ 3″+, in what time does it pass through one degree? Ans. 3′ 55″ 20‴

45. If the earth performs its diurnal revolution in a solar day,† or 24 hours; in what time does it move one degree? Ans. 4′

46. Sold a cargo of flax seed in Ireland, for £.1795 10s. Irish money; what does that amount to, in Massachusetts currency, £.81 5s. Irish being equal to £.100 Massachusetts.

Irish. Mass. Irish Mass.
As £.81¼ : £.100 :: £.1795½ : £.2209 16s. 11d. Ans.

Or, As £.13 : £.1795½ :: £.16 : £.2209 16s. 11d. as before, because £.13 Irish are equal to £.16 Massachusetts.

47. My correspondent in Maryland purchased a cargo of flour for me, for £.437 that currency; how much Massachusetts money must I remit him, £.125 Maryland being equal to £.100 Massachusetts or 5 Mar.=4 Mass. Ans. £.349 12s.

48. A bill of exchange was accepted at Newburyport for the payment of £.345 10, for the like value delivered in New York, at £.133⅓ New-York currency for £.100 Massachusetts ditto; how much money was paid in New-York?

Mass. N. Y. Mass. N. Y.
As £.75 : £.100 :: £.345 10s. : £.460 13s. 4d. Ans.

49. When the exchange from Massachusetts to Georgia is £.83⅓ Georgia per £.100 Massachusetts, how much Massachusetts money must be paid in Boston to balance £.457 Georgia currency?

Ans. £.548 8s. Mass.

50. A merchant delivered at Boston £.320 Massachusetts currency, to receive £.400 in Philadelphia; what was the Massachusetts pound valued at? Ans. £.1 5s. Phil.

51. If I draw a bill of exchange for £.537 10s. 6d. Massachusetts, to be paid in Ireland, at £.123$\frac{1}{13}$ Massachusetts, per £.100 Irish, or 16 Mass. for 13 Irish; for how much Irish money must I draw the bill? Ans. £.436 14s. 9½d. Irish.

52. Suppose a bill is drawn in Ireland, and payable in Boston, for £.673 12s. 6d. Irish; how much Massachusetts money comes it to, the exchange at £.81¼ Irish, per £.100 Massachusetts?

Ans. £.829 1s. 6$\frac{6}{13}$d. Mass.

The

* A sidereal day is the space of time which happens between the departure of a star from, and its return to the same meridian again.

† The solar day is that space of time which intervenes between the sun's departing from any one meridian, and its return to the same again.

The value of any quantity of silver in any of the currencies of the United States may be found by the following proportion.

As the number of grains, contained in £.1, is to £.1; so are the grains, in any given quantity, to its value.

53. What is the value of 1lb. of silver in Massachusetts currency; the pound, or 20 shillings, containing $1393\frac{1}{2}$ grains? £. s. d.

As $1393\frac{1}{2}$: 1 :: 5760 : 4 2 8.

All questions in the Rule of Three, whether direct or inverse, may be solved by the following rule.

Let that number, which is of the same name or quality as the number sought, be the third term; then, consider whether the number sought should be more or less than the third; if *more*, let the greater of the two other terms be the *middle* term, and the *less* the *first*; but if the fourth number ought to be *less* than the third, then give the *less* the *second* place, and the *greater*, the first. The question being thus stated, the proportion will be; as the first term is to the second, so is the third to the fourth, or number sought. Euclid's Elements V. 14.

Note. The first and second terms must always be brought into one name, and the third into the lowest mentioned, then proceed as in the common method, by multiplying the second and third terms together, and dividing the product by the first, and the quotient will be the answer, in the same name as the third term was reduced into.

54. If 15 yards of cloth cost £6, how many yards may I have for £125?

```
  £.   £.    yds.
As 6 : 125 :: 15
        15
       ----
       625
      125
      ----
    6)1875
      ----
      312½ yards Ans.
```

55. If 12 men can do a jobb in 20 days; in what time will 18 men do it?

```
   M.   M.   D.
As 18 : 12 :: 20
             20
            ---
      18)240(13⅓ days Ans.
         18
         --
          60
          54
          --
           6
```

56. If I give 1 D. 75 c. for 3 yards, how many yards may I have for 180 D?

D. c. D. yds. yds. qrs. n.

As 1 75 : 180 :: 3 : 308 2 $1\frac{1}{7}$ Ans.

Or thus,

State the question in the usual way, and let the second term keep its proper, or natural place; then, multiply it by the greater or less extreme, that is, by the first or third number accordingly, as the answer ought to be greater or less; divide the product by the other term, and the quotient will be the answer.

RULE

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.*

Having made the necessary preparations, as directed in Multiplication and Division of Vulgar Fractions, state your question as in whole numbers, and invert the first term of the proportion ; then multiply the three terms continually together, and the product will be the answer.

1. If $\frac{5}{8}$ of a yard cost $\frac{5}{7}$ of a £. what will $\frac{9}{15}$ of an Ell Eng. cost ?

$\frac{5}{8}$ yd.$=\frac{5}{8}$ of $\frac{4}{1}$ of $\frac{1}{5}=\frac{5\times4\times1}{8\times1\times5}=\frac{1}{2}$ Ell. Eng.

E. Eng. £. E. Eng.
As $\frac{1}{2}$: $\frac{5}{7}$ :: $\frac{9}{13}$: $\frac{2}{1}\times\frac{5}{7}\times\frac{9}{13}=\frac{2\times5\times9}{1\times7\times15}=\frac{90}{105}$£.=17s. $1\frac{1}{2}$d. $\frac{6}{8}$ Ans.

2. If $\frac{3}{5}$ yd. cost $\frac{7}{8}$ D. what will $40\frac{1}{2}$ yds. come to ?
Ans. D. 59 6c. $2\frac{1}{2}$m.

3. If 70 bushels of corn cost £.$12\frac{3}{5}$, what is it per bushel ?
Ans. 3s. $7\frac{1}{5}$d.

4. If $\frac{7}{16}$ of a ship cost £.51, what are $\frac{3}{32}$ of her worth ?
Ans. £.10 18s $6\frac{3}{4}$d. $\frac{3}{7}$.

5. At D.$3\frac{5}{6}$ per cwt. what will $9\frac{2}{3}$lb. come to ? Ans. 31c. 3m.—

6. A person having $\frac{4}{5}$ of a vessel, sells $\frac{2}{3}$ of his share for D.$1080\frac{2}{3}$; what is the whole vessel worth ? Ans. D 2026 25c.

7. A merchant sold $5\frac{1}{2}$ pieces of cloth, each containing $12\frac{2}{3}$yds. at $12\frac{2}{3}$c. per yard ; what did the whole amount to ? Ans. D.8 $82\frac{4}{9}$c.

8. A buys of B £.$560\frac{3}{4}$ bank stock, at £.$85\frac{2}{3}$ per cent. what comes it to ?
Ans. £.480 7s. $6\frac{1}{7}$d.

9. A merchant makes insurance upon a vessel and cargo, valued at £3750 16s. at $15\frac{1}{2}$ guineas per cent. what does the premium amount to ? Ans. 813l 18s. $5\frac{1}{2}$d.

10. A merchant in Holland draws a bill upon his correspondent in Boston for 3750 ducats at 8s. $4\frac{1}{3}$d. : How much Massachusetts currency must he receive ? Ans. 1565l. 12s. 6d.

11. A gentleman from Boston being in England, where the price of silver is to that of gold, as 1 to $15\frac{1}{14}$, exchanged $158\frac{1}{4}$lb. of silver for gold ; on his return to Massachusetts, where the price of silver is to that of gold, as 1 to $15\frac{15}{31}$, a friend, wanting his gold, gave him the value thereof in silver ; what weight of silver did he gain by the exchange ?

lb. S. G. lb. S. lb. G. G. S G. lb. S.
As $15\frac{1}{14}$: $\frac{1}{1}$ :: $158\frac{1}{4}$: $10\frac{1}{2}$ As $\frac{1}{1}$: $15\frac{15}{31}$ :: $10\frac{1}{2}$: $162\frac{36}{62}$ Ans. $4\frac{41}{124}$lb.

12. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of 7 dollars for 5 yards, and sold them out at the rate of 11 dollars for 7 yards ; and gained 200 dollars by the bargain ; how many bales were there ?

Yds.

* This rule and the next, depend upon the same principle as the Rule of Three in whole numbers.

Yds. Dol. Yds. Dol.	
As 7 : 11 :: 5 : $7\frac{6}{7}$	Sold 5 yards for $7\frac{6}{7}$ Dollars.
	Bought 5 yds. for 7 Dollars.
	In 5 yards gained $\frac{6}{7}$ Dollar.

Dol. Yds. Dol. Yds. Yds. B Yds. B.
As $\frac{6}{7}$: 5 :: 200 : $1166\frac{2}{3}$, and, As $129\frac{1}{2}\frac{7}{7}$: $\frac{1}{1}$ :: $1166\frac{2}{3}$: 9 Ans.

Although the method before laid down be universally applicable, yet there are other methods more ready and expeditious in some particular cases.

RULE I.

If the first and third terms be fractions, and the second a whole number, reduce the first and third to one common denominator, then, rejecting the denominators, make the numerator of the first, the first term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{5}{8}$ yard cost 9s. what cost $\frac{7}{12}$ yard at that rate ?

$\frac{5}{8}=\frac{15}{24}$ and $\frac{7}{12}=\frac{14}{24}$. Now, As 15 : 9s. :: 14 : 8s. $4\frac{1}{4}$d. Ans.

RULE II.

If of the first and third terms, one be 1, and the other a fraction : put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land cost £.12, what what will $\frac{5}{8}$ of an acre cost, at that rate ?

Den. £. Num. £. s.
As 8 : 12 :: 5 : 7 10 Ans.

RULE III.

If the second term be a fraction likewise, (that is, if all the terms be fractions) having reduced the first and third to one common denominator, multiply the numerator of the first term by the denominator of the second, for a divisor ; and the numerator of the third by the numerator of the second, for a dividend ; divide the last product by the first, and the quotient will be the answer.

If $\frac{1}{2}$ yard of cloth cost £.$\frac{3}{4}$ what cost $\frac{7}{8}$ yard ?

$\frac{1}{2}=\frac{4}{8}$, which reduces it to a common denominator ; then,

As 4 : $\frac{3}{4}$ :: 7
4 3
——
16 16)21($1\frac{5}{16}$£. = 26s. 3d. Ans.
16
——
5

To find the value of Gold in Massachusetts currency.

PROB. 1. Given the weight of any quantity of gold, to find its value.

Oz. £. Oz. £. pwt. s. gr. d.
THEOREM 1. As 1 : $5\frac{1}{3}$ :: 12 : 64 :: 1 : $5\frac{1}{3}$ :: 1 :: $2\frac{2}{3}$(Case 1.)$=\frac{2\frac{2}{3}}{1}$

(Case 2.)$=\frac{5\frac{1}{3}}{2}$ (Case 3.) $=\frac{8}{3}$, Therefore,

Rule

Rule 1.—If the given quantity be in grains; say, As the denominator is to the number of grains; so is the numerator to their value in pence.

1. What is the value of 18 grains of gold?

By Case 1.	By Case 2.	By Case 3.
Gr.	Gr.	Gr
As 1 : 18 :: $2\frac{2}{3}$	As 2 : 18 :: $5\frac{1}{3}$	As 3 : 18 :: 8
$2\frac{2}{3}$	$5\frac{1}{3}$	8
36	90	3)144
12	6	48d.=4s.
12)48(4s. Ans.	2)96(48d.=4s.	

Rule 2.—If the given quantity consist of ounces, pennyweights and grains, halve the grains, and then proceed as in multiplication of pounds, shillings and pence, making the numerator in Case 2d, the multiplier.

1. What is the value of 7 oz. 8 pwt. 16 gr. of gold?

Gr. 16 ÷ 2 = 8 gr., then, 7 oz. 8 pwt. 8 gr.

7	8	8
		$5\frac{1}{3}$
37	3	4
2	9	$6\frac{2}{3}$
£39	12	$10\frac{2}{3}$ Ans.

Rule 3.—If the given quantity consist of pounds only, multiply by 64, and the product will be the answer; but, if it consist of pounds, ounces, &c. it will be most convenient to reduce the pounds to ounces and proceed by Rule 2.

1. What is the value of 36lb. of gold, at £.64 per lb.?

$$\begin{array}{r} 36 \\ 64 \\ \hline 144 \\ 216 \\ \hline \pounds.2304 \text{ Ans.} \end{array}$$

2. What is the value of 15lb. 9oz. 12pwt. 18gr. of gold?

12

	pwt	gr.	
oz. 189	12	9	= 18 gr. ÷ 2
		$5\frac{1}{3}$	
948	3	9	
63	4	3	
£1011	8	0 Ans.	

Prob. 2.

PROB. 2. To ascertain the value of any given quantity of gold in Spanish milled dollars, or federal money.

THEOREM 2. 1pwt. of gold $= 5\frac{1}{3}$s. 1 dollar $=$ 6s. And,

$$\frac{5\frac{1}{3}}{6} = \frac{16}{18} = \frac{8}{9}. \text{ Therefore,}$$

Rule. Reduce the given quantity of gold to pennyweights ; then, as the denominator is to the given quantity ; so is the numerator to the answer in dollars. Or,

Divide by the denominator, and multiply the quotient by the numerator. Or,

Divide by the denominator and subtract the quotient from the dividend. In either case, you will have the answer.

1. What is the value of 6oz. 6pwt. of gold, in Spanish dollars ?

```
                               20
                              ----
        pwt.                  126 pwt.                  Or,
As 9 : 126 :: 8                                        9)126
              8               Or,                      —14
            ----               9)126                   ----
       9)1008                 -----                    112 Ans.
          ----                 14×8=112 Ans.           ----
  Ans. 112 Dolls.
          ----
```

2. In 7oz. 13pwt. 17gr. how many dollars ?

```
                oz. pwt. gr.
                 7   13  17
                 20
              ----
               153 17/24
                24
              ----
               619
              307
              ----
              3689
```

$$\text{As } \frac{9}{1} : \frac{3689}{24} :: \frac{8}{1} :: \frac{29512}{216}$$

```
              216)29512(136 doll.
                  216
                  ----
                   791
                   648
                  ----
                   1432
                   1296
                   ----
                    136
```

To

To find the value of this remainder.

1. In shillings, &c.

```
     136
       6
     ---
216)816(3s.
     648
     ---
     168
      12
     ---
216)2016(9d.
    1944
    ----
      72
       4
     ---
216)288(1⅓qr.
     216
```

$\frac{72}{216}=\frac{1}{3}$

2. In Federal money.

Annex cyphers, as in division of decimals ; the two quotient places next to dollars, will be cents ; the third mills ; the others, decimals of a mill ; or the remainder with the divisor will form a fraction of a mill.

```
216)1360(62c. 9 17/27 m.
    1296
    ----
     640
     432
    ----
    2080
    1944
```

$\frac{136}{216}=\frac{17}{27}$

PROB. 3. To ascertain the weight of gold equivalent to any given sum, currency.

Rule 1. If the given sum be in pence, reverse Rule 1. Theorem 1. that is ; As the numerator 8 is to the given sum in pence ; so is the denominator 3 to the weight required, in grains.

What weight of gold is equal to 4s. ?

```
         d.               12
As 8 : 48 :: 3            --
          3               48
        ---
      8)144
        ---
```

Ans. 18 grains.

Rule 2. If the given sum be in pounds, shillings and pence.

As $\frac{5\frac{1}{3}}{1}$ is equal to $\frac{16}{3}$; therefore, divide the given sum by 8, and that quotient by 2 ; add the two quotients together, double the last denomination, and you will have the answer.

What quantity of gold is equivalent to 45l 13s 4d.

Mark the pounds, shillings and pence, as oz. pwt. and gr.

```
  oz. pwt. gr.
8)45   13   4
  -----------
2)5    14   2 }
  2    17   1 } Add.
  -----------
  8    11   3+3
  -----------
Oz. 8  11   6 Ans.
```

PROB. 4. To find the value of gold equivalent to any given sum in Federal money.

Rule. As the numerator 8 is to the number of dollars ; so is the denominator 9 to the answer in pennyweights: Or, divide the dollars by the numerator 8, and add the quotient to the dividend.

Or, divide as before, and multiply the quotient by the denominator 9. In either case you will have the answer.

1. Required the weight of gold equal to 76 dollars.

As 8 : 76 :: 9
9
8)684
Ans. $85\frac{1}{2}$pwt. = 4 oz. 5 pwt. 12 gr.

Or thus, 8)76
$9\frac{1}{2}$
Ans. $85\frac{1}{2}$pwt.

Or, $9\frac{1}{2}\times9=85\frac{1}{2}$pwt.

2. Required the weight of gold equal D.159 75c.

As 8 : 159·75 :: 9 : 179pwt. $17\frac{1}{4}$ gr. Ans.
9
8)1437·75
179·71875
24
287500
143750
17·25 grains.

Or, $\overline{159{\cdot}75\div8} + 159{\cdot}75 = 179$pwt. $17\frac{1}{4}$ gr. Ans.

Or, $\overline{159{\cdot}75\div8} \times 9 = 179$pwt. $17\frac{1}{4}$gr. as before.

RULE OF THREE DIRECT IN DECIMALS.

Rule.

Having reduced your fractions to decimals, and stated your question as in whole numbers, multiply the second and third together ; divide by the first, and the quotient will be the answer.

Examples.

1. If $\frac{5}{8}$ of a yard cost $\frac{7}{12}$ of a pound ; what will $9\frac{2}{3}$ yards come to ?

$\frac{5}{8} = {\cdot}625$, $\frac{7}{12} = {\cdot}583 +$, and $\frac{2}{3} = {\cdot}667$ —.

As ·625 : ·583 :: 9·667
·583
29001
77336
48335
·625)5635861(9·017 + = £.9 0s. 4d.+. Ans.
5625
1086
625
4611
4375
236

2. If

2. If 1oz. of silver cost 6s. 8d. what is the price of a bowl, which weighs 1lb. 7oz. 13gr.? Ans. £.6 6s. 10d.

3. If $9\frac{3}{4}$ yards cost D.11 25c. what will $\frac{1}{2}$yard come to?
Ans. 57c. $6\frac{12}{13}$m.

4. If 1hhd. sugar, weighing 9cwt. 3qrs. 14lb. cost £.27 13s. 7d. what will 3cwt. 1qr. 17lb. come to? Ans £.9 10s. $8\frac{1}{4}$d.

5. A tobacconist bought 5hhds. of tobacco, each weighing 8cwt. 2qrs. 19lb. for D.534 5c. what was it per ounce? Ans. $6\frac{2}{3}$m.

6. There is a cistern, which has 3 cocks, the first will empty it in $\frac{1}{4}$ of an hour, the second in $\frac{3}{4}$, and the third in $1\frac{1}{2}$ hour: in what time will it be emptied, if all three run together?

$$\text{As}\begin{cases} \overset{\text{h.}}{\cdot25} : \overset{\text{Cist.}}{1} :: \overset{\text{h.}}{1} : \overset{\text{Cist.}}{4} \\ \cdot75 : 1 :: 1 : 1\cdot333+ \\ 1\cdot5 : 1 :: 1 : 0\cdot667- \end{cases} \qquad \text{As } \overset{\text{Cist.}}{6} : \overset{\text{h.}}{1} :: \overset{\text{Cist.}}{1} : \cdot1667- = 10 \text{ min. Ans.}$$

6 Cist.

7. A conduit has a cock, which, running into a cistern, will fill it in 12 minutes: This cistern has three cocks; the first will empty it in $1\frac{1}{4}$ hour, the second in $37\frac{1}{2}$ minutes, and the third in $\frac{1}{2}$ an hour: In what time will the cistern be filled, if all four run together?

$$\text{As}\begin{cases} \cdot2 : 1 :: 1 : 5 \quad \text{the filling Cock.} \\ \left.\begin{matrix} 1\cdot25 : 1 :: 1 : 0\cdot8 \\ \cdot625 : 1 :: 1 : 1\cdot6 \\ \cdot5 : 1 :: 1 : 2 \end{matrix}\right\} \text{emptying Cocks.} \end{cases} \qquad \begin{cases} 5 \text{ cist. filled in an hour.} \\ 4\cdot4 \text{ do. emptied in do.} \\ \cdot6 \text{ cist. difference.} \end{cases}$$

4·4

Then, as $\overset{\text{Cist.}}{\cdot6} : \overset{\text{h.}}{1} :: \overset{\text{Cist}}{1} : \overset{\text{h.}}{1\cdot67-} = \overset{\text{h. m.}}{1\ 40}$ Ans.

8. If 19 yards cost $\overset{\text{D. d. c.}}{25\cdot\ 7\ 5}$ what will $435\frac{1}{2}$ yards come to?

```
        yds.  D. d. c.    yds.
As  19 : 25· 7 5 :: 435·5
                    25·75
                   ------
                   217 75
                  3048 5
                 21775
                 8710
                 -------  D.  d. c. m.
              19)11214·125(590· 2  1  7 2/19 Ans.
```

9. If 345 yards of tape cost D.5· 1d. 7c. 5m. what will 1 yd. cost?

As $\overset{\text{yds.}}{345} : \overset{\text{D. d. c. m.}}{5\cdot\ 1\ 7\ 5} :: \overset{\text{yds.}}{1} : \overset{\text{c. m.}}{\cdot0\ 1\ 5}$ Ans.

10. If I give D.12· 8d. 2c. 5m. for 675 tops, how many tops will 19 mills buy? Ans. 1 top.

RULE

RULE OF THREE INVERSE, OR RECIPROCAL PROPORTION,

Teaches, by having three numbers given, to find a fourth, which shall have the same proportion to the second, as the first has to the third.

Therefore, the greater the third term is, in respect to the first, the less will the fourth term be, in respect to the second ; or, the less the third term is in proportion to the first, the greater the fourth must be in proportion to the second ; and this is called *reciprocal, inverted,* or *indirect* Proportion.

The principal difficulty that will embarrass the learner will be, to distinguish when the proportion is direct. and when indirect. This is done by an attentive consideration of the sense and tenour of the question proposed : For if thereby it appears that, when the third term of the stating is less than the first, the answer must be less than the second ; or when the third is greater than the first, the answer must be greater than the second ; then the proportion is direct : But, if the third be less than the first, and yet the sense of the question requires the fourth to be greater than the second ; or if the third, being greater than the first, the answer must be less than the second, the proportion is inverse.

Rule.*

State and reduce the terms as in the Rule of Three Direct ; then, multiply the first and second terms together, and divide the product by the third ; the quotient will be the answer in the same denomination as the middle term was reduced into.

If there be fractions in your question, they must be stated as before directed, and if they be vulgar, invert the third term : Then multiply the three terms continually together, and the product will be the answer.

Examples.

1. How much shalloon, that is $\frac{3}{4}$ yard wide, will line $6\frac{3}{4}$ yards of cloth which is $1\frac{1}{4}$ yard wide ?

$$\begin{array}{rcccl} & \text{yd.} & \text{yds.} & \text{qrs.} \\ \text{As} & 1\frac{1}{4} : & 6\frac{3}{4} & :: 3 \\ & 4 & 4 & \\ \hline & 5 & 27 & \end{array}
\qquad
\begin{array}{rcccl} & \text{qrs.} & \text{qrs.} & \text{qrs.} \\ \text{As} & 5 : & 27 & :: 3 \\ & & 5 & \\ \hline & 3) & 135 & \\ \hline & 4) & 45 & \\ \hline & & 11\frac{1}{4} & \text{yards, Ans.} \end{array}$$

The

* The reason of this rule may be explained from the principles of Compound Multiplication and Compound Division, in the same manner as the direct rule.— *For example,* If 4 men can do a piece of work in 12 days, in what time will 8 men do it ?

As 4 men : 12 days :: 8 men : $\frac{4\times 12}{8}=6$ days, the Answer.

And here the product of the first and second terms, that is, 4 times 12, or 48, is evidently the time in which one man would perform the work. Therefore, 8 men will do it in one eighth part of the time, or 6 days.

The same by Vulgar Fractions.

First. $1\frac{1}{4}=\frac{5}{4}$, $6\frac{3}{4}=\frac{27}{4}$, and 3qrs.$=\frac{3}{4}$. Then, yds.

As $\frac{5}{4} : \frac{27}{4} :: \frac{3}{4}$. And $\frac{5}{4}\times\frac{27}{4}\times\frac{4}{3}=\frac{5\times27\times4}{4\times4\times3}=\frac{540}{48}=\frac{45}{4}=11\frac{1}{4}$ Answer.

The same by Decimal Fractions.

$1\frac{1}{4}=1{\cdot}25$, $6\frac{3}{4}=6{\cdot}75$ and 3qrs.$={\cdot}75$. Then,

As 1·25 : 6·75 :: ·75

```
      1·25
      ----
      3375
     1350
    675
    -----
·75)8·4375(11·25 yds. Ans.
    7 5
    ---
      93
      75
      ---
      187
      150
      ---
       375
       375
```

2. What length of board 7 inches wide, will make a square foot?

In. br. in. len. in. br. in. len.

As 12 : 12 :: $7\frac{1}{2}$: $19\frac{1}{5}$ Ans.

3. How many yards of carpet, $2\frac{3}{4}$ feet wide will cover a floor, which is 18 feet long and 16 feet wide?

ft. ft. ft. ft. yds.

As 16 : 18 :: $\overline{2\frac{3}{4}\times3}$: $34\frac{10}{11}$ Ans.

Note, I multiply $2\frac{3}{4}$ by 3, because 3 feet=1 yard.

4. Suppose I lend a friend £.350 for 5 months, he promising the like kindness; but, when requested, can spare but £.125, how long may I keep it to balance the favour?

£. Mo. £. Mo.

As 350 : 5 :: 125 : 14 Ans.

5. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain 9 months?

Mo. M. Mo. M. M.

As 5 : 450 :: 9 : 250, and 450 — 250=200 men, Ans.

6. If a man perform a journey in 15 days, when the day is 12 hours long, in how many days will he do it, when the day is but 10 hours?

Ans. 18 days.

7. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide must it be, when it is but 19 rods long to make an acre?

Ans. 8rods 6ft. $11\frac{7}{19}$in.

8. If when wheat is D.1 per bushel, the two penny loaf weigh 9·6oz. what ought it to weigh, when wheat is D.1 25c. per bushel?

Ans. 7oz. 13pwt. 14·4gr.

9. If

9. If a piece of board be 30 inches in length, what breadth will make $1\frac{1}{2}$ square foot ?

Ans. 7·2 inches.

10. If 9 men can build a house in 5 months, by working 14 hours per day, in what time will the same number of men do it, when they work only 10 hours per day ?

Ans. 7 months.

11. A wall, which was to be built 24 feet high, was raised 8 feet by 6 men, in 12 days : How many men must be employed to finish the wall in four days ?

ft. m. ft. m.

As 8 : 6 :: 24—8 : 12 to finish it in 12 days. And,

d. m. d. m.

As 12 : 12 :: 4 : 36 to finish in 4 days.

12. There is a cistern having a pipe, which will empty it in 6 hours : How many pipes of the same capacity, will empty it in 20 minutes ?

h. pi. mi. pi.

As 6 : 1 :: 20 : 18 Ans.

13. What number of men must be employed to finish in 9 days, what 15 men would be 30 days about ?

Ans. 50 men.

14. If a field will feed 6 cows 91 days, how long will it feed 21 cows ?

Ans. 26 days.

15. How much in length, that is $8\frac{4}{7}$ inches broad, will make a foot square ?

Ans. $16\frac{32}{61}$ inches.

16. How much in length, that is $13\frac{7}{8}$ poles in breadth, will make a square acre ?

Ans. $11\frac{59}{111}$ poles.

17. A regiment of soldiers, consisting of 745 men, is to be clothed, each suit to contain $3\frac{1}{2}$ yards of cloth, which is $1\frac{3}{8}$ yard wide, and lined with shalloon $\frac{7}{8}$ yard wide ; how many yards of shalloon will line them ?

As $745 \times 3\frac{1}{2}$: $1\frac{3}{8}$: $\frac{7}{8}$: $4097\frac{1}{2}$ yards, Ans.

18. If a suit of clothes can be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide ; how many yards of coating $\frac{7}{8}$ of a yard wide, will it require for the same person ?

Ans. 6yds. 1qr. $3\frac{5}{7}$n.

Abbreviations.

To know whether a fraction, when abbreviated, be equivalent in all respects to the original fraction.

Rule.

As the numerator of the fraction, in its lowest terms, is to its denominator ;

nominator ; so will the numerator of the original fraction be to its own denominator.

Or, as one numerator is to the other ; so will one denominator be to the other, &c.

A owes B 75l. 13s. 6d ; now 100l. of A's money is equal to 140l. of B's ; what must A pay to satisfy the said debt ?

$$\frac{100}{140}=\frac{5}{7}, \text{ therefore, } \begin{array}{r} £.\ \ \text{s.}\ \ \text{d.} \\ 75\ \ 13\ \ 6 \\ 5 \\ \hline 7)378\ \ \ 7\ \ 6 \\ \hline £.54\ \ \ 1\ \ 0\tfrac{6}{7}\ \text{Ans.} \end{array}$$

Now, to prove whether $\frac{5}{7}$ be equal $\frac{100}{140}$.

Num. Den. Num. Den. Num. Num. Den. Den.

As 5 : 7 :: 100 : 140——Or, as 5 : 100 :: 7 : 140.

COMPOUND PROPORTION,

OR DOUBLE RULE OF THREE,

TEACHES to resolve such questions as require two, or more, statings by simple proportion ; and that, whether direct or inverse : It is composed (commonly) of 5 numbers to find a sixth, which if the proportion be direct, must bear such proportion to the 4th and 5th as the 3d bears to the 1st and 2d ; but if inverse, the 6th number must bear such proportion to the 4th and 5th, as the first bears to the 2d, and 3d.

First Method.*

By two, or more, proportions in the Single Rule of Three.

Rule.

1. Let either of the two numbers, of which the question is raised, be put in the third place, and the correspondent number, of the same name or kind, in the first ; the second will be that, which has no correspondent number given.

2. Three of the five given numbers being thus stated, find a fourth proportional.

3. Put

* The reason of this rule may be shewn from the nature of direct and inverse proportion :—For, in this rule, every row is a particular stating in one of those rules ; and, therefore, if all the separate dividends be collected into one dividend, and all the divisors into one divisor, their quotients must be the answer sought : Thus, in example 1st.

$$\begin{array}{l} \quad\ \text{D.}\quad\ \ \text{D.}\quad\ \text{D.} \qquad\qquad\qquad\qquad\quad \text{Mo.}\quad\ \text{D. D.}\qquad\quad \text{D.} \\ \text{As } 100 : 6 :: 400 : \dfrac{400\times6}{100}, \text{ and as } 12 : \dfrac{400\times6}{100} :: 9 : \dfrac{400\times6\times9}{100\times12} \text{ by the} \end{array}$$

Rule of Three Direct.

3. Put this fourth number for a second number of a second stating, the remaining number of which the question is raised, the third, and its correspondent number of the same name, the first, then will the fourth number resulting be the answer.

EXAMPLES.

If a principal of D.100 gain D.6 interest in a year; what will a principal of D.400 gain in 9 months?

Here of the five given numbers D.100 principal, D.6 interest, and a year or 12 months, are conjoined in form of a supposition, and thereupon a question is raised concerning D.400 for 9 months; wherefore, let either of the two numbers, D.400 or 9 months, be put for the third number of the first stating, and its corresponding term D.100 or 12 months, for the first.

```
   D.  D.   D.                          Mo.  D.  Mo.
As 100 : 6 :: 400                    As 12 : 24 :: 9
              6                                    9
            ----                                ----
    1|00)24|00                              12)216
            ----                                ----
       D. 24                               D. 18  Ans.
                          Or thus,
 Mo.  D.  Mo.  D.                   D.    D.     D.    D.
As 12 : 6 :: 9 : 4½.        And, As 100 : 4½ :: 400 : 18
```

Such questions as, when stated, are found to have both statings direct, may be solved more readily by one compound stating, thus: Place the two terms, of which the question is raised, under one another in the third place, their correspondent terms under each other in the first, and the remaining term in the middle: Then multiply both these first terms together, and the third terms together, and so the double stating is reduced to a simple one of the *Rule of Three Direct*; viz. the product of the two first terms is the first of a simple stating; the second term is the second, and the product of the two third terms is the third, to find a fourth proportional—Thus,

$$\text{As} \left\{ \begin{array}{r} 100 \\ 12 \end{array} \right\} : 6 :: \left\{ \begin{array}{r} 400 \\ 9 \end{array} \right\}$$

So the first example will stand thus:

```
D. 100 }          { 400 D. }
Mo. 12 } : D.6 :: {   9 Mo.}
 -----              -----
 12|00              36|00
                        6
                     ----
                  12)216
                     ----
                  D.18 Ans.
```

SECOND

Second Method.

Always place the three conditional terms in this order : That number, which is the principal cause of gain, loss or action, possesses the first place ; that, which denotes the space of time, distance of place, rate, medium or mean of action, the second ; and that, which is the gain, loss or action, the third : This being done, place the other two terms which move the question, under those of the same name, and if the blank place, or term sought, fall under the third place, then the question is in direct proportion : therefore,

Rule 1.—Multiply the three last terms together, for a dividend, and the two first for a divisor :—But if the blank fall under the first or second place ; then, the proportion is inverse ; therefore,

Rule 2.—Multiply the first, second and last terms together for a dividend, and the other two for a divisor, and the quotient will be the answer.

Examples.

1. If D.100 gain D.6 in a year ; what will D.400 gain in 9 months ?

D P. Mo. D. Int.
100 : 12 :: 6 Terms in the supposition, or conditional terms.
400 : 9 Terms which move the question.

Here, the blank falling under the third place, the question is in direct proportion, and the answer must be found by the first Rule ; therefore,

400× 9×6=21600 For the dividend, and,
100×12 =1200 For the divisor.

See the work at large.

```
          D. Pr. Mo.  D. Int.
           100 : 12 :: 6
           400 :  9
                  9
                ----
   100         3600
    12            6
  ----         ----
12|00)216|00(18D. Ans.
      12
      --
       96
       96
       --
```

2. If D.100 will gain D.6 in a year ; in what time will D.400 gain D.18 ?

D. Mo. D.
100 : 12 :: 6 Terms in the supposition.
400 : :: 18 Terms which move the question.

Here

Here, the blank falling under the 2d place, the question is in reciprocal or inverse Proportion, and the answer must be sought by the second rule; therefore,

100×12×18=21600 For the dividend.
400× 6 = 2400 For the divisor.

D. Pr.	Mo.	D. Int.
100 :	12 ::	6
400 :	::	18
6		12
2400		216
		100

24|00)216|00(9 months, Ans.
216

3. What principal, at 6 per cent. per ann. will gain D.18 in 9 months?

Pr.	Mo.	Int.
100 :	12 ::	6
	9 ::	18
		12
	9	216
	6	100

D.
54)21600(400 Ans.
216

00

4. If D.400 gain D.18 in 9 months; what is the rate per cent. per anunum?

Pr.	Mo.	Int.
400 :	9 ::	18
100 :	12 ::	D.6 Ans.

Here, the blank falling under the first place, the proportion is inverse, and the answer found by the second rule, as in the last example.

5. If 8 men spend £.32 in 13 weeks; what will 24 men spend in 52 weeks? Ans. £.384

6. If the freight of 9hhds of sugar, each weighing 12cwt. 20 leagues, cost D.50; what must be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$cwt. 100 leagues? Ans. D.289 35c. $1\frac{23}{27}$m.

7. There was a certain edifice completed in a year by 20 workmen; but the same being demolished, it is necessary that just such an one should be built in 5 months. I demand the number of men to be employed about it? Ans. 48 men.

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick?

m.	da.	ft.
6 :	16 ::	20×6×4
24 :	::	200×8×6 80 days, Ans.

COMPARISON

COMPARISON OF WEIGHTS AND MEASURES.

EXAMPLES.

1. If 78 pence Massachusetts be worth 1 French crown, how many Massachusetts pence are worth 320 French crowns?

F. cr. d. F. cr.
As 1 : 78 :: 320
78

2560
2240

24960 Ans.

2. If 24 yards at Boston make 16 ells at Paris, how many ells at Paris will make 128 yards at Boston?

Bost. Par. Bos. Par.
As 24yds. : 16ells. :: 128yds. : $85\frac{1}{3}$ells, Ans.

3. If 60lb. at Boston make 56lb. at Amsterdam, how many lb. at Boston will be equal to 350 at Amsterdam?

Ans. 375lb. Boston.

4. If 95lb. Flemish make 100lb. American, how many American lbs. are equal to 550lb. Flemish?

Ans. $578\frac{18}{19}$lb. American.

CONJOINED PROPORTION

IS when the coins, weights or measures of several countries are compared in the same question; or, in other words, it is joining many proportions together, and by the relation, which several antecedents have to their consequents, the proportion between the first antecedent and the lost consequent is discovered, as well as the proportion between the others in their several respects.

This rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble, and it may be proved by as many statings in the Single Rule of Three, as the nature of the question may require.

CASE I.

When it is required to find how many of the first sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the last.

RULE.

Place the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right, and let the last number stand on the left hand; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

EXAMPLES.

EXAMPLES.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouse, and 100 canes of Thoulouse=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many yards of America are equal to 379 ells of Hamburgh?

Antecedents.	Consequents.	Abriged. Ant.	Con.
100 of America	= 100 of England.		
100 of England	= 50 of Thoulouse.	5	8
100 of Thoulouse	= 160 of Geneva.	379	
100 of Geneva	= 200 of Hamburgh.		
379 of Hamburgh?			

Therefore, $\frac{379\times5}{8}=236\frac{7}{8}$yds. of America=379ells of Hamburgh.

ILLUSTRATION.

The two 100s of both sides cancel each other. Let the last cyphers of the three next antecedents and consequents be cancelled, which is dividing by 10 Then divide the second antecedent and consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8. 10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2. Now, there being 2 left on each side, they cancel each other, and as there is no farther room for abridging by reason of the odd number 379, the operation is finished. and the answer found, as before.

2. If 20lb at Boston make 23lb at Antwerp, and 155 at Antwerp make 180 at Leghorn: How many at Boston are equal to 144 at Leghorn?

Ans. $107\frac{19}{23}$lb.

3. If 12lb. at Boston make 10lb. at Amsterdam, 10lb. at Amsterdam 120lb. at Paris: How many lb. at Boston are equal to 80lb. at Paris?

Ans. 80lb.

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American yards: How many Venetian braces are equal to 32 American yards?

Ans. $52\frac{4}{15}$.

5. If 40lb. at Newburyport make 36 at Amsterdam, and 90lb. at Amsterdam make 116 at Dantzick: How many lb. at Newburyport are equal to 260lb. at Dantzick?

Ans. $224\frac{4}{29}$.

CASE II.

When it is required to find how many of the last sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the first.

RULE.

Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand; then multiply the first row for a divisor, and the second for a dividend.

EXAMPLES.

EXAMPLES.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouse, and 100 canes of Thoulouse=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many ells of Hamburgh are equal to $236\frac{7}{8}$ yards of America?

Ant.		Con.		Abridged. Ant.	Con.
100 Amer	=	100	Eng.	5.	8
100 Eng.	=	50	Thoul.		$236\frac{7}{8}$
100 Thoul.	=	160	Gen.		
100 Gen.	=	200	Hamb.		
		$236\frac{7}{8}$	Amer.		

$$\frac{236\frac{7}{8}\times 8}{5} = 379 \text{ Ham. Ans.}$$

This needs no further illustration. The learner will readily see, that, this case being the reverse of the former, they are proofs to each other.

2. If 20lb. at Boston make 23lb at Antwerp, and 155 at Antwerp make 180 at Leghorn: How many at Leghorn are equal 144 at Boston? Ans 144lb.

3. If 12lb. at Boston make 10lb. at Amsterdam, and 100lb. at Amsterdam 120lb at Paris: How many at Paris are equal to 80lb. at Boston? Ans. 80lb.

4 If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American yards: How many American yards are equal to $52\frac{4}{15}$ Venetian braces?

Ans. 32 yards.

5. If 40lb. at Newburyport make 36 at Amsterdam, and 90lb. at Amsterdam make 116 at Dantzick: How many lb. at Dantzick are equal to 244 at Newburyport? Ans. $283\frac{12}{25}$lb.

ARBITRATION OF EXCHANGES.

By this term is understood how to choose, or determine the best way of remitting money from abroad with advantage; which is performed by conjoined proportion: Thus,

Suppose a merchant has effects at Amsterdam to the amount of 3530 dollars, which he can remit by way of Lisbon at 840 rees per dollar, and thence to Boston, at 8s. 1d per milree (or 1000 rees:) Or, by way of Nantz, at $5\frac{2}{3}$ livres per dollar, and thence to Boston at 6s. 8d. per crown, It is required to arbitrate these exchanges, that is, to choose that which is most advantageous?

1 dollar at Amsterdam = 840 rees at Lisbon.
1000 rees at Lisbon = 97d. at Boston.
3530 dollars at Amsterdam.

$$\frac{840\times 97\times 3530}{1000\times 1} = £1198\ 8s.\ 8\tfrac{4}{10}d. \text{ by way of Lisbon.}$$

1 dollar at Amsterdam = $5\frac{2}{3}$ livres at Nantz.
6 livres at Nantz = 80 pence at Boston.
3530 dollars at Amsterdam.

$$\frac{5\frac{2}{3}\times 80\times 3530}{1\times 6} = £.1059 \text{ by way of Nantz.}$$

Here

Here it may be observed that the difference is £139 8s. $8\frac{4}{10}$d in favour of remitting by way of Lisbon rather than by Nantz, which depends on the *course* of exchange, at that time ; but the *course* may vary so, that, in a short time by way of Nantz may be better ; hence appears the necessity and advantage of an extensive correspondence, to acquire a thorough knowledge in the *courses* of exchange, to make this kind of remittance.

FELLOWSHIP.

THE Rules of Fellowship are those by which the accompts of several merchants or other persons, trading in partnership, are so adjusted, that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the joint stock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP

Is, when the stocks are employed for any certain equal time.

Rule.*

As the whole stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain, or loss.

Proof. Add all the particular shares of the gain or loss together, and, if it be right, the sum will be equal to the whole gain or loss.

Examples.

1. Divide the number 360 into four such parts, which shall be to each other, as 3, 4, 5 and 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{\begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array}\right\} \text{ Answer.}$$

$$\overline{360} \text{ Proof.}$$

2. A, B, C and D companied ; A put in £.145 ; B, £.219 ; C, £.378, and D, £.417, with which they gained £.569 : What was the share of each ?

$$\begin{array}{l} \text{Whole stock.} \qquad \text{Gain.} \\ \text{As } 145+219+378+417 : 569 :: \end{array} \left\{\begin{array}{lrrll} & £. & \text{s.} & \text{d.} & \\ 145 : & 71 & 3 & 8\frac{1}{2}\ \frac{1058}{1159} & \text{A's sha.} \\ 219 : & 107 & 10 & 3\frac{3}{4}\ \frac{375}{1159} & \text{B's dit.} \\ 378 : & 185 & 11 & 6\ \ \frac{552}{1159} & \text{C's dit.} \\ 417 : & 204 & 14 & 5\frac{1}{4}\ \frac{133}{1159} & \text{D's dit.} \end{array}\right.$$

£.569 — Proof. 3. A.

* That their gain or loſs, in this rule, is in proportion to their ſtocks is evident: For, as the times, in which the ſtocks are in trade, are equal, if I put in ½ of the whole ſtock, I ought to have ½ of the gain : If my part of the ſtock be ¼, my ſhare of the gain or loſs ought to be ¼ alſo. And generally the ſame ratio that the whole ſtock has to the whole gain or loſs, muſt each perſon's particular ſtock have to his reſpective gain or loſs.

3. A, B, C and D are concerned in a joint stock of D.1000; of which A's part is D. 150; B's D. 250; C's D.275, and D's D.325.—Upon the adjustment of their accompts, they have lost D.337 50c. What is the loss of each? Ans. A's loss D.50 62½c. B's D.84 37½c. C's D.92 81¼c. and D's D.109 68¾c.

4. A and B companied; A put in £.45, and took ⅗ of the gain; What did B put in? 5—3=2. Then, As 3 : 45 :: 2 : 30 Ans.

5. A, B and C freighted a ship with 68900 feet of boards: A put in 16520 feet; B 28750; and C the rest; but, in a storm, the Captain threw overboard 26450 feet: How much must each sustain of the loss? Ans. A, 6341¾ feet. B, 11036¼ and C, 9071½ do.

6. A gentleman died, leaving three sons and a daughter, to whom he bequeathed his estate in the following manner: To the eldest son he gave 312 moidores, to the second 312 guineas, to the third 312 pistoles, and to the daughter 312 dollars; but when his debts were paid, there were but 312 half joes left: What must each have in proportion to the legacies which had been bequeathed them?

Ans. 1st son £.293 0s. 3d.—2d. son £.227 17s. 10¾d.— 2d. son £.179 1s. 2½d. and the daughter £.48 16s. 8¼d.

7. A ship, worth D.3000, being lost at sea, of which ⅙ belonged to A, ½ to B, and the rest to C: What loss will each sustain, supposing D.450 to have been insured upon her?

Ans.	A's loss	D.312 50c.
	B's	937 50
	C's	625

8. A and B venturing equal sums of money, cleared by joint trade D.140: By agreement, as A executed the business, he was to have 8 per cent. and B was to have 5 per cent.: What was A allowed for his trouble?

D. D. D. D. D. D. D. D.
As 8+5 : 140 :: 8 : $86\frac{2}{13}$ And, as 8+5 : 140 :: 5 : $53\frac{11}{13}$.

Ans. D.32 30c. $7\frac{9}{13}$m.

9. A bankrupt is indebted to A £.120, to B £.230, to C £.340, and to D £.450, and his whole estate amounts only to £.560: How must it be divided among the creditors?

Ans. A, £.58 18s. 11¼d. B, £.112 19s. 7¾d. C, £.167 0s. 4d. and D, £.221 1s. 0½d.

10. A, B and C put their money into a joint stock; A put in D.40; B and C together, D.170: They gained D.126, of which B took D.42; What did A and C gain, and B and C put in respectively?

As D.210 the whole stock : D.126 the whole gain :: D.40 A's stock : D.24 A's gain.

As D.24 A's gain : D.40 A's stock :: D.42 B's gain : D.70 B's stock. Then D.170—D.70=D.100 C's stock; and whole gain D.126—D.66 A's and B's gain=D.60 C's gain.

11. A, B and C companied;—A put in £.40; B 60, and C a sum unknown: They gained £.72; of which C took £.32 for his share: What did A and B gain, and C put in?

The

The whole gain £.72—C's gain £.32=£.40. A's and B's gain : Then, As £.100, A's and B's stock : £.40 their gain :: £.40 A's stock : £.16, his gain. Again, As £.16 A's gain : £.40, his stock :: £.32, C's gain : £.80, his stock.

12. A, B and C put in D.720, and gained D.540, of which, so often as A took up D.3, B took 5, and C 7 : What did each put in and gain?

$$\begin{array}{r} \text{D. D. D. D.} \\ \text{As } 3+5+7 : 540 :: \end{array} \left\{ \begin{array}{l} \text{D.} \quad \text{D.} \\ 3 : 108 \text{ A's gain.} \\ 5 : 180 \text{ B's ditto} \\ 7 : 252 \text{ C's ditto.} \end{array} \right\}$$

$$\begin{array}{r} \text{D.} \\ \text{And, as } 3+5+7 : 720 :: \end{array} \left\{ \begin{array}{l} 3 : 144 \text{ A's stock.} \\ 5 : 240 \text{ B's ditto.} \\ 7 : 336 \text{ C's ditto.} \end{array} \right\}$$

Or, you may find a common multiplier to multiply the proportions by, or multiplicand to be multiplied by the given proportions, thus, 15)720(48 multiplicand to find the stocks.—And 15)540(36 multiplicand to find the gains.

$$\left. \begin{array}{l} \text{D.} \\ 48\times3=144 \text{ A's stock.} \\ 48\times5=240 \text{ B's ditto.} \\ 48\times7=336 \text{ C's ditto.} \end{array} \right\} \text{And} \left\{ \begin{array}{l} \text{D.} \\ 36\times3=108 \text{ A's gain.} \\ 36\times5=180 \text{ B's ditto.} \\ 36\times7=252 \text{ C's dittto. as before.} \end{array} \right.$$

13. A, B, C and D companied ; and gained a sum of money of which A, B and C took £.120, B, C and D, £.180, C, D, and A, £ 160, and D, A and B, £.140 ; What distinct gain had each?
The sum of these 4 numbers is £.600, and as each man's money is named 3 times, therefore $\frac{1}{3}$, viz. £.200 is the whole gain—Therefore £.200—£.120 A's, B's and C's gain=£.80 D's gain ;—And £ 200—£.180 B's, C's and D's gain=£.20 A's gain.—£.200—£.160 C's, D's and A's gain=£ 40 B's gain.—And £.200—£.140 D's, A's and B's gain=£.60 C's gain.

14. Two merchants companied ; A put in £.40, and B 288 ducats. They gained £.135, of which A took £.60. What was the value of a ducat?
As £.60, A's gain : £.40, his stock :: £.135 the whole gain—£.60, A's gain : £.50, B's stock.

$$\begin{array}{lccccc} & \text{Duc.} & \text{£.} & \text{Duc.} & \text{s.} & \text{d.} \\ \text{And, as} & 288 & : 50 :: & 1 & : 3 & 5\tfrac{2}{5} \text{ Ans.} \end{array}$$

15. Four men spent, at a reckoning, 20 shillings, of which they agreed that A should pay $\frac{3}{4}$, B, $\frac{1}{2}$, C, $\frac{1}{4}$, and D, $\frac{1}{8}$. What must each pay in that proportion?

$$\begin{array}{r} \text{£. £. £. £. s.} \\ \text{As } \frac{3}{4}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8} : 20 :: \end{array} \left\{ \begin{array}{rrcrl} \text{s.} & \text{d.} & & \text{s.} & \text{d.} \\ 15 & 0 & : & 9 & 2\frac{10}{13} \\ 10 & 0 & : & 6 & 1\frac{11}{13} \\ 5 & 0 & : & 3 & 0\frac{12}{13} \\ 2 & 6 & : & 1 & 6\frac{6}{13} \end{array} \right\} \text{Answer.}$$

DOUBLE

DOUBLE FELLOWSHIP,*

Or, *Fellowship with Time*, is occasioned by the shares of partners being continued unequal times.

Rule.

Multiply each man's stock, or share, by the time it was continued in trade. Then,

As the whole sum of the products, is to the whole gain or loss, so is each man's particular product, to his particular share of the gain or loss.

Examples.

1. A, B and C hold a pasture in common, for which they pay 40l. per annum. A put in 9 oxen for 5 weeks; B, 12 oxen for 7 weeks, and C 8 oxen for 16 weeks. What must each pay of the rent?

9×5=45. 12×7=84, and 8×16=128, then 128+84+45=257

As 257 : 40 :: 45	As 257 : 40 :: 84	As 257 : 40 :: 128
45	84	40
200	160	257)5120(19
160	320	257
257)1800(7	257)3360(13	2550
1799	257	2313
1	790	237
20	771	20
257)20(0	19	257)4740(18
12	20	257
257)240(0	257)380(1	2170
4	257	2056
257)960(3	123	114
771	12	12
189	257)1476(5	257)1368(5
	1285	1285
	191	83
	4	4
	257)764(2	257)332(1
	514	257
	250	75

2. Four

* When times are equal, the ſhares of the gain or loſs are evidently as the ſtocks, as in Single Fellowſhip; and when the ſtocks are equal, the ſhares are as the times; wherefore, when neither are equal, the ſhares muſt be as their products.

T

2. Four merchants traded in company ; A put in D.400 for five months, B, D.600 for 7 months, C, D.960 for 8 months, and D, D.1200 for 9 months ; but by misfortunes at sea, they lost D.750. What must each man sustain of the loss ?

Answer, $\left\{\begin{array}{llll} \text{A, D.94} & \text{93c.} & 6\frac{56}{79}\text{m.} & \text{C D.227 84c. } 8\frac{8}{79}\text{m.} \\ \text{B } 142 & 40 & 5\frac{5}{79} & \text{D } 284\ 81\ 0\frac{10}{79} \end{array}\right\}$

3. A, with a capital of 100l. began trade January 1st 1787, and meeting with success in his business, he took in B as a partner, on the 1st day of March following, with a capital of 150l. Three months after that, they admit C as a third partner, who brought into stock 180l. and after trading together until the 1st of January 1788, they found there had been gained since A's commencing business, 177l. 13s. How must this be divided among the partners ?

Ans. A, 53l. 16s. 8d. B, 67l. 5s. 10d. C, 56l. 10s. 6d.

4. Two merchants entered into partnership for 18 months ; A, at first, put into stock D.400, and at the end of 8 months he put in D.200 more ; B, at first, put in D.1100, and at 4 months' end took out D.280. Now at the expiration of the time, they found they had gained D.1052. What is each man's just share ?

Ans. A, D.385 90c. B, D.666 10c.

5. A and B companied ; A put in the 1st of January, 150l. ; but B could not put in any until the 1st of May : What did he then put in, to have an equal share with A at the year's end ?

$$\begin{array}{l} \quad\;\; \text{M.} \quad \text{£} \quad\;\; \text{M.} \\ \text{As } 12 : 150 :: 8 : \dfrac{150\times12}{8} = \text{£}.225 \text{ Ans.} \end{array}$$

6. A, B and C companied ; A put in, the first of March, 30l. B, the 1st of May, put in 80 yards of broadcloth ; and on the 1st of June C put in 120 dollars. On the 1st of January following, they reckoned their gains, of which A and B took 228l. B and C 215l. 10s. and C and A 187l. 10s. What was the whole gain, and the gain of each ? What did they value a yard of cloth at ? and, what was C's dollar worth ?

228l. + 215l. 10s + 187l. 10s. = 631l. and 631 ÷ 2 = 315l. 10s. the whole gain ; then, 315l. 10s.—228 = 87l. 10s. C's gain. 315l. 10s.—215l. 10s. = 100l. A's gain, and 315l. 10s.—187l. 10s. = 128l. B's gain. To find the value of one yard of cloth, say, As 100l. A's gain : 30l. his stock :: 128l. B's gain : 38l. 8s. ; then, inversely, As 10 months : 38l. 8s. :: 8 months : 48l. the value of the whole cloth.

As 80yds. : 48l. :: 1yd. : 12s. answer. Now, to find the value of a dollar. As 100l. A's gain : 30l. his stock :: 87l. 10s. C's gain : 26l. 5s. ; then, inversely, As 10 months : 26l. 5s :: 7 months : 37l. 10s. = 120 dollars. Lastly : As 120 dollars : 37l. 10s. :: 1 dollar : 6s. 3d. Answer.

FELLOWSHIP

FELLOWSHIP BY DECIMALS.

RULE.*

Divide the whole gain, or loss, by the whole stock, or sum of all the products, as the case requires, and the quotient multiplied severally, by each man's stock, or product, will give the gain or loss of each.

EXAMPLES.

1. A, B and C companied, A put in 40l. 5s.; B 80l. 10s. and C, 161l. with which they gained 120l.: What is each man's share of the gain?

A's Stock = 40·25
B's ditto = 80·5
C's ditto = 161·

Sum total = 281·75)120·000000(·4259+

·4259	·4259	·4259
40·25	80·5	161
21295	21295	4259
8518	34072	25554
17036	£.34·28495	4259
£.17·142475	20	£.68·5699
20	5·69900	20
2·849500	12	11·3980
12	8·388	12
10·194000	4	4·776
4	1·552	4
0·776000		3·104

Proof. A's gain 17l. 2s. 10d. + B's gain 34l. 5s. 8¼d. + C's gain 68l. 11s. 4¾d. = 119l. 19s. 11d.

2. A, B and C companied; A put in D.400 for 8 months; B D.300 for 9 months; and C D.175 for 12 months; with which they gained D.720. Required the share of each?

	D.		Mo.		Prod.
A	400	×	8	=	3200
B	300	×	9	=	2700
C	175	×	12	=	2100

Sum of products = 8000)720·(·09 = quotient.

D.

3200 × ·09 = 288 = A's share,
2700 × ·09 = 243 = B's ditto, Ans.
2100 × ·09 = 189 = C's ditto,

3. A.

* This is no more than Division of Decimals.

3. A, B, C and D trade, and gain 200l. which is to be divided in the following manner, viz. so often as A has 6l. B must have 10l. C, 14l. and D, 20l. What is the share of each ?

$6 + 10 + 14 + 20 = 50$, and $\frac{200}{50} = 4$, quotient ; then $6 \times 4 = 24$l. A's gain ; $10 \times 4 = 40$l. B's gain ; $14 \times 4 = 56$l C's ; and $20 \times 4 = 80$l. D's gain.

4. An insolvent estate amounting to D.633 60c. is indebted to A D.312 75c. to B D.297, to C D.50 25c. to D 25c. to E D.200, to F D.142 50c. and to G D.21 25c. ; what proportion will each creditor receive ?

$$\frac{633{\cdot}6}{312{\cdot}75 + 297 + 50{\cdot}25 + {\cdot}25 + 200 + 142{\cdot}5 + 21{\cdot}25} = {\cdot}61875 \text{ ; or,}$$

61c, $8\frac{3}{4}$m. on a dollar. And,

$$\begin{array}{r} \text{D. c.} \\ 312{\cdot}75 \\ 297{\cdot} \\ 50{\cdot}25 \\ {\cdot}25 \\ 200{\cdot} \\ 142\ 50 \\ 21{\cdot}25 \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\\1\end{array}}\right\} \times {\cdot}61875 = \left\{\begin{array}{rrl} \text{D. c.} & \text{m.} & \\ 193{\cdot}51 & 4\frac{1}{16} & = \text{A's proportion.} \\ 183{\cdot}76 & 8\frac{3}{4} & = \text{B's ditto.} \\ 31{\cdot}09 & 2\frac{3}{16} & = \text{C's ditto.} \\ {\cdot}15 & 4\frac{11}{16} & = \text{D's ditto.} \\ 123{\cdot}75 & & = \text{E's ditto.} \\ 88{\cdot}17 & 1\frac{7}{8} & = \text{F's ditto.} \\ 13{\cdot}14 & 8\frac{7}{16} & = \text{G's ditto.} \end{array}\right.$$

Proof. D.633·60

PRACTICE

IS a contraction of the Rule of Three Direct, when the first term happens to be an unit, or one ; and has its name from its daily use among merchants and tradesmen, being an easy and concise method of working most questions which occur in trade and business.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

General Rule.

1. Suppose the price of the given quantity to be 1l. or 1s. &c. then will the quantity itself be the answer at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price, or of one another, and the sum of the quotients belonging to each will be the true answer required.

Example.

What is the value of 468 yards, at 2s. $9\frac{1}{4}$d. per yard ?

	£.468 s. d	Answer at £.1 s. d.
2s. 6d. is $\frac{1}{8}$ =	58 10 0	ditto at 0 2 6
3d. is $\frac{1}{10}$ =	5 17 0	ditto at 0 0 3
$\frac{1}{4}$d. is $\frac{1}{12}$ =	0 9 9	ditto at 0 0 $0\frac{1}{4}$
The full price =	£.64 16 9	0 2 $9\frac{1}{4}$

In

In this example it is plain, that the quantity 468 is the answer at 1l.; consequently as 2s. 6d. is $\frac{1}{8}$ of a pound, $\frac{1}{8}$ part of that quantity, or 58l. 10s. is the price at 2s. 6d; in like manner, as 3d is the $\frac{1}{10}$ part of 2s 6d. so $\frac{1}{10}$ part of 58l. 10s. or 5l. 17s. is the answer at 3d. and as $\frac{1}{4}$d. is $\frac{1}{12}$ of 3d. so $\frac{1}{12}$ of 5l. 17s. or 9s. 9d is the answer at $\frac{1}{4}$d.—Now, as the sum of all these parts is equal to the whole price (2s. 9$\frac{1}{4}$d.) so the sum of the answers belonging to each price will be the answer at the full price required, and the same will be true in any example whatever.

General Rule,

To find the value of goods in Federal Money.—Multiply the price and quantity together; point off in the product, for denominations lower than dollars, as many places as there are in the given price; or, if there be decimal places in the quantity, (or lower denominations previously reduced to decimals,) according to multiplication of decimals.

Examples.

1. What cost 823 yards, at D.1 29c. per yard?

D c. D. c.
823 × 1·29 = 1061·67, Ans.

2. What cost 56 yds. 2 qrs. at D.3·11 per yard?

56 yds. 2qrs. = 56·3 yds.; and 56·5 × 3·11 = D.175 71c. 5m. Ans.

Before the questions, hereafter given, can be wrought, the following Tables must be perfectly gotten by heart.

TABLES.

Aliquot, or even Parts of Money.

Pts. of a shil. of a £

d.	s.	£.
6	$=\frac{1}{2}$	$=\frac{1}{40}$
4	$=\frac{1}{3}$	$=\frac{1}{60}$
3	$=\frac{1}{4}$	$=\frac{1}{80}$
2	$=\frac{1}{6}$	$=\frac{1}{120}$
1$\frac{1}{2}$	$=\frac{1}{8}$	$=\frac{1}{160}$
1	$=\frac{1}{12}$	$=\frac{1}{240}$
$\frac{3}{4}$	$=\frac{1}{16}$	$=\frac{1}{320}$
$\frac{1}{2}$	$=\frac{1}{24}$	$=\frac{1}{480}$
$\frac{1}{4}$	$=\frac{1}{48}$	$=\frac{1}{960}$

Parts of 2 Shill.

d.		2s.
1	=	$\frac{1}{24}$
1$\frac{1}{2}$	=	$\frac{1}{16}$
2	=	$\frac{1}{12}$
3	=	$\frac{1}{8}$
4	=	$\frac{1}{6}$
6	=	$\frac{1}{4}$
8	=	$\frac{1}{3}$

Parts of a Pound.

s.	d.		£.
10	0	=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$
3	4	=	$\frac{1}{6}$
2	6	=	$\frac{1}{8}$
1	8	=	$\frac{1}{12}$
1	4	=	$\frac{1}{15}$
1	3	=	$\frac{1}{16}$
1	0	=	$\frac{1}{20}$
0	10	=	$\frac{1}{24}$
0	8	=	$\frac{1}{30}$
0	5	=	$\frac{1}{48}$
0	2$\frac{1}{2}$	=	$\frac{1}{96}$

Parts of a Dollar.

c.		D.
50	=	$\frac{1}{2}$
33$\frac{1}{3}$	=	$\frac{1}{3}$
25	=	$\frac{1}{4}$
20	=	$\frac{1}{5}$
16$\frac{2}{3}$	=	$\frac{1}{6}$
12$\frac{1}{2}$	=	$\frac{1}{8}$
8$\frac{1}{3}$	=	$\frac{1}{12}$
6$\frac{1}{4}$	=	$\frac{1}{16}$
5	=	$\frac{1}{20}$
4	=	$\frac{1}{25}$
2	=	$\frac{1}{50}$

Aliquot, or even Parts of Weight,

Parts of a Cwt.			Parts of $\frac{1}{2}$ Cwt.		Parts of $\frac{1}{4}$ Cwt.		Parts of a Ton,		
Qrs.	lb.	Cwt.	lb.	$\frac{1}{2}$ Cwt.	lb.	$\frac{1}{4}$ Cwt.	Cwt.	qr.	T.
2	0	= $\frac{1}{2}$	28	= $\frac{1}{2}$	14	= $\frac{1}{2}$	10	0	= $\frac{1}{2}$
1	0	= $\frac{1}{4}$	14	= $\frac{1}{4}$	7	= $\frac{1}{4}$	5	0	= $\frac{1}{4}$
0	16	= $\frac{1}{7}$	8	= $\frac{1}{7}$	4	= $\frac{1}{7}$	4	0	= $\frac{1}{5}$
0	14	= $\frac{1}{8}$	7	= $\frac{1}{8}$	2	= $\frac{1}{14}$	2	2	= $\frac{1}{8}$
0	8	= $\frac{1}{14}$	4	= $\frac{1}{14}$			2	0	= $\frac{1}{10}$
0	7	= $\frac{1}{16}$					1	1	= $\frac{1}{16}$
0	4	= $\frac{1}{28}$					1	0	= $\frac{1}{20}$

Another Table of aliquot Parts of Money.

Parts of a Shill.

d.	s.
10	= $\frac{5}{6}$
9	= $\frac{3}{4}$
8	= $\frac{2}{3}$
$7\frac{1}{2}$	= $\frac{5}{8}$
$4\frac{1}{2}$	= $\frac{3}{8}$

Parts of a Pound.

s.	d.	£.
18	0	= $\frac{9}{10}$
17	6	= $\frac{7}{8}$
16	8	= $\frac{5}{6}$
16	0	= $\frac{8}{10}$
15	0	= $\frac{3}{4}$
14	0	= $\frac{7}{10}$
13	4	= $\frac{2}{3}$
12	6	= $\frac{5}{8}$
12	0	= $\frac{6}{10}$
8	0	= $\frac{4}{10}$
7	6	= $\frac{3}{8}$
6	0	= $\frac{3}{10}$

Parts of a Dollar.

c.	D.
$93\frac{3}{4}$	= $\frac{15}{16}$
$91\frac{2}{3}$	= $\frac{11}{12}$
90	= $\frac{9}{10}$
$87\frac{1}{2}$	= $\frac{7}{8}$
$83\frac{1}{3}$	= $\frac{5}{6}$
$81\frac{1}{4}$	= $\frac{13}{16}$
80	= $\frac{4}{5}$
75	= $\frac{3}{4}$
70	= $\frac{7}{10}$
$68\frac{3}{4}$	= $\frac{11}{16}$
$66\frac{2}{3}$	= $\frac{2}{3}$
60	= $\frac{3}{5}$
$58\frac{1}{3}$	= $\frac{7}{12}$
$56\frac{1}{4}$	= $\frac{9}{16}$
$43\frac{3}{4}$	= $\frac{7}{16}$
$41\frac{2}{3}$	= $\frac{5}{12}$
40	= $\frac{2}{5}$
$37\frac{1}{2}$	= $\frac{3}{8}$
$31\frac{1}{4}$	= $\frac{5}{16}$
30	= $\frac{3}{10}$
$18\frac{3}{4}$	= $\frac{3}{16}$

A TABLE OF DISCOUNT PER CENT.

£.	s. d.		£.	s. d.		£.	s d.	
$1\frac{1}{4}$ per cent	=0 3	on the Pound.	$8\frac{3}{4}$ per cent.	=1 9	on the Pound.	$22\frac{1}{2}$ per cent.	= 4 6	on the Pound.
$2\frac{1}{2}$ ———	=0 6		10 ———	=2 0		25 ———	= 5 0	
$3\frac{3}{4}$ ———	=0 9		$12\frac{1}{2}$ ———	=2 6		30 ———	= 6 0	
5 ———	=1 0		15 ———	=3 0		35 ———	= 7 0	
$6\frac{1}{4}$ ———	=1 3		$17\frac{1}{2}$ ———	=3 6		40 ———	= 8 0	
$7\frac{1}{2}$ ———	=1 6		20 ———	=4 0		45 ———	= 9 0	
						50 ———	=10 0	

Though the general rule given above is sufficient for answering any question in Practice, yet some may perhaps be answered more easily by other rules. Several cases follow.

CASE

CASE I.

When the price of 1 yd. lb. &c. is an even part of one shilling: Find the value of the given quantiy at 1s. per yard, lb. &c.; then draw a line underneath, and divide by *that even part*, and the quotient will be the answer in shillings, which must always be brought into pounds.

EXAMPLES.

1. What will 354½ yards cost, at ¼d. per yard?

		s. d.	
¼d.	$\frac{1}{48}$	354 6	value of 354½ yards, at 1s. per yard.
Ans.		£.0 7 4½	value of 354½ yards, at ¼d. per yard.

Or thus.

£. s. d s. d.
8)17 14 6=354 6

6)2 4 3¾

7 4½ Ans. as before.

Or divide by 8 and 6, thus, 8)354 6

6)44 3¾

7 4½ Ans. as bef.

2. What will 759¾ yards come to, at 3d. per yard?

3d.|¼|759 9 value at 1s. per yard.

2|0)18|9 11¼

Ans.£.9 9 11¼ value at 3d. per yard.

Or thus, |3d.|¼|37 19 9 value at 1s. per yard. (£. s. d.)

Ans. £.9 9 11¼ value of 759¾ yds. at 3d. per yard.

Questions. yds.		Answers. £. s. d.	Questions. Yds.		Answers. £. s. d.
3. 642	at ¼d. per yd.	0 13 4½	7. 685¾	at 2d.	5 14 3½
4. 918¼	— ½d.	1 18 3⅛	8. 475¼	— 4d.	7 18 5
5. 739½	— 1d.	3 1 7½	9. 913½	— 6d.	22 16 9
6. 567½	— 1½d.	3 10 11¼			

CASE II.

When the price is pence, and no even part of a shilling: Find the value of the given quantity at 1s. per yard; divide the pence into aliquot parts, for divisors, and the sum of the quotients arising from them, will be the answer.

EXAMPLES.

1. What will 487½ yards come to at 5d. per yard?

		£. s. d.	
3d.	¼	24 7 6	value of 487½ yards, at 1s. per yard.
2d.	⅙		
		6 1 10½	value of ditto, at 3d. per yard.
		4 1 3	value of ditto, at 2d. per yard.
		Ans. £.10 3 1½	value of ditto, at 5d. per yard.

Questions.

Questions. Yds.	Answers. £. s. d.	Questions. Yds.	Answer. £. s. d.
2. $568\frac{1}{4}$ at 7d. ——	16 11 $5\frac{3}{4}$	5. $649\frac{1}{4}$ at 10d. ——	27 1 $0\frac{1}{2}$
3. $683\frac{3}{4}$ — 8d. ——	22 15 10	6. $745\frac{3}{4}$ — 11d. ——	34 3 $7\frac{1}{4}$
4. $912\frac{1}{2}$ — 9d. ——	34 4 $4\frac{1}{2}$		

CASE. III.

When the price is between one and two shillings: Find the value of the quantity at 1s per yard, &c. which value being divided by *those even parts* which the pence are of 1s. and the quotient or quotients, arising therefrom, added thereto; the sum will be the answer.

Examples.

1. What will $758\frac{1}{2}$ yards, at 1s. 9d. per yard, come to?

		£.	s.	d.	
6d.	$\frac{1}{2}$	37	18	6	value at 1s. per yard.
3d.	$\frac{1}{4}$	18	19	3	value at 6d. per yard.
		9	9	$7\frac{1}{2}$	value at 3d. per yard.

Ans. £.66 7 $4\frac{1}{2}$ value of $758\frac{1}{2}$ yds. at 1s. 9d. per yard.

Questions. Yds.	Answers. £. s. d.	Questions. Yds.	Answers. £. s. d.
2. 793 at $12\frac{3}{4}$d.	42 2 $6\frac{3}{4}$	6. $896\frac{1}{4}$ at 1s. 6d.	67 4 $4\frac{1}{2}$
3. $847\frac{1}{2}$ —1s. 1d.	45 18 $1\frac{1}{2}$	7. 458 —1s. 7d.	36 5 2
4. $846\frac{1}{2}$ —1s. 4d.	56 8 8	8. $752\frac{1}{2}$ — 1s. 10d.	68 19 7
5. $647\frac{3}{4}$ —1s. 5d.	45 17 $7\frac{3}{4}$		

CASE IV.

When the price is any even number of shillings under 24: Multiply the given quantity by half the price, and double the first figure of the product for shillings. The rest of the product will be pounds.

N. B. If the price be 2s. you need only double the unit figure for shillings. The other figures will be pounds.

Examples.

1st. What will 746 yards cost at 2s. per yard?

746

Ans. £.74 12 value at 2s. per yard.

Note. The above is done, by saying twice 6 (the unit figure) is 12. The other figures, viz. 74, are pounds.

2d What will $567\frac{3}{4}$yds. at 2s. per yard come to? Ans £.56 15s. 6d.

N. B. Before I double the unit figure, viz. 7, I consider that $\frac{3}{4}$ of a yard at 2s per yard, will amount to 1s 6d. Then I double 7, which makes 14s. and 1s. 6d. added, makes 15s. 6d. The other figures are pounds.

Questions. Yds.				Answers. £.	s.	d.
3d.	$129\frac{1}{2}$ at	4s.	per yard.	25	18	0
4th	697 —	6s.	——	209	2	0
5th.	845 —	8s.	——	338	0	0
6th.	$917\frac{1}{4}$ —	10s.	——	458	12	6

CASE

CASE V.

When the price wants an even part of 2s.: First find the value of the the whole quantity at 2s. per lb. yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient from the value at 2s. The remainder will be the answer.

EXAMPLES.

1. What will $95\frac{1}{2}$ yards cost at 22d. per yard?

		£.	s.	d.	
2d.	$\frac{1}{12}$	9	11	0	value at 2s. per yard.
		0	15	11	value at 2d. per yard.
	Ans	£.8	15	1	value at 1s. 10d. per yard.

Questions.	Yds.				Answers. £.	s.	d.
2d.	64	at	23d.	per yd.	6	2	8
3d.	128	—	$22\frac{1}{2}$d.	——	12	0	0
4th.	$246\frac{1}{2}$	—	21d.	——	21	11	$4\frac{1}{2}$
5th.	$375\frac{1}{4}$	—	20d.	——	31	5	5

CASE VI.

When the price is between 2s. and 3s.: First find the value of the quantity at 2s. per yard, &c which value being divided by *those even parts* which the pence are of 2s. and those quotients added thereto, the sum will be the answer.

EXAMPLES.

1st. What will $148\frac{1}{2}$ yards come to at 2s. 7d. per yard?

		£.	s	d.	
4d.	$\frac{1}{6}$	14	17	0	value at 2s per yard.
3d.	$\frac{1}{8}$	2	9	6	ditto at 4d. per yard.
		1	17	$1\frac{1}{2}$	ditto at 3d. per yard.
	Ans.	£ 19	3	$7\frac{1}{2}$	value at 2s. 7d. per yard.

Questions.	Yds.					Answers. £.	s.	d.
2d.	$266\frac{1}{4}$	at	2s.	1d.	per. yd.	27	14	$8\frac{1}{4}$
3d.	344	—	2s.	$1\frac{1}{2}$d.	——	36	11	0
4th.	$543\frac{1}{2}$	—	2s.	2d.	——	58	17	7
5th.	813	—	2s.	5d.	——	98	4	9

CASE VII.

When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20 *:* First find the value of the quantity at the shillings per yard, &c. according to Case 4th : then suppose the quantity to stand as shillings per yard ; divide it by the *that even part,* which the pence are of 1s. and this quotient being added to the value before found, the sum will be the answer.

EXAMPLES.

1st. What will $156\frac{1}{2}$ yards come to, at 6s. 4d. per yard?

			Yds.	
			$156\frac{1}{2}$	
	s. d.		3	
4d. | $\frac{1}{3}$ |	156 6			
		£.46	19 0	value of $156\frac{1}{2}$ yards at 6s per yard.
	52s. 2d. =	2	12 2	value of ditto at 4d. per yard.
	Ans.	£.49	11 2	value of ditto at 6s. 4d. per yard.

Questions.	Yds		s.	d.		Answers. £.	s.	d.
2d.	$17\frac{1}{2}$	at	4	$0\frac{1}{2}$	per yd.	3	10	$8\frac{3}{4}$
3d.	$59\frac{3}{4}$	—	6	$0\frac{3}{4}$	———	18	2	$2\frac{3}{4}$
4th.	$68\frac{1}{4}$	—	8	1	———	27	11	$8\frac{1}{4}$
5th.	96	—	10	$1\frac{1}{2}$	———	48	12	0
6th.	$67\frac{1}{2}$	—	12	2	———	41	1	3

CASE VIII.

When the price is any odd number of shillings under 20 : Find the value of the greatest even number contained in the price, according to Case 4th, and add thereto the value of the quantity at 1s. per yard, &c. which sum will be the answer : Or, Multiply the quantity by the price, according to the 1st or 2d Case in Simple Multiplication, and divide the product by 20, the quotient will be the answer : Or, lastly, if the price be not more than 12s. find the value of the quantity at 1s. per yard, &c. and multiply it by the number of shillings in the price of 1 yard ; the product will be the answer.

EXAMPLES.

1st. What will 186 yards cost, at 3s. per yard?

£.	s.	
18	12	value at 2s. per yard.
9	6	ditto at 1s. per yard.
£.27	18	Ans.

Or thus.

	£.	s.	
	9	6	value at 1s. per yard.
		3	
Product	£.27	18	Ans.

2d. What will 647 yards cost, at 17s. per yard?

	8		
	£.517	12	value at 16s. per yard.
	32	7	ditto at 1s. per yard.
Ans.	£.549	19	ditto at 17s. per yard.

Questions.

Questions.	Yds.		s.		Answers. £.	s.	d.
3d.	$169\frac{1}{4}$	at	5	per yd.	42	6	3
4th.	$248\frac{3}{4}$	—	7	——	87	1	3
5th.	139	—	9	——	62	11	0
6th.	782	—	25	——	977	10	0

CASE IX.

When the price is an even part of a pound: Find the value of the given quantity, at one pound per yard, &c. then draw a line underneath, and divide it by *that part*; the quotient will be the answer.

EXAMPLES.

1st. What will $156\frac{3}{4}$ yards of cloth come to, at 3s. 4d. per yard?

s. d.		£. s. d.	
3 4	$\frac{1}{3}$	156 15 0	price at 1l. per yard.
	Ans.	£.26 2 6	price at 3s. 4d. per yard.

Questions.	Yds.		s.	d.		Answers. £.	s.	d.
2d.	$516\frac{3}{4}$	at	1	0	per yd.	25	16	9
3d.	624	—	1	3	——	39	0	0
4th.	$719\frac{1}{2}$	—	1	4	——	47	19	4
5th.	648	—	1	8	——	54	0	0

CASE X.

When the price wants an even part of a pound: First find the value of the given quantity at 1l. per yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient therefrom; the remainder will be the answer.

EXAMPLES.

1st. What will $167\frac{1}{2}$ yards cost, at 17s. 6d. per yard?

s. d.		£. s. d.	
2 6	$\frac{1}{8}$	167 10 0	value at 1l. per yard.
		20 18 9	ditto at 2s. 6d. per yard.
	Ans.	£.146 11 3	value at 17s. 6d. per yard.

Questions.	Yds.		s.	d.		Answers. £.	s.	d.
2d.	$347\frac{1}{2}$	at	13	4	per yd.	231	13	4
3d.	$485\frac{3}{4}$	—	15	0	——	364	6	3
4th.	614	—	16	0	——	491	4	0
5th.	$912\frac{1}{4}$	—	17	0	——	798	4	$4\frac{1}{2}$

CASE XI.

When the price is shillings, pence and farthings, and not an even part of a pound: Multiply the given quantity by the shillings in the price of 1 yard,

1 yard, &c. and take parts of parts from the quantity for the pence, &c. then add them together, and their sum will be the answer, in shillings, &c. Or you may let the given quantity stand as pounds per yard, &c. then draw a line underneath, and take parts of parts therefrom; which add together, and their sum will be the answer.

N. B. I advise the learner to work the following examples both ways, by which means he will be able to discover the most concise method of performing such questions, in business as may fall under this case.

EXAMPLES.

1. What will 248½ yards, at 7s. 6d. per yard, come to?

|6|½|248s. 6d. value of 248½ yards, at 1s. per yard.
7

1739 6 value of ditto at 7s. per yard.
124 3 value of ditto at 6d. per yard.

2|0)186|3 9

Ans £.93 3 9 value of ditto at 7s. 6d. per yard.

Or thus,

|6|½|12 8 6 value of 248½ yards, at 1s. per yard,
Multiply by 7

86 19 6 value of of ditto at 7s. per yard.
6 4 3 value of ditto at 6d. per yard.

Ans. £.93 3 9

By the latter part of this case,

		£. s. d.	
5 0	¼	248 10 0	value of 248½ yards, at 1 l. per yard.
2 6	½	62 2 6	value of ditto at 5s. per yard.
		31 1 3	value of ditto, at 2s. 6d. per yard.

Ans. £.93 3 9 value of ditto at 7s. 6d. per yard.

Questions.	Yds.		s.	d.		Answers. £.	s.	d.
2.	68½	at	4	6	per yard.	15	8	3
3.	124	—	5	8	———	35	2	8
4.	146	—	14	9	———	107	13	6
5.	218½	—	12	6	———	136	11	3

CASE XII.

When the price of the yard, lb. *&c. is pounds, shillings and pence*: First, multiply the quantity by the pounds, and if the shillings and pence be an even part of a pound, divide the given quantity by *that part*, and add

add the quotient to the product for the answer. But if they be not an even part of a pound, you must take parts of parts, and add them together as before. Or, reduce the pounds and shillings into shillings, and multiply the quantity thereby, after which, take parts for the pence, and add the whole together, and their sum will be the answer in shillings, &c.

N. B. The learner should work the following questions both ways.

EXAMPLES.

1. What will 156 yards of broadcloth come to, at 3l. 6s. 8d. per yard?

6s.8d. $\frac{1}{3}$	156 0 0	value at 1l.
	3	per yard.
	468 0 0	
	52 0 0	
Ans.	£.520 0 0	

Or thus.

4	$\frac{1}{3}$	156	value at 1s. per yard.
4	$\frac{1}{3}$	66	shillings in the price of 1yd.
		936	
		936	
		10296	value at 3l. 6s. per yard.
		52	
		52	
		2|0)1040|0	
		£.520 0 0	

Questions.					Answers.		
	Yds.		£. s. d.		£.	s.	d.
2.	345$\frac{1}{2}$	at	6 5 0	per yard.	2159	7	6
3.	59$\frac{3}{4}$	—	3 6 8	———	199	3	4
4.	75	—	5 3 4	———	387	10	0
5.	68	—	4 6 0	———	292	8	0

CASE XIII.

When the quantity is any number less than 1000, *and the price not more than* 12d. *per yard, &c.* : Find the value of the whole quantity at 1d. per yard, which may be done by dividing it by 12, *mentally*, setting down the quotient only in pounds, or shillings, or both. Then multiply this sum by the pence in the price of 1 yard, and the product will be the answer.

EXAMPLES.

1. What will 759$\frac{1}{2}$ yards cost, at 7d. per yard

	£.	s.	d.	
	0	63	3$\frac{1}{2}$	value at 1d. per yard
Or,	3	3	3$\frac{1}{2}$	value at 1d. per yard.
Multiply by			7	
Ans.	£.22	3	0$\frac{1}{2}$	value of 759$\frac{1}{2}$ yards, at 7d. per yard.

Questions.					Answers.		
	Yds.		d.		£.	s.	d.
2.	975$\frac{1}{2}$	at	2	per yard.	8	2	7
3.	846	—	3$\frac{1}{2}$	———	12	6	9

CASE

CASE XIV.

When the price of one hundred weight &c. is of several denominations, and the quantity likewise: Multiply the price by the integers, and take parts for the rest from the price of an integer; which, added together, will be the answer.

EXAMPLES.

1. What will 9Cwt. 3qrs. 14lb. of sugar come to, at 4l. 17s. 4d. per Cwt.?

qrs.	lb.		£.	s.	d.		Cwt.	qr.	lb.
2	0	$\frac{1}{2}$	4	17	4	price of 1Cwt.			
1	0	$\frac{1}{2}$			9				
0	14	$\frac{1}{2}$							
			43	16	0	price of	9	0	0
			2	8	8	——	0	2	0
			1	4	4	——	0	1	0
			0	12	2	——	0	0	14
		Ans.	£.48	1	2	price of	9	3	14

Questions.									Answers.		
	Cwt.	qrs.	lb.		£.	s.	d.		£.	s.	d.
2.	8	1	16	Tobacco, at	5	17	9	per cwt.	49	8	$2\frac{3}{4}$
3.	7	3	19	————	7	12	8	————	60	9	$0\frac{1}{2}$
4.	12	1	24	————	3	18	10	————	49	2	$7\frac{1}{4}$
5.	16	2	17	————	2	15	11	————	46	11	1
6.	72	3	27	————	8	11	5	————	625	11	$10\frac{3}{4}$
7.	59	1	14	Sugar, at	1	8	7	————	84	17	$1\frac{1}{2}$
	lb.	oz.									
8.	27	10		Coffee, at	0	1	4	per lb.	1	16	10
	lb. oz. pwt. gr.										
9.	13 10 12 8			Silv. at	4	7	6	per lb.	60	14	$11\frac{1}{2}$
	oz. pwt. gr.										
10.	17 6 16			Gold, at	3	16	8	per oz.	66	8	$10\frac{2}{3}$

CASE XV.

When the price is at any of the rates in the second Practice Table of aliquot parts: Multiply the given quantity by the numerator, and divide that product by the denominator; if the price be pence, the quotient will be the answer in shillings; if shillings, the answer will be pounds.

EXAMPLES.

1. What will 379 yards, at $4\frac{1}{2}$d. per yard come to?

```
      379
        3
    -----
   8)1137
    -----
  2|0)14|2  1½
    -----
Ans. £.7 2 1½
```

2. What will 149 yards, at 6s. per yard come to?

```
      149
        3
    -----
  1|0)44|7
    -----
Ans. £.44 14
```

Questions.

Questions.						Answers.		
	Yds.		s.	d.		£.	s.	d.
3.	127	at	0	$7\frac{1}{2}$	per yard,	3	19	$4\frac{1}{2}$
4.	159	—	0	8	———	5	6	0
5.	173	—	0	9	———	6	9	9
6.	241	—	0	10	———	10	0	10
7.	249	—	7	6	———	93	7	6
8.	357	—	12	6	———	223	2	6

CASE XVI.

When the price is any even number of shillings, if it be required to know what quantity of any thing may be bought for so much money: Annex a cypher to the money, and divide it by half the price, and the quotient will be the quantity to be purchased.

Examples.

1. How many yards of cloth, at 18s. per yard, may I have for £.345?

Half the price = 9)3450 = money with a cypher annexed.

$383\frac{1}{3}$ yards, Ans.

Questions.			Answers.
	s.	£.	Yds.
2. How many yds. at	2 per yd. for	427?	4270
3.	4	312	1560
4.	6	917	$3056\frac{2}{3}$
5.	8	195	$487\frac{1}{2}$

CASE XVII.

To find the value of goods sold by particular quantities, viz. I. By the score. II. Round timber. III. By 5 score to the hundred. IV. By 112 to the hundred. V. By 6 score to the hundred. VI. By the great gross. VII. By the thousand.

I. *To find the value of goods sold by the score.*

The price of one is given, to find the price of one score.

If the given price be shillings and pence, or only pence, divide the given price, in pence, by 12. The quotient will be the answer in pounds, and the remainder will be so many times 1s. 8d.

Examples.

1. At 9d. each: What is that per score?

12)9d.(·75=£.0 15 0 Ans.

Or by inverting the question.

1 score=20=1s. 8d.

9

15s.0

2. At 4s. 9d. each: What is that per score?

4s. 9d.

12

12)57d.

£.4 15 Ans.

It

It may be remarked, that when the price is shillings and pence, the answer will be just so many pounds as there are shillings, and so many times 1s. 8d. as there are pence. If farthings are given, for $\frac{1}{4}$d. reckon 5d. for $\frac{1}{2}$d. 10d. and for $\frac{3}{4}$d. 1s. 3d.

TABLE of Aliquot Parts. 20 the Integer.

2 is $\frac{1}{10}$	6 is $\frac{3}{10}$	12 is $\frac{6}{10}$	16 is $\frac{8}{10}$
4 — $\frac{1}{5}$	8 — $\frac{4}{10}$	14 — $\frac{7}{10}$	18 — $\frac{9}{10}$
5 — $\frac{1}{4}$	10 — $\frac{1}{2}$	15 — $\frac{3}{4}$	

3. What cost 7; at 2s. 9d. per score?

		s. d.
5	$\frac{1}{4}$	2 9
2	$\frac{1}{10}$	0 $8\frac{1}{4}$
		0 $3\frac{1}{4}$
7	=	0 $11\frac{1}{2}$

4. What cost 17; at 19s. 10d. per score?

		s. d.
10		19 10
5	$\frac{1}{2}$	
2	$\frac{1}{4}$	9 11
	$\frac{1}{10}$	4 $11\frac{1}{2}$
		1 $11\frac{3}{4}$
17	=	16 $10\frac{1}{4}$

II. *Round Timber.*

Forty feet make a load or ton of round timber.

If the given price of a foot be shillings,

Rule.

Multiply the given price by 2, and the product will be the answer in pounds.

5. What cost a ton at 3s. per foot? 3s ×2=6l. Ans.

6. What cost a ton at 9s. per foot? 9s.×2=18l. Ans.

If the given price of 1 foot be pence only, or shillings and pence, divide the given price, in pence, by 6 The quotient will be the answer in pounds, and the remainder will be so many times 3s. 4d.

7th. What cost 40 feet, at 17d. per foot?

6)17

£.2 16 8 Ans.

8th. At 1s. 9d. per foot: What cost a ton?

6)21

£ 3 10 Ans.

If the given price of a foot be farthings only, or pence and farthings, divide the given price in farthings, by 6; then divide *that quotient* by 4, and *this last quotient* will be the answer.

9th. At $\frac{3}{4}$d. per foot: What cost a ton?

6)3

4)0 10

£.0 2 6 Ans.

10th. At $13\frac{1}{4}$ per foot: What cost a ton?

$13\frac{1}{4}$
4

6)53

4)8 16 8

£.2 4 2 Ans.

Or,

Or, suppose every shilling in the price to be 2l. every penny to be 3s. 4d. and every farthing to be 10d

11th. What cost 40 feet at $\frac{3}{4}$d. per foot ?

$\frac{3}{4}$d. × 10 £0 2 6 Ans.

12th. What cost 40 at 15$\frac{1}{2}$d. per foot ?

s.	d.			£	s.	d.
1	0	× 2 =		£.2	0	0
3	4	× 3 =		0	10	0
0	$\frac{1}{2}$	× 10 =		0	1	8
				£.2	11	8

III.*. *To find the value of goods sold by 5 score to the hundred.*

1st. If the given price be pounds and shillings, or shillings only.

RULE.

Multiply the given price in shillings, by 5, and the quotient will be the answer in pounds.

13th. At 19s. per yard, what cost 100 yards ?

19s.
5
£ 95 Ans.

14th. At 4l. 13s per cwt. what cost 100 cwt. or 5 tons ?

4 13
20
93
5
£ 465 Ans.

2d. If the given price of 1 be pence only, or shillings and pence.

RULE.

Multiply the given price, in pence, by 5 ; then divide that product by 12. The quotient will be pounds ; and the remainder so many times 1s. 8d.

15th. If

* *In Federal Money.*—Remove the decimal point two places to the right for the anſwer.

EXAMPLES.

1. What cost 100 yards at D.2 50c. per yard ?
D.2·50 × 100 = D.250·, Ans.
2. What cost 100 yards at 75c. per yard ?
D.·75 × 100 = D.75·, Ans.
3. What cost 100 yards at 5c. 6$\frac{1}{4}$m per yard ?
D.·05625 × 100 = D.5·625, Ans.
4. What cost 100 yards at 37c. 5m. per yard ?
Ans. D.37 50c.
5. What cost 100 yards at 68c. 7$\frac{1}{2}$m. per yard ?
Ans. D.68 75c.

15th. If 1 yard cost 9d. what cost 100 yards?

```
   9
   5
   —
12)45
  ——
£.3 15 Ans.
```

16th. What cost 100 bushels, at 35s. 4d. per bushel?

```
  s. d.                    Or,
  35 4                  35s. 4d.
  12        | 4d. | 1/3 |  5
 ———                    ———
 424                    175
   5                      1 13 4
 ———                    ————————
12)2120                £.176 13 4
 ————————               ————————
£.176 13 4 Ans.
 ————————
```

Here 5 is divided by $\frac{1}{3}$.

3. If the given price of 1 be shillings and pence: Multiply the price by 5, and the product under the place of shillings, will be the answer in pounds, and the product under the place of pence, will be so many times 1s. 8d.

17th. At 2s. 5d per bushel: what cost 100 bushels?

```
  s. d.
  2  5
     5
 ——————
 12  1
 ————————
£.12  1  8 Ans.
```

18th. At 25s. 3d. per ton: what cost 100 tons?

```
  s.  d.
  25  3     1s. 8d × 3 = 5s.
      5
 ————————
 126  3
 ————————
£.126  5 Ans.
```

4.* To find the price of *one* at so much per hundred of 5 score.

GENERAL RULE.

Multiply the given price by 12; divide the product by 5, and the quotient will be the answer in pence.

But if the price be pounds only:

RULE.

Divide the given price by 5, and the quotient will be the answer in shillings.

19th. If

* *In Federal Money.*—Remove the decimal point two places to the left for the anfwer.

EXAMPLES.

1. If 100 yards cost D.250, what cost 1 yard?
   D.250 ÷ 100 = D.2·50 Ans.
2. If 100 yards cost D.75·, what cost 1 yard?
   D.75· ÷ 100 = D.·75, Ans.
3. If 100 yards cost D.5 62c. 5m. what cost 1 yard?
   D.5·625 ÷ 100 = D.·05625 = 5c. $6\frac{1}{4}$m. Ans.
4. If 100 yards cost D.37 50c. what cost 1 yard?
   Ans. 37c. 5m.
5. If 100 yards cost D.68 75c. what cost 1 yard?
   Ans. 68c. $7\frac{1}{2}$m.

19th. If 100 yds cost 65l. what cost 1 yd. ?

5)65
—
13s. Ans.

20th If 100 yds cost 2l. 18s. 4d. what is that per yard ?

£.	s.	d.
2	18	4
		12
5)35	0	0

7d. Answer.

21st. If 100 yards cost 11l. 7s. 9d. what cost 1 yard ?

£.	s.	d.
11	7	9
		12
5)136	13	0
12)27	6	7

2s. $3\frac{1}{4}$d. Ans.

In dividing 27 by 12 (in the 21st question) the quotient is 2s. and the remainder 3d. the 6 is $\frac{6}{20}$ of a penny = one farthing, and the 7 is of no account.

TABLE of Aliquot Parts. 100 the Integer.

5 is	$\frac{1}{20}$	25 is	$\frac{1}{4}$	50 is	$\frac{1}{2}$	75 is	$\frac{3}{4}$
10 ——	$\frac{1}{10}$	30 ——	$\frac{3}{10}$	60 ——	$\frac{6}{10}$	80 ——	$\frac{8}{10}$
20 ——	$\frac{1}{5}$	40 ——	$\frac{4}{10}$	70 ——	$\frac{7}{10}$	90 ——	$\frac{9}{10}$

22d.* At 3l. 7s. 6d. per 100 : What will 23 cost ?

		£. s d.	
20	$\frac{1}{5}$	3 7 6	
		0 13 6	
2	$\frac{1}{10}$	0 1 4	Add.
1	$\frac{1}{2}$	0 0 8	
23	=	£.0 15 6 Ans.	

23d. At

* *To find the value of any number at a given price per* 100, *in federal money.*—Multiply the price per 100 by the given quantity, and point off two right hand figures, in the product more than required by multiplication of decimals. Or, point off the two right hand places in the given quantity, and multiply, and point, as in multiplication of decimals.

EXAMPLES.

1. What cost 56 yards at D.87 50c. per 100 yards ?

$\frac{D.87{\cdot}5 \times 56}{100}$ = D.49, Ans. Or, D.87·5 × ·56 = D.49, as before.

2. What cost $45\frac{1}{2}$ lb. beef at D.$5\frac{1}{2}$ per 100 ?

$\frac{D.5{\cdot}5 \times 45{\cdot}5}{100}$ = D.2·5025 = D.2 50c. $2\frac{1}{2}$m. Ans. Or, D.5·5 × ·455lb. = D.2·5025, as [before

3. What cost 375 yards at D.375 per 100 yards ? Ans. D.1404 25c.
4. What cost 54 yards at D.16 per 100 ? Ans. D.8 64c.
5. What cost 512 yards at D.6 25c. per 100 yards ? Ans. D.32.

23d. At 2l. 1s. 10d. per 100: What cost 18?

		£.	s.	d.	
20	$\frac{1}{5}$	2	1	10	
		0	8	$4\frac{2}{5}$	Sub.
2	$\frac{1}{10}$	0	0	10	
18	=	£.0	7	$6\frac{2}{5}$	Ans.

24th. At 5l. 9s. 6d. per 100: What cost 35?

		£.	s.	d.	
		5	9	6	
				3	
		10)16	8	6	
5	$\frac{1}{6}$	1	12	10	Add.
			5	$5\frac{2}{3}$	
		£. 1	18	$3\frac{2}{3}$	Ans.

IV. *To find the value of goods, sold by* 112lb. *the Cwt.*

The price of 1lb. is given to find the value of 1 cwt.

RULE.

For a farthing, account 2s. 4d per cwt. For a half a penny, 4s. 8d. For three farthings, 7s. And for every penny 9s. 4d per cwt.

25th. What cost 1cwt. at $3\frac{1}{2}$d. per lb.?

		s.	d.	
At 1d per-lb.		s.	d.	
1cwt. costs		9	4	
			3	
At 3d.	£.1	8	0	Add.
At $\frac{1}{2}$d.	0	4	8	
	£.1	12	8	Ans.

26th. At $8\frac{3}{4}$d. per lb.: What cost 1 cwt?

		s.	d.	
At 1d. per lb.		s.	d.	
1 cwt. costs		9	4	
			8	
At 8d.	£.3	14	8	Add.
At $\frac{3}{4}$	0	7	0	
	£.4	1	8	Ans.

V. *To find the value of goods sold by* 6 *score to the hundred.*

The price of 1 is given, to find the price of 1 hundred.

RULE.

Suppose every penny in the price to be so many pounds, and for the farthings, such a part of a pound, as they are of a penny; then, half of that sum will be the answer.

27th. At $4\frac{1}{2}$d. per yard: What cost 120 yards?

£. s.
2)4 10
£.2 5 Ans.

28th. At 16s. $9\frac{1}{4}$d. per yard: What cost 120 yards?

s. d.
16 $9\frac{1}{4}$
12
2)201l. 5s.
£.100 12 6 Ans.

To find the price of *one*, at so much per hundred of 6 score.

RULE.

Multiply the price by 2, then call the pounds so many pence, and the shillings, such a part of a penny, as they are of a pound, and you will have the answer.

29th. If

29th. If 120 yds. cost 3l. 12s.: What cost 1 yard?

£.	s.
3	12
	2
7	4

Ans. $7\frac{1}{5}$d.

30th. If 120 yds. cost 5l. 18s. 6d.: What cost 1 yard?

£	s.	d.
5	18	6
		2
11	17	0

Ans. $11\frac{3}{4}$d. + $\frac{2}{5}$ of farthing.

TABLE of Aliquot Parts. 120 the Integer.
Also,

6	is	$\frac{1}{20}$	24	is	$\frac{1}{5}$	36	is	$\frac{3}{10}$	72	is	$\frac{3}{5}$	96	is	$\frac{4}{5}$
10	—	$\frac{1}{12}$	30	—	$\frac{1}{4}$	45	—	$\frac{3}{8}$	75	—	$\frac{5}{8}$	100	—	$\frac{5}{6}$
12	—	$\frac{1}{10}$	40	—	$\frac{1}{3}$	48	—	$\frac{2}{5}$	80	—	$\frac{2}{3}$	105	—	$\frac{7}{8}$
15	—	$\frac{1}{8}$	60	—	$\frac{1}{2}$	50	—	$\frac{5}{12}$	84	—	$\frac{7}{10}$	108	—	$\frac{9}{10}$
20	—	$\frac{1}{6}$				70	—	$\frac{7}{12}$	90	—	$\frac{3}{4}$	110	—	$\frac{11}{12}$

31st. At 3l. 17s. 6d. per hundred. what cost 14?

		£. s. d.	
12	$\frac{1}{10}$	3 17 6	
		0 7 9	Add.
2	$\frac{1}{6}$	0 1 $3\frac{1}{2}$	
14		= £.0 9 $0\frac{1}{2}$ Ans.	

32. At 2l. 13s. $6\frac{1}{2}$d. per hundred, what cost 49?

		£. s. d.
40	$\frac{1}{3}$	2 13 $6\frac{1}{2}$
8	$\frac{1}{5}$	0 17 10 $0\frac{2}{3}$
1	$\frac{1}{8}$	0 3 6 $3\frac{1}{3}$
		0 0 5 1
49		= £.1 1 10 1 Ans.

33. At 1l. 19s. 3d. per hundred, what cost 75?

£.	s.	d.
1	19	3
		5
8)9	16	3
£.1	4	$6\frac{1}{4}$ Ans.

VI. *To find the value of goods sold by the great gross.*

NOTE. 12 make 1 dozen, 12 dozen 1 small gross, 12 small gross 1 great gross.

The price of 1 dozen being given, in pence, to find the price of a great gross.

RULE.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c.

For proof, do the contrary.

N. B. If the price of 1 be given, the price of 1 small gross is found after the same manner.

34. What

34. What cost 1 great gross, at 18d. per dozen?

```
        3
       ——
     5)54
     ————
  £.10 16
```

35. At 4s. 3d. per dozen, what cost 1 great gross?

```
     4s. 3d.              Or,               Or,
     12                   s. d.             £. s.
     —                    4 3          144=7  4
     51d.                 12                  4
     3                  —————           ——————
     —                  2 11 0   | 3d. | 1/4 | 28  16 } Add.
   5)153                  12                   1  16 }
   ————                 —————           ——————
Ans. £.30 12         £.30 12 0         £.30  12
```

TABLE of Aliquot Parts. 144 the Integer.

Also,

12	is	$\frac{1}{12}$	36	is	$\frac{1}{4}$	32	is	$\frac{2}{9}$	84	is	$\frac{7}{12}$	128	is	$\frac{8}{9}$
16	—	$\frac{1}{9}$	48	—	$\frac{1}{3}$	60	—	$\frac{5}{12}$	96	—	$\frac{2}{3}$	132	—	$\frac{11}{12}$
18	—	$\frac{1}{8}$	72	—	$\frac{1}{2}$	64	—	$\frac{4}{9}$	108	—	$\frac{3}{4}$			
24	—	$\frac{1}{6}$				80	—	$\frac{5}{9}$	120	—	$\frac{5}{6}$			

36. At 2l. 12s. 9d. per great gross, what cost 45 dozen?

```
    Doz.      £. s.  d.
  | 36 | 1/4 | 2  12  9
  |    |     ——————————
  |    |     | 0  13  2 1/4 } Add.
  |  9 | 1/4 | 0   3  3 1/2 }
   ——        ——————————
   45 = £.0  16  5 3/4 Ans.
```

37. What cost 117 dozen, at 9l. 13s. 7d. per great gross?

```
             £. s. d.
              9 13 7
                   3
Doz.         ———————
| 108 | 3/4 | 29  0 9
|     |     ———————
|   9 | 1/12 | 7  5 2 1/4 } Add.
|     |        12 1       }
 ———         ———————
 117   = £.7 17 3 1/4 Ans.
```

38. At 3l. 16s. 8d. per great gross, what cost 7 great gross and 96 dozen?

```
                      £. s. d.
                       3 16 8
                            2
                      ———————
                     3)7 13 4
           Doz.       ———————
           96 =        2 11 1 1/4
  Top line ×7 =       26 16 8
                      ———————
                   £.29  7 9 1/4
```

VII.* *To find the value of goods sold by the thousand.*

The price of 1 is given to find the price of 1000.

RULE.

Multiply the given price, in pence, by 50, then divide the product by 12, and the quotient will be the answer in pounds, &c.

39. At

* See note on next page.

39. At 6d. each ; what cost 1000 ?

```
      6
     50
    ---
 12)300
    ---
  £.25 Ans.
```

Or, as 1000s. are 50l. take parts, for the pence out of 50.

```
|6d.|½|50
       ---
Ans.   25
```

40. What cost 1000, at $2\frac{1}{4}$d. each ?

```
|2d.|⅙| 50
|   |  | -------
| ¼ |⅛ | 8 6  8 }
|   |  | 1 0 10 } +
         -------
      £.9 7  6 Ans.
```

VIII.* *To find the price of one at so much per thousand.*

Rule.

Multiply the price by 12 ; divide the product by 50 ; then take the pounds for so many pence, and the shillings for such a part of a penny as they are of a pound, which will be the answer.

41. At 5l. 4s. 2d. per 1000, what cost 1 ?

```
        £.  s.  d.
         5  4   2
               12
        -----------
50 { 5)62  10   0
   {    -----------
   {10)12  10
       -------
      £.1   5
       -------
  Ans. 1¼d
```

42. At 354l. 3s. 4d. per 1000, what cost 1 ?

```
       £.    s.  d.
       354   3   4
                12
    ----------------
50 { 10)4250  0   0
   {   -------------
   {  5)425
       ---
        85
  Ans. 7s. 1d.
```

Or,

```
              £.   s.  d.
|100|1/10| 354   3   4
|   |    | -----------
| 10|1/10|  35   8   4
|   |    | -----------
|  1|1/10|   3  10  10
           -----------
      Ans.   0   7   1
```

TABLE

* *In Federal Money*—Remove the decimal point three places to the right, or left, as the case requires, for the answer.

Examples.

1. What cost 1000 yards at 5 cents per yard ? ·05×1000=050·=50D. Ans.
2. What cost 1000 yards, at 12 cents 5 mills per yard ? Ans. D.125
3. If 1000 yards cost D.37 50c. what cost 1 yard ?
D.37·5÷1000 = D.·0375 = 3c. $7\frac{1}{2}$m.or, $3\frac{3}{4}$c. Ans.
4. If 1000 yards cost D.1625, what cost 1 yard ? Ans. D. 1 62c. 5m.

TABLE of Aliquot Parts. 1000 the Integer.

50 is $\frac{1}{20}$	200 is $\frac{1}{5}$	300 is $\frac{3}{10}$	700 is $\frac{7}{10}$
100 — $\frac{1}{10}$	250 — $\frac{1}{4}$	375 — $\frac{3}{8}$	750 — $\frac{3}{4}$
125 — $\frac{1}{8}$	500 — $\frac{1}{2}$	400 — $\frac{2}{5}$	800 — $\frac{4}{5}$
		600 — $\frac{3}{5}$	875 — $\frac{7}{8}$
		625 — $\frac{5}{8}$	900 — $\frac{9}{10}$

43. * At 1l. 17s. 9d. per 1000, what cost 115?

		£.	s	d.	
100	$\frac{1}{10}$	1	17	9	
10	$\frac{1}{10}$	0	3	$9\frac{1}{2}$	Add.
		0	0	$4\frac{1}{2}$	
5	$\frac{1}{2}$	0	0	$2\frac{1}{4}$	
115	=	£.0	4	4 Ans.	

44th. At 2l. 1s. 8d. per 1000, what cost 875?

£. s. d.
2 1 8
7

8)14 11 8

£.1 16 $5\frac{1}{2}$ Ans.

45th What cost 33, at 24s. 8d. per 1000?

		£.	s.	d.	
50	$\frac{1}{20}$	1	4	8	
25	$\frac{1}{2}$	0	1	$2\frac{3}{4}$	
		0	0	$7\frac{1}{4}$	Add.
5	$\frac{1}{5}$	0	0	$1\frac{1}{4}$	
30	=	0	0	$8\frac{1}{2}$	Add.
3	$\frac{1}{10}$	0	0	$\frac{3}{4}$	
33	=	0	0	$9\frac{1}{4}$.	

CASES IN FEDERAL MONEY.

CASE I.

When the price of 1 is an aliquot part of a dollar.—Divide the quantity by the denominator of the fraction, which the price is of a dollar, and the quotient will be the answer in dollars, &c.

EXAMPLES.

1. What cost 227 yards, at 50 cents per yard?

c. D. D.
50 | $\frac{1}{2}$ | 227=price at D.1 per

D.113 50c. Ans.

2. What

* *To find the value of any number, at a given price per 1000, in federal money.*—Multiply the price per 1000 by the given quantity, and point off three right hand figures in the product more than required by multiplication of decimals. Or, point off the three right hand places in the given quantity: and multiply and point as in multiplication of decimals.

EXAMPLES.

1. What cost 875 at D.13 per 1000?
875×13=11375; and 11375÷1000=11·375=D.11 37c. 5m. Ans.
2. What cost 39175 feet of boards, at D.16 per 1000? Ans. D.626 80c.
3. What cost 325 nails at D.1 50c. per 1000? Ans. 48c. $7\frac{1}{2}$m. or, $48\frac{3}{4}$c.

2. What cost 265 yards at 12c. 5m. per yard ? Ans. D.33 12c. 5m.

3. What cost 269½ yards at 16⅔c. per yard ?

16⅔c. | ⅙D. | 269·5=price at D.1.

D.44·91c. 7m. Ans.

4. What cost 1050 yards, at 6¼c. per yard ? Ans.D.65 62c. 5m.

CASE II.

When the price of 1 *is two or more aliquot parts of a dollar added together:* Divide the given number first for one aliquot part, then for another, &c. the quotients added together will be the answer.

EXAMPLES.

1. What cost 298 yards at 75 cents per yard ?

c.	D.	D.		
50	½	298	= price at D.1	
25	¼	149	= ditto at	·50
		74 50	=ditto at	·25
	Ans.	D.223 50c.	=ditto at	·75

2. What cost 927 yards, at 53⅓c. per yard ? Ans. D.494 40c.

3. What cost 618 yds. at 87½c. per yd. ? Ans. D.540 75c.

4. What cost 328 yds. at 57c. 5m. per yd. ? Ans. D.188 60c.

CASE III.

When the price of 1 *is the difference between two aliquot parts of a dollar :* Find the price at the greater aliquot part, and then at the less, and their difference will be the answer.

EXAMPLES.

1. What cost 328 yards, at 13⅓ cents per yard ?

c.	D.	D. c.		
33⅓	⅓	328	= price at D.1	
20	⅕	109 33⅓	= ditto at	33⅓c.
		65 60	= ditto at	20
		Ans. 43 73⅓	= ditto at	13⅓c.

2. What cost 817 yards at 30c. per yard ? Ans. D.245 10c.

3. What cost 296 yards at 15c. per yard ? Ans. D.44 40c.

CASE IV.

When the price of 1 *is any sum less than a dollar :* Divide the given price into aliquot parts, either of a dollar, or of each other ; find the price at each, and add them together for the answer.

EXAMPLES.

1. What cost 279 yards at 31c. per yard ?

25c.	¼D.	279·	= price at D.1	
5	⅕ of 25c.	69·75	= ditto at	25c.
1	⅕ of 5	13·95	= ditto at	5
		2·79	= ditto at	1
		Ans. D.86·49	= ditto at	31c.

2. What

2. What cost 953 yards at 57c. per yard ? Ans. D.543 21c.
3. What cost 839 yards at 36c. per yard ? Ans. D.302 4c.

CASE V.

When the price of 1 *is any sum between D.*1 *and D.*2 : The quantity itself in dollars is the price at D.1 then, finding, by the preceding rules, the price at the parts of D.1, the sum of the whole is the answer.

Examples.

1. What cost 386 yards at D.1·65c. per yard ?

		D. c.		
50c.	$\frac{1}{2}$D.	386	= price at D.1	
10	$\frac{1}{5}$ of 50c.	193	= ditto at	50c.
5	$\frac{1}{2}$ of 10	38 60	= ditto at	10
		19 30	= ditto at	5
		Ans. D.636 90	= ditto at D.1	65

2. What cost 849 yards at D.1·72 per yard ? Ans. D.1460·28.
3. What cost 294 yards at D.1·18 per yard ? Ans. D.346·92.

CASE VI.

When the price of 1 *is any number of dollars and parts of a dollar :* Multiply the quantity by the number of dollars ; and, finding, by the preceding rules, the price at the parts of D.1, the sum of the whole is the answer.

Examples.

1. What cost 395 yards at D.3 24c. per yard ?

c.		D. c.		
20	$\frac{1}{5}$D.	395	= price at	D.1
		3		
4	$\frac{1}{5}$ of 20c.	1185	= ditto at	3
		79	= ditto at	20c.
		15 80	= ditto at	4
		Ans. D.1279 80	= ditto at D.3	24c.

2. What cost	269 yards at	D.2 60c. per yard ?	Ans. D. 699 40c.
3. ——	694 ——	12 10 ————	8397 40
4.	318	4 $12\frac{1}{2}$	1311 75
5.	175	4 44	777

CASE VII.

When the price of 1 *contains the same aliquot part of a dollar any number of times exactly ; or, in other words, when the price of* 1 *has an aliquot part, which is also an aliquot part of a dollar :* First, find the value of the given quantity at the aliquot part ; then multiply this by the number of times which the aliquot part is contained in the given sum, for the answer. Or,

Since the price in this case is always such a number, as, being divided by the aliquot part, will make the numerator of a fraction, of which the denominator is the denominator of that fraction, which the aliquot part is of a dollar ;—Multiply the quantity by the numerator, and divide

divide the product by the denominator, (or, when convenient, divide the quantity by the denominator, and multiply the quotient by the numerator,) for the answer.*

EXAMPLES.

1. What cost 384 yards at $87\frac{1}{2}$ cents per yard ?

$12\frac{1}{2}$c. = $\frac{1}{7}$ of ·875 = | D.$\frac{1}{8}$ |	384·	=	price at	D.1
	48·	=	ditto at	·$12\frac{1}{2}$
	× 7			× 7
Ans.	D.336·	=	ditto at	·$87\frac{1}{2}$

Or thus,

·875=D.$\frac{7}{8}$, and $384 \times \frac{7}{8} = \frac{384\times7}{8}$ $(=\frac{384}{8}\times7)$=D.336, Ans. as before.

2. What cost 842 yards at $66\frac{2}{3}$c. per yard ? Ans. D.561 $33\frac{1}{3}$c.
3. What cost 912 yards at 55c. per yard ? Ans. D.501 60c.

MISCELLANEOUS QUESTIONS IN PRACTICE.

1. What cost 300 yards	at 27c. per yard ?	Ans. D. 81
917	D.1 12 5m.	1031 62c. 5m.
$35\frac{1}{5}$	35	12 32
$862\frac{1}{6}$ ft. boards	at D.12 per M. ?	10 34 6
32159	13 75c.	442 18 $6\frac{1}{4}$.

PRACTICE BY DECIMALS.

I. Since 2s. is $\frac{1}{10}$ of £.1, the decimal of 2s. is ·1 : Wherefore any quantity being given at 2s. per lb. yard, &c. the price is found in pounds and decimal parts of a pound, by separating the unit figure of the given quantity from the rest, for a decimal.

Let it be required to find the value of 356 yards at 2s. per yard ?

By pointing off the unit figure 6 for a decimal, I find the amount to be £.35·6, which is known to be equal to 35l. 12s. } £.35·6

II. Consequently, if the price be a multiple of 2s. (viz. any even number of shillings) the amount at 2s. being first found in pounds and decimal parts, as above, and that amount multiplied by the number which shows how often 2s. is contained in the given price, the product will be the amount required in pounds and decimal parts of a pound.

What cost 427 gallons of wine, at 8s. per gallon ?

£.42·7 amount at 2s. per gallon.
4

Ans. £.170·8 or 170l. 16s.

The

* Some of the prices which apply to this case, are to be found in the second table of parts of a dollar.

The examples in Case 4th. may be worked in this manner. Likewise, if the price be pounds and even shillings.

754 yards at 1l. 8s.

75·4 amount at 2s.
14×2=28s.

3016
754

Ans. £.1055·6=1055l. 12s.

Or,

$\left.\begin{array}{r}754\\ 75{\cdot}4\times4=301{\cdot}6\end{array}\right\}$ Add.

£.1055·6

III *If the price be an aliquot part of 2s.* : Find the amount at 2s. and divide it by the denominator of the part, and the quotient will be the answer.

At 8d. per lb. : What cost 976 lb. ?

| 8d. | $\frac{1}{3}$ | 97·6

£32·533 = £32 10 8 Ans.

IV. *If the price be an aliquant (that is, uneven) part* : Divide it into aliquot parts.

7235 yards, at 7d.

4d.	$\frac{1}{6}$	723·5
3d.	$\frac{1}{8}$	120·583
		90·437
		211·02 = £.211 0 $4\frac{3}{4}$ Ans.

V. *If the price be pounds and shillings, or pounds, shillings and pence* : Reduce the shillings, &c. to the decimal of a pound, and multiply the quantity thereby, or the price by the quantity.

At 15l. 12s. 6d. per Cwt. : What cost 75 Cwt. ?

£.15 12 6 = £.15·625
75

78125
109375

1171·875

£.1171 17 6 Ans.

VI. *If the quantity likewise be of divers denominations* : Reduce the less denominations to the decimal of that, whereof the price is given.

9lb.

9lb. 10oz. of silk, at £.4 5 9 = £.4·287
9lb. 10oz. = 9 625

```
     21435
     8574
   25722
  38583
  ---------
  41·262375
  ---------
£.41  5  3 Ans.
```

Cases 11th. and 12th. may be wrought in this manner.

Or, You may take parts for the lower denominations:

```
| 8oz. | 1/2 | 4·287
| 2oz. | 1/4 |     9
            --------
            38·583
             2·1435
              ·535875
            ---------
            41·262375
            ---------
          £.41  5  3
```

VII. When the price is any *odd* number of shillings: If it be required to know what quantity of any thing may be bought for any sum of money, in pounds: Annex *two* cyphers to the money, and divide it by half the price.

Note. As half a shilling (or 6 pence) is ·5, therefore, to halve any odd number of shillings, is only to annex ·5 to half of the greatest even number in the price.

1st. How many yds. at 7s. per yd. may I have for 435l.?

```
Half = 3·5)43500(1242 30/35 yds. Ans.
           35
           ---
           85
           70
           ---
           150
           140
           ---
            100
             70
            ---
             30
```

2d. How many pounds of tea, at 5s. per lb for 37l.?

```
2·5)3700(148lb. Ans.
    25
    ---
    120
    100
    ---
     200
     200
    ---
```

3d. How many yards at 9s. per yard may I have for 540l.?

Ans. 1200 yards.

BILL

BILL OF PARCELS.

Newburyport, January 1st, 1808.

Mr. Timothy Huckster

Bought of *Samuel Merchant*,

$25\frac{1}{2}$lb Bohea tea, at 3s. 6d. per lb.
48lb. Cheese, at 9d. per lb.
15 Pair worsted hose, at 5s. 8d. per pair.
$4\frac{1}{2}$ Dozen women's gloves, at 36s. 6d. per dozen.
19 Dozen knives and forks, at 5s. 9d. per dozen.
9 Grindstones at 15s 9d. per stone.
$\frac{1}{2}$ Cwt. Brown sugar, at 51s. per cwt.
31 lb. Loaf Sugar, at 1s. $0\frac{1}{2}$d. per lb.

£.34 3 $3\frac{1}{2}$

Received payment in full,
Samuel Merchant.

TARE AND TRET

TARE and Tret are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of 4lb. in every 104lb. for waste, dust, &c.

Cloff is an allowance of 2lb. upon every 3 Cwt.

Gross weight is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. which contains them.

Suttle is, when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

CASE I.*

When the tare is at so much per box, barrel or bag, &c : Multiply the number of boxes, barrels, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight required.

EXAMPLES.

1. In 6 hogsheads of sugar, each weighing 9cwt. 2qrs. 10lb. gross, tare 25lb. per hogshead ; how mnch neat ?

Cwt.	qr.	lb.		Cwt.	qr.	lb.	
25×6=1	1	10		9	2	10	gross wt. of 1 hhd.
						6	
				57	2	4	gross.
				1	1	10	tare.
			Ans.	56	0	22	neat.

2. In

* This, as well as every other case in this rule, is only an application of the rules of Proportion and Practice.

2. In 5 bags of cotton, marked with the gross weight as follows, tare 23lb. per bag; what neat weight?

	Cwt.	qr.	lb.
A =	7	1	19
B =	3	3	27
C =	5	1	12
D =	6	0	15
E =	8	1	0

Cwt. qr. lb.
Ans. 30 0 14 neat.

3. What is the neat weight of 15 hogsheads of tobacco, each 7cwt. 1qr. 13lb. tare 100lb. per hogshead? Ans. 97cwt. 0qr. 11lb.

CASE II.

When the tare is at so much per cwt.: Divide the gross weight by the aliquot parts of a cwt. subtract the quotient from the gross, and the remainder will be the neat weight.

EXAMPLES.

1. In 129cwt. 3qrs. 16lb. gross, tare 14lb. per cwt. what neat weight?

		Cwt.	qr.	lb.	
14lb.	$\frac{1}{8}$	129	3	16	gross.
		16	0	$26\frac{1}{2}$	tare.
	Ans.	113	2	$17\frac{1}{2}$	neat.

2. In 97cwt. 1qr. 7lb. gross, tare 20lb. per cwt. what neat weight?

lb.		Cwt.	qr.	lb.	
16	$\frac{1}{7}$	97	1	7	gross.
4	$\frac{1}{4}$	13	3	17	Add.
		3	1	25	
Subtract		17	1	14	tare.
	Ans.	79	3	21	neat.

3. What is the neat weight of 9 barrels of potash, each weighing 305lb. gross, tare 12lb. per cwt.? Ans. 2450lb. 14oz. $4\frac{4}{7}$dr.

4. What is the value of the neat weight of 7hhds. of tobacco, at 5l. 7s. 6d. per cwt. each weighing 8cwt. 3qrs. 10lb. gross, tare 21lb. per cwt.? Ans. £.270 4 $4\frac{1}{2}$ reckoning the odd ounces.

CASE III.

When tret is allowed with tare: Divide the suttle weight by 26, and the quotient will be the tret, which subtract from the suttle, and the remainder will be the neat.

EXAMPLES.

1. In 247cwt. 2qrs. 15lb. gross, tare 28lb. per cwt. and tret 4lb. for every 104 lb. what neat weight?

28	$\frac{1}{4}$	247C.	2qr.	15lb.		gross.
		61	3	17	12	tare, subtract.
4	$\frac{1}{26}$	185	2	25	4	suttle.
		7	0	16	0	tret, subtract.
	Ans.	178	2	9	4	neat.

2. What

2. What is the neat weight of 4 hhds. of tobacco, weighing as follow: The 1st. 5cwt. 1qr. 12lb. gross, tare 65lb. per hhd.; the 2d. 3cwt. 0qr. 19lb. gross, tare 75lb.; the 3d. 6cwt. 3qrs. gross, tare 49lb.; and the 4th 4cwt. 2qrs. 9lb. gross, tare 35lb. and allowing tret to each as usual? Ans. 17cwt. 0qr. 19lb.+

CASE IV.

When tare, tret and cloff are allowed: Deduct the tare and tret as before, and divide the suttle by 168, and the quotient will be the cloff, which subtract from the suttle, and the remainder will be the neat.

EXAMPLES.

1. What is the neat weight of 1hhd. of tobacco, weighing 16cwt. 2qrs. 20lb. gross, tare 14lb. per cwt. tret 4lb. per 104, and cloff 2lb. per 3cwt.?

14lb. is $\frac{1}{8}$)	16	2	20	0	gross.
	2	0	9	8	tare, subtract.
4lb. is $\frac{1}{26}$)	14	2	10	8	
	0	2	6	13	tret, subtract.
2lb. is $\frac{1}{168}$)	14	0	3	11	suttle.
	0	0	9	5	cloff, subtract.
Ans.	13	3	22	6	neat.

2. If 9hhds. of tobacco, contain 85cwt. 0qr. 2lb. tare 30lb. per hhd. tret and cloff as usual, what will the neat weight come to at $6\frac{1}{2}$d. per per lb. after deducting for duties and other charges, 51l. 11s. 8d.? Ans. £.187 18s. 5d.

INVOLUTION,

OR TO RAISE POWERS.

A POWER is the product arising from multiplying any given number into itself continually a certain number of times, thus:

$3\times3=9$ is the 2d. power, or square of 3. $=3^2$

$3\times3\times3=27$ is the 3d. power, or cube of 3. $=3^3$

$3\times3\times3\times3=81$ is the 4th. power, or the biquadrate of 3, &c. $=3^4$

The number denoting the power is called the *index*, or the *exponent* of that power. Thus, the fourth power of 3 is 81, or 3^4; the second power of 5 is 25, or 5^2, &c.

$2\times2=4$, the square of 2; $4\times4=16=$4th. power of 2; $16\times16=256=$ 8th. power of 2, &c.

RULE.

Multiply the given number, root, or first power continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

Note. Whence, because fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved

volved by raising each of their terms to the power required, and if a mixed number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

EXAMPLES.

1. What is the 5th. power of 9 ?

$$\begin{array}{r l} 9 & \\ 9 & \\ \hline 81 & =2\text{d. power.} \\ 9 & \\ \hline 729 & =3\text{d. power.} \\ 9 & \\ \hline 6561 & =4\text{th. power.} \\ 9 & \\ \hline 59049 & =5\text{th. power, or answer}=9^5. \end{array}$$

2. What is the 5th power of $\frac{3}{5}$? Ans. $\frac{243}{3125}$.

3. What is the fourth power of ·045 ? Ans. ·000004100625.

Here we see, that in raising a fraction to a higher power, we decrease its value.

EVOLUTION,
OR THE EXTRACTION OF ROOTS.

THE Root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d. 3d. 4th. root, &c. accordingly as it is, when raised to the 2d, 3d. &c. power, equal to that power. Thus, 4 is the square root of 16, because $4\times4=16$, and 3 is the cube root of 27, because $3\times3\times3=27$, and so on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which precise roots can never be determined. But, by the help of decimals, we can approximate towards the root to any assigned degree of exactness.

The roots, which approximate, are called *surd roots*, and those which are perfectly accurate, are called *rational roots*.

Roots are sometimes denoted by writing the character $\surd$ before the power, with the index of the power over it ; thus the 3d. root of 36 is expressed $\sqrt[3]{36}$, and the 2d. root of 36 is $\sqrt{36}$, the index 2 being omitted when the square root is designed.

If the power be expressed by several numbers, with the sign + or — between them, a line is drawn from the top of the sign over all the parts of it. Thus the 3d root of 47+22 is $\sqrt[3]{47+22}$, and the 2d. root of 59 — 17 is $\sqrt{59-17}$, &c.

Sometimes roots are designed like powers, with fractional indices. Thus, the square root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and 4th. root of 37 — 20 is $\overline{37-20}^{\frac{1}{4}}$, &c.

A TABLE OF POWERS.

	Powers									
Roots, - - -	or 1st. Pow.	1	2	3	4	5	6	7	8	9
Squares, - -	or 2d. Pow.	1	4	9	16	25	36	49	64	81
Cubes, - - -	or 3d. Pow.	1	8	27	64	125	216	343	512	729
Biquadrates, - -	or 4th. Pow.	1	16	81	256	625	1296	2401	4096	6561
Surfolids, - -	or 5th. Pow	1	32	243	1024	3125	7776	16807	32768	59049
Square cubes, -	or 6th. Pow	1	64	729	4096	15625	46656	117649	262144	531441
Second Surfolids,	or 7th. Pow	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Sqd.	or 8th. Pow.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed, -	or 9th. Pow.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Surfolids fquared,	or 10th. Pow	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Surfolids,	or 11th. Pow	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square Cubes Sqd.	or 12th. Pow	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
Fourth Surfolids,	or 13th. Pow	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
2d. Surfolids Sqd.	or 14th. Pow	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
Surfolids Cubed,	or 15th. Pow.	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

THE EXTRACTION OF THE SQUARE ROOT.

RULE.

*1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand, period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor: Multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend : To the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it) and from these, find the next figure in the root as last directed, and continue the operation, in the same manner, till you have brought down all the periods.

Note 1. If when the given power is pointed off as the power requires, the left hand figure should be deficient, it must nevertheless stand as the first period.

Note 2. If there be decimals in the given number, it must be pointed both ways from the place of units : If, when there are integers, the

* In order to ſhew the reaſon of the rule, it will be proper to premiſe the following Lemma. The product of any two numbers can have, at moſt, but ſo many places of figures as are in both the factors, and at leaſt but one leſs.

Demonſtration. Take two numbers conſiſting of any number of places ; but let them be the leaſt poſſible of thoſe places, viz. Unity with cyphers, as 100 and 10 : Then their product will be 1 with ſo many cyphers annexed as are in both the numbers, viz. 1000 ; but 1000 has one place leſs than 100 and 10 together have : And ſince 100 and 10 were taken the leaſt poſſible, the product of any other two numbers, of the ſame number of places, will be greater than 1000 ; conſequently, the product of any two numbers can have, at leaſt, but one place leſs than both the factors.

Again, take two numbers, of any number of places, which ſhall be the greateſt poſſible of thoſe places, as 99 and 9. Now, 99×9 is leſs than 99×10 ; but 99×10 (=990) contains only ſo many places of figures as are in 99 and 9 ; therefore, 99×9, or the product of any other two numbers, conſiſting of the ſame number of places, cannot have more places of figures, than are in both its factors.

Corollary 1. A ſquare number cannot have more places of figures than double the places of the root, and at leaſt but one leſs.

Corollary 2. A cube number cannot have more places of figures than triple the places of the root, and at leaſt but two leſs.

the first period in the decimals be deficient, it may be completed by annexing so many cyphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing cyphers.

EXAMPLES.

1st. Required the square root of 30138696025?

```
                        30138696025(173605 the root.
                        1
                        --
  1st. Divisor=27)201
                  189
                  ----
  2d. Divisor=343)1238
                  1029
                  -----
  3d. Divisor=3466)20969
                   20796
                   -------
  4th. Divisor=347205)1736025
                      1736025
                      -------
```

2d. Required the square root of 575·5?

```
          575·50(23·98+, the root.
          4
          --
      43)175
         129
         ----
     469)4650
         4221
         -----
    4788)42900
         38304
         -----
          4596 Remainder.
```

3d. What is the square root of 10342656? Ans. 3216.

4th. What is the square root of 964·5192360241? Ans. 31·05671.

5th. What is the square root of 234·09? Ans. 15·3.

6th. What is the square root of ·0000316969? Ans. ·00563.

7th. What is the square root of ·045369? Ans. ·213.

RULES FOR THE SQUARE ROOT OF VULGAR FRACTIONS AND MIXED NUMBERS.

After reducing the fraction to its lowest terms, for this and all other roots; then,

1st. Extract

1st. Extract the *root* of the *numerator* for a *new numerator*, and the *root* of the *denominator* for a *new denominator*, which is the best method, provided the denominator be a complete power. But if it be not,

2d. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional part required.* Or,

3d. Reduce the vulgar fraction to a decimal, and extract its root.

4th. Mixed numbers may either be reduced to improper fractions, and extracted by the first or second rule, or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

EXAMPLES.

1st. What is the square root of $\frac{144}{15129}$?

By Rule 1.

$\frac{144}{15129}=\frac{16}{1681}$

16(4 root of the numerator.
16
—

1681(41 root of the denominator.
16
—
81)81 Therefore, $\frac{4}{41}$ = the root of the given fraction.
81

By Rule 2.

$16\times1681=26896$, and $\sqrt{26896}=164$. Then,

$\frac{164}{1681}=\frac{16}{164}=\frac{4}{41}=\cdot09756+$

By Rule 3.

$1681)16(\cdot0095181439+$. And $\sqrt{\cdot0095181439}=\cdot09756+$.

2d. What is the square root of $\frac{2793}{8208}$? Ans. $\frac{7}{12}$.

3d. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.

Note. In extracting the square or cube root of any surd number, there is always a remainder or fraction left, when the root is found. To find the value of which, the common method is, to annex *pairs* of cyphers to the resolvend, for the square, and *ternaries* of cyphers to that of the cube, which makes it tedious to discover the value of the remainder, especially in the cube, whereas this trouble might be saved if the true denominator could be discovered

As in division the divisor is always the denominator to its own fraction, so likewise it is in the square and cube, each of their divisors being the denominators to their own particular fractions or numerators. In

* That is, ſuppoſe $a=7$, and $b=2$, the rule may be thus expreſſed: $\sqrt{\frac{a}{b}}=\frac{\sqrt{ab}}{b}=\frac{a}{\sqrt{ab}}$, Or, numerically thus: $\sqrt{\frac{7}{2}}=\frac{\sqrt{7\times2}}{2}=\frac{7}{\sqrt{7\times2}}=1\cdot87+$, and this rule will ſerve whether the root be *finite* or *infinite*.

In the square the quotient is always doubled for a new divisor; therefore, when the work is completed, the root doubled is the true divisor or denominator to its own fraction; as, if the root be 12, the denominator will be 24, to be placed under the remainder, which vulgar fraction, or its equivalent decimal, must be annexed to the quotient or root, to complete it.*

If to the remainder, either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

APPLICATION AND USE OF THE SQUARE ROOT.

PROB. I *To find a mean proportional between two numbers.*

RULE. Multiply the given numbers together, and extract the square root of the product; which root will be the mean proportional sought.

EXAMPLE.

What is the mean proportional between 24 and 96?

$\sqrt{96\times24}=48$ Answer.

PROB. II. *To find the side of a* square *equal in* area *to any given* superficies *whatever.*

RULE. Find the *area,* and the square root is the side of the square sought.

EXAMPLES.

1st. If the area of a circle be 184·125, What is the side of a square equal in area thereto?

$\sqrt{184{\cdot}125}=13{\cdot}569+$ Answer.

2d. If the area of a triangle be 160, What is the side of a square equal in area thereto $\sqrt{160}=12{\cdot}649+$ Answer.

PROB. III. A certain general has an army of 5625 men: pray How many must he place in rank and file, to form them into a square? $\sqrt{5625}=75$ Answer.†

PROB. IV. Let 10952 men be so formed, as that the number in rank may be double the file.

$\sqrt{\frac{10952}{2}}=74$ in file, and $74\times2=148$ in rank.

PROB V. If it be required to place 2016 men so as that there may be 56 in rank and 36 in file, and to stand 4 feet distance in rank, and as much in file, How much ground do they stand on?

To answer this, or any of the kind, use the following proportion: As unity : to the distance :: so is the number in rank less by one : to a fourth number; next, do the same by the file, and multiply the two

* Although these denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use, and are much more expeditious than the operation with cyphers.

† If you would have the number of men be double, triple, or quadruple, &c. as many in rank as in file, extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c. and the product will be the number in rank.

two numbers together, found by the above proportion, and the product will be the answer.*

As 1 : 4 :: 56 — 1 : 220. And, as 1 : 4 :: 36 — 1 : 140. Then, $220\times140=30800$ square feet, the Answer.

PROB. VI. Suppose I would set out an orchard of 600 trees, so that the length shall be to the breadth as 3 to 2, and the distance of each tree, one from the other, 7 yards: How many trees must it be in length, and how many in breadth? and, How many square yards of ground do they stand on?

To resolve any question of this nature, say, as the ratio in length : is to the ratio in breadth :: so is the number of trees : to a fourth number, whose square root is the number in breadth. And as the ratio in breadth : is to the ratio in length :: so is the number of trees : to a fourth, whose root is the number in length.

As 3 : 2 :: 600 : 400. And $\sqrt{400}=20=$ number in breadth.

As 2 : 3 :: 600 : 900. And $\sqrt{900}=30=$ number in length.

As 1 : 7 :: 30 — 1 : 203. And as 1 : 7 :: 20 — 1 : to 133. And $203\times133=26999$ square yards, the Answer.

PROB. VII. Admit a leaden pipe $\frac{3}{4}$ inch diameter will fill a cistern in 3 hours; I demand the diameter of another pipe which will fill the same cistern in 1 hour.

RULE. As the given time is to the square of the given diameter, so is the required time to the square of the required diameter.

$\frac{3}{4}=\cdot75$: and $\cdot75\times\cdot75=\cdot5625$. Then, as 3h. : ·5625 :: 1h. : 1·6875 inversely, and $\sqrt{1\cdot6875}=1\cdot3$ inch nearly, Ans.

PROB VIII. If a pipe whose diameter is is 1·5 inch, fill a cistern in 5 hours, in what time will a pipe whose diameter is 3·5 inches fill the same?

$1\cdot5\times1\cdot5=2\cdot25$; and $3\cdot5\times3\cdot5=12\cdot25$. Then, as 2·25 : 5 :: 12·25 : ·918+ hour, inversely, =55 min. 5 sec. Answer.

PROB. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water, In what time will 3 pipes, each four inches bore, be in discharging double the quantity?

$6\times6=36$. $4\times4=16$, and $16\times3=48$. Then, as 36 : 4h. :: 48 : 3h. inversely, and as 1w. : 3h. :: 2w. : 6h. Answer.

PROB. X. Given the diameter of a circle to make another circle, which shall be 2, 3, 4, &c. times greater or less than the given circle.

RULE. Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the root of the product will be the required diameter. But if the required circle be less, divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

There is a circle whose diameter is 4 inches; I demand the diameter of a circle 3 times as large?

$4\times4=16$; and $16\times3=48$; and $\sqrt{48}=6\cdot928+$ inches Answer.

PROB.

* The above rule will be found useful in planting trees, having the distance of ground between each given.

PROB. XI. To find the diameter of a circle equal in area, to an ellipsis, (or oval) whose transverse and conjugate diameters are given.*

RULE. Multiply the two diameters of the ellipsis together, and the square root of that product will be the diameter of a circle equal to the ellipsis.

Let the transverse diameter of an ellipsis be 48, and the conjugate 36: What is the diameter of an equal circle?

$48\times36=1728$, and $\sqrt{1728}=41{\cdot}569+$ the Answer.

Note. The square of the hypothenuse, or the longest side of a right angled triangle, (by 47th B. 1. Euc.) is equal to the sum of the squares of the other two sides; and consequently the difference of the squares of the hypothenuse and either of the other sides is the square of the remaining side.

PROB XII. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad. The height of the wall is required?

$36\times36=1296$; and $24\times24=576$. Then, $1296-576=720$, and $\sqrt{720}=26{\cdot}83+$yards, the Answer.

PROB. XIII. The height of a tree growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet. What is the breadth of the stream, provided the land on each side of the water be level?

$256\times256=65536$; and $75\times75=5625$: Then, $65536-5625=59911$ and $\sqrt{59911}=244{\cdot}76+$ and $244{\cdot}76-\frac{44}{2}=222{\cdot}76$ feet, Answer.

PROB. XIV. Suppose a ladder 60 feet long be so planted as to reach a window 37 feet from the ground, on one side of the street, and without moving it at the foot, will reach a window 23 feet high on the other side; I demand the breadth of the street?

$60\times60=3600$. $37\times37=1369$. $23\times23=529$: Then, $3600-1369=2231$, and $\sqrt{2231}=47{\cdot}23+$, and $3600-529=3071$, and $\sqrt{3071}=55{\cdot}41+$, then, $47{\cdot}23+55{\cdot}41=102{\cdot}64$ feet, the Answer.

PROB. XV. Two ships sail from the same port; one goes due north 45 leagues, and the other due west 76 leagues: How far are they asunder?†

$45\times45=2025$. $76\times76=5776$. Then, $5776+2025=7801$ and $\sqrt{7801}=88{\cdot}32$ leagues, the answer.

EXTRACTION

* The transverse and conjugate are the longest and shortest diameters of an ellipsis; they pass through the centre, and cross each other at right angles

† The square root may in the same manner be applied to navigation; and, when deprived of other means of solving problems of that nature, the following proportion will serve to find the course.

As the sum of the hypothenuse (or distance) and half the greater leg (whether difference of latitude or departure) is to the less leg; so is 86, to the angle opposite the less leg.

EXTRACTION OF THE CUBE ROOT.

A cube is any number multiplied by its *square*. To extract the cube root, is to find a number which, being multiplied into its square, shall produce the given number.

First Method.

Rule.

† 1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend*.

4. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor*.

5. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure and call their sum the *subtrahend*.

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.

Note. The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root.

Examples.

1st. Required the cube root of 436036824287 ?

436036824287(7583 root.	7×7×300 =	14700=1ſt. Trip.ſq.
343	7× 30 =	210=1ſt. do. quo.
1ſt. Divis.=14910)93036=1ſt. Dividend.		14910=1ſt. diviſor.
73500	14700×5=	73500
5250	5×5×210=	5250
125	5×5×5 =	125
78875=1ſt. Subtrahend.		78785=1ſt. Subtra.
2d.Div.=1689750)14161824=2d.Divid.	75×75×300=	1687500=2d.Trip. ſq.
	75×30 =	2250=2d. do. quo.
13500000		
144000		1689750=2d. Diviſor.
512		
	1687500×8=	13500000
13644512=2d. Subtra.	2250×8×8=	144000

Carried over.

† The reaſou of pointing the given number, as directed in the rule, is obvious from *Coroll.* 2, to the *Lemma* made uſe of in demonſtrating the ſquare root.

Brought over. 13644512=2d. Subtra.	$2250 \times 8 \times 8 =$ 144000
	$8 \times 8 \times 8 =$ 512
3d.Div.=172391940)517312287=3d. Divid.	
	13644512=2d. Subtra.
517107600	
204660	$758 \times 758 \times 300 =$ 172369200=3d.Trip. ſq.
27	758×30 22740=3d. do. quo.
517312287=3d. Subtra.	172391940=3d. Diviſor.
....	$172369200 \times 3 =$ 517107600
	$22740 \times 3 \times 3 =$ 204660
	$3 \times 3 \times 3 =$ 27
	517312287=3d. Subtra.

2d. What is the cube root of 34965783? Ans. 327.
3d. What is the cube root of 84·604519? Ans. 4·39.
4th. What is the cube root of ·008649? Ans. ·2052+.
5th. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finished.

In the extraction of the cube root, the quotient is said to be squared and tripled for a new divisor; but is not really so, till the triple number of the quotient be added to it; therefore when the operation is finished, it is but squaring the quotient, or root, then multiplying it by 3, and to that number adding the triple number of the root, when it will become the divisor, or true denominator to its own fraction, which fraction must be annexed to the quotient, to complete the root.

Suppose the root to be 12, when squared it will be 144, and multiplied by 3, it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.*

SECOND METHOD.

RULE.

1. Having pointed the given number into periods of three figures each, find the greatest cube in the left hand period, subtracting it therefrom

* It may not be amiſs to remark here, that the denominators, both of the ſquare and cube, ſhew how many numbers they are denominators to, that is, what numbers are contained between any ſquare or cube number and the next ſucceeding ſquare or cube number, excluſive of both numbers, for a complete number, of either, leaves no fraction, when the root is extracted, and conſequently has no uſe for a denominator, but all the numbers contained between them have occaſion for it:—Suppoſe the ſquare root to be 12, then its ſquare is 144, and the denominator 24, which will be a denominator to all the ſucceeding numbers, until we come to the next ſquare number, *viz.* 169, whoſe root is 13, with which it has nothing to do, for between the ſquare numbers 144 and 169 are contained 24 numbers excluding both the ſquare numbers. It is the ſame in the cube; for, ſuppoſe the root to be 6, the cube number is 216, and its denominator 126 will be a denominator to all the ſucceeding numbers, until we come to the next cube number, *viz.* 343, whoſe root is 7, with which it has nothing to do, as ceaſing then to be a denominator; for between the cube 343 and 216 are 126 numbers, excluding both cubes. And so it is with all other denominators, either in the ſquare or cube.

therefrom and placing its root in the quotient; to the remainder bring down the next period and call it the *dividend.*

2. Under this dividend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former; and under the said triple square, write the triple root, removed one place to the right hand, and call the sum of these the *divisor*.

3. Seek how often the divisor may be had in the dividend, exclusive of the place of units, and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient figure, setting down the unit's place of this line, under the place of tens in the divisor; under this line, write the product of the triple root by the square of the last quotient figure, so as to be removed one place beyond the right hand figure of the former; and, under this line, removed one place forward to the right hand, write down the cube of the last quotient figure, and call their sum the *subtrahend.*

5. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on until the whole be finished.

EXAMPLE.

Required the Cube Root of 16194277?

16194277(253=Root.

8	
8194	= First dividend.
12	= Triple square of 2.
6	= Triple of 2.
126	= First divisor.
60	= Triple square of 2, multiplied by 5.
150	= Triple of 2 multiplied by the square of 5.
125	= Cube of 5.
7625	= First Subtrahend.
569277	= Second dividend.
1875	= Triple square of 25.
75	= Triple of 25.
18825	= Second divisor.
5625	= Triple square of 25 multiplied by 3.
675	= Triple of 25 multiplied by the square of 3.
27	= Cube of 3.
569277	= Second subtrahend.

First Method by Approximation.

Rule.

1. Find, by trial, a cube near to the given number, and call it the *supposed* cube.

2. Then as twice the supposed cube, added to the given number, is to twice the given number, added to the supposed cube, so is the root of the supposed cube, to the true root, or an approximation to it.

3. By taking the cube of the root, thus found, for the supposed cube, and repeating the operation, the root will be had to a greater degree of exactness.

Example.

It is required to find the cube root of 54854153 ?

Let 64000000=supposed cube, whose root is 400 :

```
Then, 64000000     54854153
             2            2
      ---------    ---------
      128000000    109708306
       54854153     64000000
      ---------    ---------
As    182854153 :  173708306 :: 400
                         400
                   ---------
      182854153)69483322400(379=root nearly.
```

Again, let 54439939 = supposed cube, whose root is 379.

```
Then, 54439939     54854153
             2            2
     ---------    ---------
     108879878    109708306
      54854153     54439939
     ---------    ---------
As   163734031 :  164148245 :: 379
                        379
                 ----------
                 1477334205
                1149037715
                492444735
                -----------
     163734031)62212184855(379·958793+= root corrected.
```

Second Method by Approximation.

Rule.

1. Divide the resolvend by three times the assumed root, and reserve the quotient.

2. Subtract one twelvth part of the square of the assumed root from the quotient.

3. Extract the square root of the remainder.

4. To this root add one half of the assumed root, and the sum will be the true root, or an approximation to it ; take this approximation as the assumed root, and, by repeating the process, a root farther approximated will be found, which operation may be farther repeated,

as

as often as necessary, and the root discovered to any assigned exactness.

Note. In order to find the value of the first assumed root, in this or any other power, divide the resolvend into periods by beginning at the place of units, and including in each period, so many figures as there are units in the exponent of the root; viz. 3 figures in the cube root; 4 for the biquadrate, and so on; then, by a table of powers, or otherwise, find a figure, which (being involved to the power whose exponent is the same with that of the required root) is the nearest to the value of the first period of the resolvend at the left hand, and to that figure annex so many cyphers as there are periods remaining in the integral part of the resolvend; this figure, with the cyphers annexed, will be the assumed root, and equal to r in the theorem; and it is of no importance whether the figure thus chosen be, when involved, greater or less than the left hand period, as the theorem is the same in both cases.

1st. What is the cube root of $43\dot{6}03\dot{6}82\dot{4}28\dot{7}$?

7000=assumed root.
3

21,000)436036824287(20763658·2994
Subtract 7000×7000÷12=4083333·3333

$\sqrt{16680324\cdot9661}=4084\cdot15$
Add $\frac{1}{2}$ the assumed root=3500

And it gives the approximated root=7584·15

For the second operation, use the approximated root as the assumed one, and proceed as above.

Third Method by Approximation.

1. Assume the root in the usual way, then multiply the square of the assumed root, by 3, and divide the resolvend by this product; to this quotient add $\frac{2}{3}$ of the assumed root, and the sum will be the true root, or an approximation to it.

2. For each succeeding operation let the last approximated root be the assumed root, and proceeding in this manner, the root may be extracted to any assigned exactness.

1st. What is the cube root of 7 ?

Let the assumed root be 2. Then, $\overline{2\times2}\times3=12$ the divisor.

12)7·0(·583 to this add $\frac{2}{3}$ of 2=1·333, &c. that is, ·583+1·333=1·916 approximated root.

Now assume 1·916 for the root. Then, by the second process, the the root is $\sqrt{\frac{7}{3\times1\cdot916}-\frac{1}{9}\times\overline{1\cdot916}^2}+\frac{1}{3}\times1\cdot916=1\cdot9126$, &c.

2d. What is the cube root of 9 ? Let 2 be the assumed root as before. Then, $\frac{9}{12}+\frac{2}{3}\times2=2\cdot08$ the approximated root. Now assume 2·08. Then, $\sqrt{\frac{9}{3\times2\cdot08}-\frac{1}{9}\times\overline{2\cdot08}^2}+\frac{1}{3}\times2\cdot08=2\cdot08008$, &c.

3d. What

2d. What is the cube root of 282? Let 6 be the assumed root.—Then, 6×6×3=108)282(2·611, &c. and 2·611+$\frac{2}{3}$ of 6=6·611 approximated root. Now assume 6·611, and it will be 6·611×6·611×3=131·116)282(2·1507, &c. and 2·1507+$\frac{2}{3}$ of 6·611=6·558 a farther approximated root.

4th. What is the cube root of 1728?——Here the assumed root is 10. Then, 10×10×3=300)1728(5·76, and 5·76+$\frac{2}{3}$ of 10=12·426.——Now assume 12·426, then 12·426×12·426×3=463·216428)1728(3·732, and 3·732+$\frac{2}{3}$ of 12·426=12·014 a farther approximated root, and so on.

APPLICATION AND USE OF THE CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

Rule.—1. Divide the *greater* by the *less*, and extract the cube root of the quotient.

2. Multiply the root, so found, by the least of the given numbers, and the product will be the *least*.

3. Multiply this product by the same root, and it will give the greatest.

Examples.

1st. What are the two mean proportionals between 6 and 750?

750÷6=125, and $\sqrt[3]{125}$=5. Then. 5×6=30=least, and 30×5=150= greatest. Answer 30 and 150.

Proof. As 6 : 30 :: 150 : 750.

2d. What are the two mean proportionals between 56 and 12096?

Answer 336 and 2016.

Note. The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

3d If a bullet 6 inches diameter weigh 32lb.; What will a bullet of the same metal weigh, whose diameter is 3 inches?

6×6×6=216. 3×3×3=27. As 216 : 32lb. :: 27 : 4lb. Ans.

4th. If a globe of silver of 3 inches diameter, be worth £.45, What is the value of another globe, of a foot diameter?

3×3×3=27 12×12×12=1728. As 27 : 45 :: 1728 : £.2880 Ans.

The side of a cube being given, to find the side of that cube which shall be double, triple, &c. in quantity to the given cube.

Rule.—Cube your given side, and multiply it by the given proportion between the given and required cube, and the cube root of the product will be the side sought

5th If a cube of silver, whose side is 4 inches, be worth £.50, I demand the side of a cube of the like silver, whose value shall be 4 times as much?

4×4×4=64, and 64×4=256. $\sqrt[3]{256}$=6·349+inches, Ans.

6th. There is a cubical vessel, whose side is 2 feet; I demand the side of a vessel, which shall contain three times as much?

2×2×2=8, and 8×3=24. $\sqrt[3]{24}$=2·884=2ft. 10$\frac{3}{5}$inches, Ans.

7th. The

7th.* The diameter of a bushel measure being $18\frac{1}{2}$ inches, and the height 8 inches, I demand the side of a cubic box, which shall contain that quantity? Ans. 12·907 + inches.

8. Suppose a ship of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold: What are the dimensions of a ship of 200 tons, of the same mould and shape?

89×89×89=704969=cubed keel.

As 500 : 200 :: 704969 : 281987·6 cube of the required keel.

$\sqrt[3]{281987{\cdot}6}=65{\cdot}57$ feet the required keel.

As 89 : 65·57 :: 36 : 26·522=$26\frac{1}{2}$ feet, beam, nearly.

As 89 : 65·57 :: 16 : 11·7 feet, depth of the hold, nearly.

9. From the proof of any cable to find the strength of any other.

Rule.—The strength of cables, and consequently the weights of their anchors, are as the cubes of their peripheries.

If a cable, 12 inches about, require an anchor of 18cwt. Of what weight must an anchor be, for a 15 inch cable?

Cwt. Cwt.

As 12×12×12 : 18 :: 15×15×15 : 35·15625 Ans.

10. If a 15 inch cable require an anchor 35·15625cwt. : What must the circumference of a cable be, for an anchor of 18cwt.?

As 35·15625 : 15×15×15 :: 18 : 1728, and $\sqrt[3]{1728}=12$ Ans.

EXTRACTION OF THE BIQUADRATE ROOT.

Rule.

Extract the square root of the resolvend, and then the square root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736?

```
20736(144            144(12 root required.
1                    1
—                    —
24)107           22)44
   96               44
  —                 —
284)1136             .
    1136
```

TWO METHODS OF EXTRACTING THE BIQUADRATE ROOT BY APPROXIMATION.

Rule I.

1. Divide the resolvend by *six* times the square of the assumed root, and from the quotient subtract $\frac{1}{18}$ part of the square of the assumed root.

2. Extract

* Multiply the square of the diameter by ·7854, and the product by the height; the cube root of the last product is the answer. See *Mensuration of Superficies and Solids. Art.* 30.

2. Extract the square root of the remainder.

3. Add $\frac{2}{3}$ of the assumed root to the square root, and the sum will be the true root, or an approximation to it.

4. For every succeeding operation, either in this or the following method, proeeed in the same manner, as in the first, each time using the last approximated root for the assumed root.

The biquadrate root of $\dot{2}07\dot{3}6$ is required.
Here 10 is the assumed root.
$10\times10\times6=600)20736(34\cdot56$
Subtract. $10\times10\div18 = \quad 5\cdot5555$

$\sqrt{29\cdot0044}=5\cdot38$
Add $\frac{2}{3}$ of 10 $=6\cdot66$

Approximated root $12\cdot04$, to be made the assumed root for the next operation.

Rule II.

Divide the resolvend by *four* times the cube of the assumed root: to the quotient add *three fourths* of the assumed root, and the sum will be the true root, or an approximation to it.

Let the biquadrate of $\dot{2}07\dot{3}6$ be required, as before?
The assumed root is 10
$10\times10\times10\times4=4000)20736(5\cdot184$
Add $\frac{3}{4}$ of $10=7\cdot5$

Approximated root $12\cdot684$, to be made the assumed root for the next operation.

EXTRACTION OF THE SURSOLID ROOT BY APPROXIMATION.

A Particular Rule.*

1. Divide the resolvend by *five* times the assumed root, and to the quotient add *one twentieth* part of the *fourth* power of the same root.

2. From the square root of this sum subtract *one fourth* pare of the square of the assumed root.

3. To the square root of the remainder add *one half* of the assumed root, and the sum is the root required, or an approximation to it.

Note. This rule will give the root true to *five* places, at the least, (and generally to eight or nine places) at the first process.

Required

* $r \pm e = \sqrt{\sqrt{\frac{G}{5r}+\frac{r^4}{20}}-\frac{rr}{4}}+\frac{r}{2}$

Required the sursolid root of 281950621875 ?

$$
\begin{array}{rl}
 & 200 = \text{assumed root.} \\
 & 5 \\
\hline
 & 1000)\ 281950621{\cdot}875 \text{ quotient} \\
\text{Add } 200\times200\times200\times200\div20 & = 80000000 \\
\hline
 & \sqrt{361950621{\cdot}875} = 19025 \text{ nearly.} \\
\text{Subtract} & 200\times200\div4 = 10000 \\
\hline
 & \sqrt{9025} = 95 \\
\text{Add half the assumed root} & = 100 \\
\hline
 & \text{Required root } 195
\end{array}
$$

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

*1. Prepare the given number, for extraction, by pointing off from the unit's place, as the required root directs.

2. Find the first figure of the root by trial, or by inspection into the table of powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the *dividend.*

4. Involve the root to the next inferiour power to that which is given, and multiply it by the number denoting the given power, for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the *given number*, as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and, in like manner, proceed till the whole be finished.

EXAMPLES.

* The extracting of roots of very high powers will, by this rule, be a tedious operation : The following method, when practicable, will be much more convenient.

When the index of the power, whose root is to be extracted, is a composite number, take any two or more *indices*, whose product is equal to the given *index*, and extract out of the given number a root answering to another of the indices, and so on to the last.

Thus, the fourth root=square root of the square root ;—the sixth root=square root of the cube root ;—the eighth root = square root of the fourth root ;—the ninth root = the cube root of the cube root ;—the tenth root = square root of the fifth root :—the twelfth root = the cube root of the fourth, &c.

EXAMPLES.

1st. What is the cube root of 20346417?

		20346417(273	$2\times2\times2=8$ root of the 1ſt. period, or 1ſt. Subtra.
2^3	=	8 = 1ſt. Subtrah.	$2\times2=4$(=next inferiour power,) and,
		–––	$4\times3=$(the index of the given pow.)=12 1ſt. Divs.
$2^2\times3=12$)123=Dividend.			$27\times27\times27=19683$=2d. Subtra.
		–––––	$27\times27=729$ (next inferiour power) and,
27^3	=	19683=2d. Subtra.	729×3 (=index of the given pow.)=2187=2d. Ds
$27^2\times3=2187$)6634=2d. Div.			$273\times273\times273=27346417$=3d. Subtra.
		––––––––	
273^3	=	20346417=3d. Subtra.	
		––––––––	
		....	

2d. What is the biquadrate root of 34827998976? Ans. 431·9+.

3d. Extract the surſolid, or fifth root of 281950621875? Ans 195.

4th. Extract the square cubed, or sixth root of 1178420166015625? Ans. 325.

A GENERAL* RULE FOR EXTRACTING ROOTS BY APPROXIMATION.

1. Subtract *one* from the exponent of the root required, and multiply half of the remainder by that exponent, and this product by that power of the assumed root, whose exponent is *two* less than that of the root required.

2. Divide

* The general theorem for the extraction of all roots, by approximation, from whence the rule was taken, and the Theorems deducible from it, as high as the twelfth power. Let G=reſolvend whoſe root is to be extracted. § $r+e$= root required; r being aſſumed as near the true root, and m=exponent of the power—then the equation will ſtand thus.

$$r \pm e\sqrt{\frac{G}{\frac{m-1}{2}\, m\, r|^{m-2}} - \frac{\overline{m-2}\, r^2}{m\, \overline{m-1}|^2}} + \frac{m-2}{m-1}\, r. \quad \text{Hence,}$$

Theorem for the cube root. $r \pm e = \sqrt{\frac{G}{3r} - \frac{rr}{12}} + \frac{r}{2}$

For the Biquadrate - - - $\sqrt{\frac{G}{6rr} - \frac{rr}{18}} + \frac{2r}{3}$

For the Surſolid - - - - $\sqrt{\frac{G}{10r^3} - \frac{3rr}{80}} + \frac{3r}{4}$

For the ſquared cube root $\sqrt{\frac{G}{15r^4} - \frac{2rr}{75}} + \frac{4r}{5}$

For

§ *By this Theorem the fraction is obtained in numbers to the loweſt terms in all the odd powers; and in the even powers only by having the numerator and denominator found by this equation.*

2. Divide the given number by the last product; and from the quotient subtract a fraction, whose numerator is obtained by subtracting *two* from the exponent, and multiplying the remainder by the square of the assumed root; and whose denominator is found by subtracting *one* from the exponent and multiplying the square of the remainder by the exponent.

3. After this subtraction is made, extract the square root of the remainder.

4. From the exponent subtract *two*, and place the remainder as a numerator; then subtract one from the exponent, and place the remainder under the numerator for a denominator.

5. Multiply this fraction by the assumed root; add the product to the square root, before found, and the sum will be the root required, or an approximation to it.

Example.

What is the square cubed root of 117842016015625?

Let the assumed root $= 300$

Exponent of the required root is 6. Then, $\frac{6-1}{2}\times 6=15$.

$300^4=8100000000$ and this multiplied by $15=121500000000$.

$117842016015625 \div 121500000000 = 9698{\cdot}9314$, from this

Subtract $\frac{\overline{6-2}\times 300^2}{6\times\overline{6-1}^2}$ $=2400$

And $\sqrt{7298{\cdot}9314}=85{\cdot}43$

To which add $\frac{6-2}{6-1}\times 300=$ 240

And the sum is the approximated root$=$ $325{\cdot}43$

For the 2d. operation, let $325{\cdot}43$ = assumed root.

ANOTHER

For the second sursolid -	$\sqrt{\frac{G}{21r^5}-\frac{5rr}{252}}+\frac{5r}{6}$
For the squared Biquadrate	$\sqrt{\frac{G}{28r^6}-\frac{3rr}{196}}+\frac{6r}{7}$
For the cubed cube - - -	$\sqrt{\frac{G}{36r^7}-\frac{7rr}{576}}+\frac{7r}{8}$
For the squared sursolid - -	$\sqrt{\frac{G}{45r^8}-\frac{4rr}{405}}+\frac{8r}{9}$
For the third sursolid - -	$\sqrt{\frac{G}{55r^9}-\frac{9rr}{1100}}+\frac{9r}{10}$
For the squared square cube	$\sqrt{\frac{G}{66r^{10}}-\frac{5rr}{726}}+\frac{10r}{11}$ &c.

ANOTHER METHOD BY APPROXIMATION.*

Rule.

1. Having assumed the root in the usual way, involve it to that power denoted by the exponent less 1.
2. Multiply this power by the exponent.
3. Divide the resolvend by this product, and reserve the quotient.
4. Divide the exponent of the given power, less 1, by the exponent, and multiply the assumed root by the quotient.
5. Add this product to the reserved quotient, and the sum will be the true root, or an approximation.
6. For every succeeding operation, let the root last found, be the assumed root.

Example.

What is the square cubed root of 117842016601562 5?

The exponent is 6. Let the assumed root be 300.

Then

* A rational formula for extracting the root of any pure power by approximation.

Let the resolvend be called G, and let $r \pm e$ be the required root, r being assumed in the usual way.

Let $G^{\frac{1}{m}}$ be required; then $r \pm e = \dfrac{G}{mr^{m-1}} + \dfrac{m-1}{m} r$ the general Theorem.

Hence, For the cube root $r \pm e = \dfrac{G}{3r^2} + \dfrac{2}{3} r.$

For the biquadrate - - - $\dfrac{G}{4r^3} + \dfrac{3}{4} r.$

For the sursolid - - - $\dfrac{G}{5r^4} + \dfrac{4}{5} r.$

For the square cubed - - $\dfrac{G}{6r^5} + \dfrac{5}{6} r.$

For the seventh root - - $\dfrac{G}{7r^6} + \dfrac{6}{7} r.$

For the eighth - - - - $\dfrac{G}{8r^7} + \dfrac{7}{8} r.$

For the ninth - - - - $\dfrac{G}{9r^8} + \dfrac{8}{9} r.$

For the tenth - - - - $\dfrac{G}{10r^9} + \dfrac{9}{10} r.$

For the eleventh - - - $\dfrac{G}{11r^{10}} + \dfrac{10}{11} r.$

For the twelfth - - - $\dfrac{G}{12r^{11}} + \dfrac{11}{12} r.$ &c.

Then, $300^5 \times 6 = 1458000000000$.

$1458000000000)117842016601 5625(80{\cdot}824$.

Add $\frac{5}{6} \times 300 = 250$

$330{\cdot}824$=approximated root.

For the next operation, let 330·824 be the assumed root.

OF PROPORTION IN GENERAL.

NUMBERS are compared together to discover the relations they have to each other.

There must be two numbers to form a comparison : the number, which is compared, being written first, is called the *antecedent* ; and that, to which it is compared, the *consequent*.

Numbers are compared with each other two different ways : The one comparison considers the *difference* of the two numbers, and is called arithmetical relation, the difference being sometimes named the arithmetical ratio ; and the other considers their *quotient*, which is termed geometrical relation, and the quotient, the geometrical ratio. Thus, of the numbers 12 and 4, the difference or arithmetical ratio, is $12-4=8$; and the geometrical ratio is $\frac{12}{4}=3$.*

If two, or more, couplets of numbers have equal ratios, or differences, the equality is termed proportion ; and their terms, similarly posited, that is, either all the greater, or all the less taken as antecedents, and the rest as consequents, are called proportionals. So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals ; and the two couplets, 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.†

Proportion

* Ratios are, here, always conſidered as the reſult of the greater term of compariſon diminiſhed, or divided, by the leſs ; not regarding which of them be the antecedent.

† To denote numbers as being geometrically proportional, the couplets are ſeparated by a double colon, and a colon is written between the terms of each couplet ; we may, alſo, denote arithmetical proportionals by ſeparating the couplets by a double colon, and writing a colon turned horizontally between the terms of each couplet. So the above arithmeticals may be written thus, 2 ·· 4 :: 6 ·· 8, and 4 ·· 2 :: 8 ·· 6 ; where the firſt antecedent is leſs or greater than its conſequent by juſt ſo much as the ſecond antecedent is leſs or greater than its conſequent : And the geometricals thus, 2 : 4 :: 8 : 16, and 4 : 2 :: 16 : 8 ; where the firſt antecedent is contained in, or contains its conſequent, juſt ſo often, as the ſecond is contained in, or contains its conſequent.

Four numbers are ſaid to be *reciprocally* or *inverſely* proportional, when the fourth is leſs than the ſecond, by as many times, as the third is greater than the firſt, or when the firſt is to the third, as the fourth to the ſecond, and vice verſa. Thus 2, 9, 6 and 3, are reciprocal proportionals.

Note.

Proportion is distinguished into continued and discontinued. If, of several couplets of proportionals, written down in a series, the difference or ratio of each consequent, and the antecedent of the next following couplet, be the same as the common difference or ratio of the couplets, the proportion is said to be continued, and the numbers themselves, a series of continued arithmetical or geometrical proportionals. So 2, 4, 6, 8, form an arithmetical progression; for $4-2=6-4=8-6=2$; and 2, 4, 8, 16, a geometrical progression; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$

But, if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinued. So 4, 2, 8, 6, are in discontinued arithmetical proportion;

Note. It is common to read the geometricals $2 : 4 :: 8 : 16$, thus, 2 is to 4 as 8 to 16, or, As 2 to 4 ſo is 8 to 16.

Harmonical Proportion is that, which is between thoſe numbers which aſſign the lengths of muſical intervals, or the lengths of ſtrings ſounding muſical notes; and of *three numbers* it is, when *the firſt is to the third, as the difference between the firſt and ſecond is to the difference between the ſecond and third*, as the numbers 3, 4, 6. Thus, if the lengths of ſtrings be as theſe numbers, they will ſound an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between 4 numbers, when *the firſt is to the fourth, as the difference between the firſt and ſecond is to the difference between the third and fourth*, as in the numbers 5, 6, 8, 10; for ſtrings of ſuch lengths will ſound an octave 5 to 10; a ſixth greater, 6 to 10; a third greater 8 to 10; a third leſs 5 to 6; a ſixth leſs 5 to 8; and a fourth 6 to 8.

A ſeries of numbers in harmonical proportion is, reciprocally, as another ſeries in arithmetical proportion.

$$\text{As}\begin{cases}\text{Harmonical} & 10 \cdot\cdot\ 12 \cdot\cdot\ 15 \cdot\cdot\ 20 \cdot\cdot\ 30 \cdot\cdot\ 60 \\ \text{Arithmetical} & 6 \cdot\cdot\ 5 \cdot\cdot\ 4 \cdot\cdot\ 3 \cdot\cdot\ 2 \cdot\cdot\ 1\end{cases}$$

for here $10 : 12 :: 5 : 6$; and $12 : 15 :: 4 : 5$, and ſo of all the reſt. Whence thoſe ſeries have an obvious relation to, and dependence on, each other.

1. Let a, b, c, be the three numbers in muſical proportion. Then, becauſe we have $a : c :: a-b : b-c$; therefore, $ab-ac=ac-bc$; whence, if any two of the three be given, the other may be found by the following Theorems.

$$\text{I.}\ \frac{ab}{2a-b}=a. \quad \text{II.}\ \frac{2ac}{2a-b}=b. \quad \text{III.}\ \frac{cb}{2c-b}=a.$$

For Example. Suppoſe you would find a muſical mean proportional between the *monochord* $50=a$, and the *octave* $25=c$; then, by Theor. II. $\frac{2ac}{a+c}=b=\frac{2500}{75}=33{\cdot}3$, which is the length of that chord, called a *fifth.*

2. If there be four numbers in muſical proportion, as a, b, c, d; then, ſince it is $a : d :: a-b : c-d$, we have $ac-ad=ad-db$. From which equation we have the following Theorems.

$$\text{I.}\ \frac{db}{2d-c}=a. \quad \text{II.}\ \frac{a}{d}\times\overline{2d-c}=b. \quad \text{III.}\ \frac{2ad-db}{a}=c. \quad \text{IV.}\ \frac{ac}{2a-b}=d.$$

Hence, when any three of thoſe numbers are given, the fourth may be found.

Thus, let 10, 8, 6 be given to find a fourth harmonical proportion.

$$\frac{a\times c}{2a-b}=\frac{10\times6}{20-8}=\frac{60}{12}=5, \text{ the octave.}$$

This harmonical theory may be carried much farther. See Martin's Newtonian Philoſophy, Vol. II. page 123.

tion ; for 4—2=8—6=2=common difference of the couplets, 8—2=6=difference of the consequent of one couplet and the antecedent of the next ; also, 4, 2, 16, 8, are in discontinued geometrical proportion ; for $\frac{4}{2}=\frac{16}{8}=2=$ common ratio of the couplets, and $\frac{16}{2}=8=$ ratio of the consequent of one couplet and the antecedent of the next.

ARITHMETICAL PROPORTION.

THEOREM 1.

IF any four quantities a, b, c, d, (2, 4, 6, 8) be in arithmetical proportion,* the sum of the two means is equal to the sum of the two extremes.†

And if any three quantities, a, b, c, (2,4, 6,) be in arithmetical proportion, the double of the mean is equal to the sum of the extremes.

THEOREM 2.

In any continued Arithmetical Proportion (1, 3, 5, 7, 9, 11) the sum of the two extremes, and that of every other two terms, equally distant from them, are equal. Thus, 1+11=3+9=5+7.‡

When the number of terms is odd, as in the proportion 3. 8. 13. 18. 23, then, the sum of the two extremes being double to the mean or middle term, the sum of any other two terms, equally remote from the extremes, must likewise be double to the mean.

THEOREM 3.

In any continued Arithmetical Proportion, (a, $a+b$, $a+2b$, $a+3b$, $a+4b$, &c. (4, 4+2, 4+4, 4+6, 4+8, &c.) the last or greatest term is equal to the first or least more the common difference of the terms drawn into the number of all the terms after the first, or into the whole number of the terms, less one.§

THEOREM

* Although, in the compariſon of quantities according to their differences, the term *proportion* is uſed : yet the word *progreſſion*, is frequently ſubſtituted in its room, and is indeed more proper ; the former form being, in the common acceptation of it, ſynonymous with ratio, which is only uſed in the other kind of compariſon.

† For ſince $b-a$ (4−2)$=d-c$(8−6) therefore $b+c$(4+6)$=a+d$(2+8.)

‡ Since, by the nature of progreſſionals, the ſecond term exceeds the firſt by juſt ſo much as its correſponding term, the laſt but one, wants of the laſt, it is evident that when theſe correſponding terms are added, the exceſs of the one will make good the defect of the other, and ſo their ſum be exactly the ſame with that of the two extremes, and in the ſame manner it will appear that the ſum of any two other correſponding terms muſt be equal to that of the two extremes.

§ For ſince each term, after the firſt exceeds that preceding it by the common difference, it is plain that the laſt muſt exceed the firſt by ſo many times the common difference as there are terms after the firſt ; and therefore muſt be equal to the firſt, and the common difference repeated that number of times.

THEOREM 4.

The sum of any rank, or series of quantities in continued Arithmetical Proportion (1. 3. 5. 7. 9 11) is equal to the sum of the two extremes multiplied into half the number of terms.*

ARITHMETICAL PROGRESSION.

ANY rank of numbers, more than two, increasing by a common excess, or decreasing by a common difference, is said to be in Arithmetical Progression.

If the succeeding terms of a progression exceed each other, it is called an ascending series or progression; if the contrary, a descending series.

So $\begin{cases} 0. \; 2. \; 4. \; 6. \; 8. \; 10, \text{ \&c. is an ascending arithmetical series.} \\ 1. \; 2. \; 4. \; 8. \; 16. \; 32, \text{ \&c. is an ascending geometrical series.} \end{cases}$

And $\begin{cases} 10. \; 8. \; 6. \; 4. \; 2. \; 0, \text{ \&c. is a descending arithmetical series.} \\ 32. \; 16. \; 8. \; 4. \; 2. \; 1, \text{ \&c. is a descending geometrical series.} \end{cases}$

The numbers which form the series, are called the *terms* of the progression.

Note.—The first and last terms of a progression are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be easily found.

1. The *first* term.
2. The *last* term
3. The *number* of terms.
4. The *common difference.*
5. The *sum* of all the terms.

PROBLEM

* For, becaufe (by the fecond Theorem) the fum of the two extremes, and that of every other two terms, equally remote from them, are equal, the whole feries, confifting of half fo many fuch equal fums as there are terms, will therefore be equal to the fum of the two extremes, repeated half as many times as there are terms.

The fame thing alfo holds, when the number of terms is odd, as in the feries 4, 8, 12, 16, 20; for then, the mean, or middle term, being equal to half the fum of any two terms, equally diftant from it on contrary fides, it is obvious that the value of the whole feries is the fame as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the fum of the two extremes) multiplied by the whole number of terms; or to the fum of the extremes multiplied by half the number of terms.

The fum of any number of terms (x) of the arithmetical feries of odd numbers 1, 3, 5, 7, 9, &c. is equal to the fquare (x^2) of that number.

For, 0+1 or the fum of 1 term $= 1^2$ or 1
1+3 or the fum of 2 terms $= 2^2$ or 4
4+5 or the fum of 3 terms $= 3^2$ or 9
9+7 or the fum of 4 terms $= 4^2$ or 16
16+9 or the fum of 5 terms $= 5^2$ or 25, &c.

Whence, it is plain, that, let x be any number whatever, the fum of x terms will be x^2.

EXAMPLE.

The firft term, the ratio, and number of terms given, to find the fum of the feries.

A gentleman travelled 29 days, the firft day he went but 1 mile, and increafed every day's travel 2 miles; How far did he travel? $29 \times 29 = 841$ miles, Ans

Problem I.

The first term, the last term, and the number of terms being given, to find the common difference.

Rule.*

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

Examples.

1st. The extremes are 3 and 39, and the number of terms is 19: What is the common difference?

$$\left.\begin{array}{r}39\\-\ 3\end{array}\right\}\text{Extremes.}$$

Divide by the number of terms less 1=19—1=18)36(2 Ans.

$$\begin{array}{r}36\\ \hline\end{array}$$

$$\text{Or, } \frac{39-3}{19-1}=2.$$

2d. A man had 10 sons, whose several ages differed alike; the youngest was 3 years old, and the eldest 48: What was the common difference of their ages?

$$\frac{48-3}{10-1}=5 \text{ Ans.}$$

3d. A man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 37 miles: Required the daily increase?

$$\frac{37-5}{9-1}=4 \text{ Ans.}$$

Problem II.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

Rule.†—Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

Examples.

* The difference of the firſt and laſt terms evidently ſhews the increaſe of the firſt term by all the ſubſequent additions, till it becomes equal to the laſt; and as the number of thoſe additions was one leſs than the number of terms, and the increaſe, by every addition, equal, it is plain that the total increaſe, divided by the number of additions, muſt give the difference of every one ſeparately; whence the rule is manifeſt.

† Suppoſe another ſeries of the ſame kind with the given one be placed under it in an inverſe order; then will the ſum of any two correſponding terms be the ſame as that of the firſt and laſt; conſequently, any one of thoſe ſums, multiplied by the number of terms, muſt give the whole ſum of the two ſeries.

Let 1, 2, 3, 4, 5, 6, 7, 8, be the given ſeries.
And 8, 7, 6, 5, 4, 3, 2, 1, the ſame inverted.
Then, 9+9+9+9+9+9+9+9=9×8=72, and

$$1+2+3+4+5+6+7+8=\frac{72}{2}=36.$$

EXAMPLES.

1st. The extremes of an arithmetical series are 3 and 39, and the number of terms 19: Required the sum of the series?

$$\left.\begin{array}{r} 39 \\ +\ 3 \end{array}\right\} \text{Extremes.}$$

$$\begin{array}{rr} \text{Sum} = & 42 \\ \text{Number of terms} = \times & 19 \\ \hline & 378 \\ & 42 \\ \hline & 2)798 \\ \hline & 399 \text{ Ans.} \end{array}$$

Or, $\dfrac{\overline{39+3}\times 19}{2} = 399.$

2d. It is required to find how many strokes the hammer of a clock would strike in a week, or 168 hours, provided it increased 1 at each hour?

$$\frac{\overline{168+1}\times 168}{2} = 14196 \text{ Ans.}$$

3d. Suppose a number of stones were laid a yard distant from each other for the space of a mile, and the first a yard from a basket: What length of ground will that man travel over, who gathers them up singly, returning with them one by one to the basket?

$$\frac{\overline{3520+2}\times 1760}{2} = 3099360 \text{ yards} = 1761 \text{ miles, Ans.}$$

N. B In this question, there being 1760 yards in a mile, and the man returning with each stone to the basket, his travel will be doubled; therefore the first term will be 2, and the last 1760×2, and the number of terms 1760.

4th. A man bought 25 yards of linen in Arithmetical Progression; for the 4th yard he gave 12 cents, and for the last yard 75 cents: What did the whole amount to, and what did it average per yard?

$\dfrac{75-12}{22-1} = 3$ the common difference by which the first term is found to [be 3.

Then $\dfrac{\overline{75+3}\times 25}{2} =$ 9D. 75c. and the average price is 39 cents per yard.

5th. Required the sum of the first 1000 numbers in their natural order?

$$\frac{\overline{1000+1}\times 1000}{2} = 500500 \text{ Ans.}$$

PROBLEM

PROBLEM III.

Given the extremes and the common difference, to find the number of terms.

RULE.*—Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the number of terms required.

EXAMPLES.

1st. The extremes are 3 and 39, and the common difference 2: What is the number of terms?

$$\begin{array}{r l} 39 & \rbrace \text{ Extremes.} \\ -\ 3 & \\ \hline \text{Common difference} = 2)36 & \\ \hline \text{Quotient} = 18 & \\ \text{Add } 1 & \\ \hline 19 & \text{Ans.} \end{array}$$

Or, $\dfrac{39-3}{2}+1=19.$

2d. A man going a journey, travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles: How many days did he travel, and how far?

$$\frac{51-7}{4}+1=12 \text{ days, and } \frac{\overline{51+7}\times 12}{2}=348 \text{ miles, Ans.}$$

PROBLEM IV.

The extremes and common difference given, to find the sum of all the series.

RULE.—Multiply the sum of the extremes by their difference increased by the common difference, and the product divided by twice the common difference will give the sum.

EXAMPLES.

1st. If the extremes are 3 and 39, and the common difference 2: What is the sum of the series?

39+3=42 sum of the extremes

39—3=36=difference of extremes.

36+2=38=difference of extremes increased by the common difference.

$$\begin{array}{r r} & 42 \\ & \times\ 38 \\ \hline & 336 \\ & 126\ \ \\ \hline \text{Twice the common difference}=4) & 1596 \\ \hline & 399 \end{array}$$

Or,

* By the first Problem, the difference of the extremes, divided by the number of terms less 1, gave the common difference; consequently, the same divided by the common difference, must give the number of terms less 1; hence, this quotient, augmented by 1, must be the answer to the question.

Or, $\frac{\overline{39+3}\times\overline{39-3+2}}{2\times2}=399.$

2d. A owes B a certain sum, to be discharged in a year, by paying 6d. the first week, 18d. the second, and thus to increase every weekly payment by a shilling, till the last payment be 2l. 11s. 6d.: What is the debt?

$$\frac{\overline{51\cdot5+\cdot5}\times\overline{51\cdot5-\cdot5+1}}{1\times2}=£.67\ 12s.\ \text{Ans.}$$

Problem V.

The extremes and the sum of the series given, to find the common difference.

Rule.—Divide the product of the sum and difference of the extremes, by the difference of twice the sum of the series, and the sum of the extremes, and the quotient will be the common difference.

Examples.

1st. Let the extremes be 3 and 39, and the sum 399: What is the common difference?

Sum of the extremes = 39 + 3 = 42
Diff. of the extremes = 39 — 3 = × 36

252
126

$\overline{399\times2}-42=756)1512(2$ Ans.
1512

Or, $\frac{\overline{39+3}\times\overline{39-3}}{399\times2-39+3}=2.$

2d. A owes B 67l. 12s. to be discharged in a year, by weekly payments; the first payment to be 6d. and the last, 2l. 11s. 6d.: What is the common difference of the payments, and what will each payment be?

$\frac{\overline{51\cdot5+\cdot5}\times\overline{51\cdot5-\cdot5}}{1352\times2-51\cdot5+\cdot5}$ = 1s. and 6d.+1s.=1s. 6d.=2d. payment, 1s. 6d.+1s.=2s. 6d.=3d. payment, &c.

Problem VI.

The extremes and sum of the series given, to find the number of terms.

Rule.—Twice the sum of the series, divided by the sum of the extremes, will give the number of terms.

Examples.

EXAMPLES.

1st. Let the extremes be 3 and 39, and the sum of the series 399: What is the number of terms?

$$\begin{array}{rl} \text{Sum of the series} = & 399 \\ & \times\ 2 \\ \hline \text{Sum of extremes} = 39+3=42) & 798(19 \text{ Ans.} \\ & 42 \\ \hline & 378 \\ & 378 \end{array}$$

Or, $\dfrac{399\times 2}{39+3} = 19.$

2d. A owes B 67l. 12s. to be paid weekly in Arithmetical Progression, the first payment to be 6d. and the last to be 51s. 6d.: How many payments will there be, and how long will he be in discharging the debt?

$$\frac{1352\times 2}{51\cdot 5+\cdot 5} = 52 \text{ payments, and as many weeks, Ans.}$$

PROBLEM VII.

The first term, the common difference, and sum of the series given, to find the number of terms.

RULE.—To the square of the difference of twice the first term and the common difference, add the rectangle (or product) of the sum and the common difference multiplied by 8, and extract the square root of the sum, from which root take twice the first term less the common difference; divide the remainder by twice the common difference, and the quotient will be the number of terms.

EXAMPLE.

If the first term be 3, the common difference 2, and the sum of the series 399: Required the number of terms?

$3\times 2=6=$ Twice the sum of the first term.

$6-2=4=$ Difference of twice the first term and the common diff.

$4\times 4=16=$ Square of the said difference.

$399\times 2\times 8=6384=$ Rectangle of the sum and com. diff. mult. by 8.

$6384+16=6400=$ Sum of the said eightfold rectangle and the square of the aforesaid difference.

$\sqrt{6400}=80=$ Square root of the last mentioned sum.

$80-4=76=$ Difference of the said root and twice the first term less the common difference.

$\dfrac{76}{4}=19$ The number of terms.

Or, $\dfrac{\sqrt{\overline{3\times 2-2}|^2+399\times 2\times 8}-3\times 2-2}{2\times 2}=19.$

PROBLEM

Problem VIII.

The first term, the common difference, and the sum of the series given, to find the last term.

Rule.—To the square of the difference of twice the first term and the common difference, add the rectangle of the sum and the common difference, and extract the square root of their sum, from which root take the common difference; and the remainder, divided by 2, will be the last term.

Example.

If the first term be 3, the common difference 2, and the sum of the series 399: What is the last term?

3×2=6. 6—2=4. 4×4=16. 399×2×8=6384. 6384+16=6400. $\sqrt{6400}=80$. 80—2=78 And 78÷2=39 the Answer.

$$\text{Or, } \frac{\sqrt{\overline{3\times2-2}|^{2}+399\times2\times8}-2}{2}=39.$$

Problem IX.

The first term, the common difference, and the number of terms given, to find the last term.

Rule.—The number of terms less 1, multiplied by the common difference, and the first term added to the product, will give the last term.

Examples.

1st. If the first term be 3, the common difference 2, and the number of terms 19: What is the last term?

Number of terms =	19
	— 1
Number of terms less 1 =	18
Common difference =	× 2
	36
First term =	+ 3
	39 the Ans.

Or, $\overline{19-1}\times2+3=39$.

2d. A owes B a certain sum to be paid in Arithmetical Progression; the first payment is 6d. the number of payments 52, and the common difference of the payments is 12d.: What is the last payment? $\overline{52-1}\times12+6=618$d.=2l. 11s. 6d. Ans.

Problem X.

The first term, common difference, and number of terms given, to find the sum of the series.

Rule.—To the first term add the product of the number of terms less 1 by half the common difference, and their sum, multiplied by the number of terms, will give the sum of the progression.

Examples.

Examples.

1st. If the first term be 3, the common difference 2, and number of terms 19: What is the sum of the series?

First term = 3

Add the product of the number of terms less 1 by ½ common difference } = 19 — 1×1 = 18

Their sum 21
Multiply by the number of terms = 19

189
21

Or, $19\times3+\overline{19-1\times1}=399$ Ans. = 399

2d. Sixteen persons gave charity to a poor man; the first gave 7c. and the second 12c. and so on in arithmetical progression; I demand what sum the last person gave, and how much the poor man received in all?

Answer $\overline{16-1\times5}+7=82$c. the last gave.

And $16\times7+\overline{16-1\times\frac{5}{2}}=712$c.=D. 7 12c. the whole sum.

Problem XI.

Given the first term, the number of terms, and the sum of the series, to find the common difference.

Rule.—From the sum subtract the rectangle of the first term and number of terms; twice the remainder, divided by the product of the number of terms and number of terms less 1, will give the common difference.

Example.

If the first term be 3, the number of terms 19, and the sum 399: What is the common difference?

Sum of the series = 399

Subtract the product of the first term and number of terms } = 3 × 19 = 57

Remainder = 342
Multiplied by 2

Divide by the product of the number of terms and number of terms less 1 } =19×18=342)684(2 Ans.

684

...

Or, $\dfrac{2\times399-\overline{3\times19}}{\overline{19-1\times19}}=2$

Problem XII.

Given the first term, number of terms, and the sum of the series, to find the last term.

Rule.—Divide twice the sum by the number of terms; from the quotient take the first term, and the remainder will be the last.

Examples.

EXAMPLES.

1st. If the first term be 3, the number of terms 19, and the sum 399; What is the last term?

Sum of the terms	= 399
Multiply by	2
Divide by the number of terms	= 19)798
Quotient	= 42
Subtract the first term	= 3
Answer	= 39

Or, $\frac{399\times2}{19} - 3 = 39$.

2d. A merchant being indebted to 12 creditors D.2460, ordered his clerk to pay the first D.40, and the rest increasing in arithmetical progression: I demand the difference of the payments, and the last payment?

Ans. $\frac{2\times2460-40\times12}{12-1\times12}$ = 30D. = diff. and $\frac{2460\times2}{12}$ − 40 = 370D. last paymt.

PROBLEM XIII.

The common difference, the last term, and sum of the progression given, to find the first term.

RULE.—From the square of twice the last term plus the common difference, take 8 times the rectangle of the sum and common difference, and extract the square root of the remainder, which (root) either add to, or subtract from the common difference, (as the case may require) and half the sum or difference will the first term.

EXAMPLES.

1st. If the common difference be 2, the last term 39, and the sum of the terms 399: Required the first term?

Last term	39
Multiplied by	2
Product	= 78
Add the common difference	= 2
	80
Multiplied by	80
From the square of twice the last term plus the com. diff.	= 6400
Take 8 times the rectangle of the sum and common difference	= 399×2×8 = 6384
Remainder	= 16
Square root of 16	= 4
Sum of the common diff. and the square rooot of 16=2+4	= 6
And half the sum = $\frac{6}{2}$	= 3 Ans.

Or,

$$\text{Or, } \frac{2+\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2}=3.$$

2. A merchant being indebted to several persons D.1080, he ordered his clerk to pay the greatest creditor D.142, the greatest but one D.132, and so on, to decrease in Arithmetical Progression; what did the least creditor receive?

$$\text{Ans.}\frac{10-\sqrt{\overline{142\times2+10}|^2-1080\times10\times8}}{2}=\text{D.}2.$$

Problem XIV.

Given the common difference, the last term, and sum of the series, to find the number of terms.

Rule.

From the square of twice the last term plus the common difference take 8 times the rectangle of the sum and common difference, and extract the square root of the remainder, which (root) either subtract from, or add to, twice the last term plus the common difference (as the case may require) and the remainder or sum, divided by twice the common difference, will give the number of terms.

Examples.

1. If the common difference be 2, the last term 39, and the sum of the terms 399; I demand the number of terms. Last term 39

Multiply by 2

78

Add the common difference = 2

80

80

Square of twice the last term plus the common diff. = 6400

Sub. 8 times the rect. of the sum and com. diff.=399×2×8=6384

16

Square root of 16 = 4

Sum of twice the last term plus the com. diff=39×2+2=80

Sum of twice the last term and com. diff. minus the square root of 16=80—4 } = 76

Which remainder, divided by twice the com. diff. $=\frac{76}{4}=$ 19Ans.

$$\text{Or, } \frac{39\times2+2-\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2\times2}=19$$

2. A

2. A merchant being indebted to several persons D.1080, he ordered his clerk to pay the greatest creditor D.142; the greatest but one D.132, and so on, to decrease in Arithmetical Progression. How many creditors had he?

$$\text{Ans. } \frac{142\times2+10+\sqrt{\overline{142\times2+10}|^2-1080\times10\times8}}{10\times2} = 15 \text{ Creditors.}$$

Problem XV.

Given the last term, the number of terms, and the sum of the terms, to find the first term.

Rule.

Divide twice the sum by the number of terms; from the quotient subtract the last term, and the remainder will be the first.

Examples.

1. If the last term be 39, the number of terms 19, and the sum of the series 399; what is the first term?

Sum of the series = 399
Multiply by 2

Divide by the number of terms = 19)798

Quotient = 42
From the quotient take the last term = 39

Remainder = 3 Ans.

$$\text{Or, } \frac{399\times2}{19} - 39 = 3.$$

2. A man had 10 sons, whose several ages differed alike; the eldest was 48 years old, and the sum of all their ages was 255: What was the age of the youngest?

$$\frac{255\times2}{10} - 48 = 3 \text{ years, Ans.}$$

Problem XVI.

Given the last term, the number of terms, and the sum of the series, to find the common difference.

Rule.

Double the rectangle of the number of terms and the last term minus the sum of the series; divide the product by the rectangle of the number of terms and the number of terms minus 1, and the quotient will be the common difference.

Examples.

1. If the last term be 39, the number of terms 19, and the sum of the series 399; what is the common difference?

Number

Number of terms = 19
Multiply by the last term = 39

171
57

Rectangle of the number of terms, and the last term $\Big\} = 19 \times 39 = 741$
Subtract the sum of the series = 399

Remainder = 342
Multiply by 2

Divide by the rectangle of the number of terms, and number of terms minus 1 $\Big\} = 19 \times 18 = 342)684(2$ Ans.
684

$$\text{Or, } \frac{2 \times \overline{19 \times 39 - 399}}{\overline{19-1} \times 19} = 2.$$

2. Sixteen persons gave charity to a poor man in such proportion as to form an arithmetical series : the last gave 65c. and the whole sum amounted to D.5 60c. : what did each give, less than the other, from the last down to the first ?

$$\frac{2 \times 16 \times 65 - 560}{\overline{16-1} \times 16} = 4\text{c. Ans.}$$

PROBLEM XVII.

The common difference, number of terms, and the last term given, to find the first term.

RULE.—From the last term subtract the product of the terms less 1 by the common difference, and the remainder will be the first term.

EXAMPLES.

1. If the common difference be 2, the number of terms 19, and the last term 39 ; what is the first? Last term = 39

Subtract the number of terms less 1 multiplied by the common difference $\Big\} = \overline{19-1} \times 2 = 36$

Remains 3 Ans.

Or, $39 - \overline{19-1} \times 2 = 3$.

2. A man travelled 6 days, each day going 4 miles farther than on the preceding day, till the last day's journey was 40 miles ; how far did he ride the first day ?

$40 - \overline{6-1} \times 4 = 20$ miles, Ans.

PROBLEM XVIII.

The common difference, the number of terms, and last term given, to find the sum of the series.

RULE.—From the last term take the number of terms minus 1, multiplied by half the common difference, and the remainder, multiplied by the number of terms, will give the sum.

EXAMPLES.

1. If the common difference be 2, number of terms 19, and the last term 39 ; what is the sum of the series ? Last term=39

Subtract the number of terms less 1 multiplied by ½ the common difference $\Big\} = \overline{19-1} \times 1 = 18$

Remainder = 21

Carried over.

Brought over, Multiply by the number of terms = 19

$$\begin{array}{r} 189 \\ 21 \\ \hline \text{Answer, } 399 \end{array}$$

Or, $19\times39-\overline{19-1\times1}=399$

2. A man performed a journey in 6 days, and, each day, travelled 4 miles farther than on the preceding day, till his last day's travel was 40 miles ; how far did he travel in the whole ?

Ans. $6\times40-\overline{6-1\times\frac{4}{2}}=180$ miles.

PROBLEM XIX.

The sum of the terms, the number of terms, and the common difference given, to find the first term.

Rule.—Divide the sum by the number of terms ; from the quotient take half the product of the number of terms, minus unity, by the common difference, and the remainder will be the first term.

EXAMPLES.

1. If the sum of the series be 399, the number of terms 19, and the common difference 2 ; what is the first term ?

Number of terms = 19)399=sum.

Quotient = 21

Subtract ½ the product of the number of terms, less 1, by the common difference $= \dfrac{\overline{19-1}\times 2}{2} = 18$

Ans. 3

$$\text{Or, } \frac{399}{19} - \frac{2\times\overline{19-1}}{2} = 3.$$

2. A man travelled 180 miles in 6 days ; he increased his journey, each day, by 4 miles : how far did he travel the first day ?

$$\frac{180}{6} - \frac{4\times\overline{6-1}}{2} = 20 \text{ miles, Ans.}$$

PROBLEM XX.

The sum of the terms, number of terms, and the common difference given, to find the last term.

Rule.—Divide the sum of the series by the number of terms ; to the quotient add half the product of the number of terms minus unity by the common difference, and the sum will be the last term.

EXAMPLES.

1. If the sum of the series be 399, the number of terms 19, and the the common difference 2 ; what is the last term ?

Divide by the number of terms = 19)399 sum.

Quotient = 21

Add ½ the product of the number of terms, less 1, by the common difference $= \dfrac{\overline{19-1}\times2}{2} = 18$

Ans. = 39

$$\text{Or, } \frac{399}{19} + \frac{2\times\overline{19-1}}{2} = 39.$$

2. A.

2. A person bought a farm for £.510 to be paid monthly in arithmetical progression, and to be completed in a year, each payment to exceed that preceding by £.5: What were the first and last payments?

Ans. $\frac{510}{12} - \frac{5\times\overline{12-1}}{2} = 15$l. the first payment, and

$\frac{510}{12} + \frac{5\times\overline{12-1}}{2} = 70$l. the last payment.

The following Table contains a summary of the whole doctrine of Arithmetical Progression.

CASES OF ARITHMETICAL PROGRESSION.			
Case	Given	Required	Solution.
1.	*aln*	*d*	$\frac{l-a}{n-1}$
		s	$\frac{\overline{a+l}\times n}{2}$
2.	*ald*	*n*	$\frac{l-a}{d}+1$
		s	$\frac{\overline{l+a}\times\overline{l-a+d}}{2d}$
3.	*als*	*d*	$\frac{\overline{l+a}\times\overline{l-a}}{2s-l+a}$
		n	$\frac{2s}{a-l}$
4.	*ads*	*n*	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-\overline{2a-d}}{2}$
		l	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-d}{2}$

Case	Given	Required	Solution.
5.	*adn*	l	$\overline{n-1}\times d+a$
		s	$n\times\overline{a+\overline{n-1}\times\frac{d}{2}}$
6.	*ans*	d	$\frac{2\times\overline{s-an}}{\overline{n-1}\times n}$
		l	$\frac{2s}{n}-a$
7.	*lds*	a	$\frac{d\pm\sqrt{\overline{2l+d}\vert^2-8ds}}{2}$
		n	$\frac{2l+d\pm\sqrt{\overline{2l+d}\vert^2-8ds}}{2d}$
8.	*lns*	a	$\frac{2s}{n}-l$
		d	$\frac{2\times\overline{nl-s}}{\overline{n-1}\times n}$
9.	*lnd*	a	$l-\overline{n-1}\times d$
		s	$n\times\overline{l-\frac{\overline{n-1}\times d}{2}}$
10.	*dns*	a	$\frac{s}{n}-\frac{d\times\overline{n-1}}{2}$
		l	$\frac{s}{n}+\frac{d\times\overline{n-1}}{2}$

Here $\begin{cases} a = \text{first term.} \\ l = \text{last term.} \\ n = \text{number of terms.} \\ d = \text{common difference.} \\ s = \text{sum of all the terms.} \end{cases}$

GEOMETRICAL

GEOMETRICAL PROPORTION.

Theorem 1.

IF four quantities, *a. b. c. d.* (2. 6. 4. 12.) be in Geometrical Proportion, the product of the two means, *b c* (6×4) will be equal to that of the two extremes, *a d* (2×12) whether they are continued, or discontinued,* and, if three quantities, *a. b. c.* (2. 4. 8.) the square of the mean is equal to the product of the two extremes.

Theorem 2.

If four quantities, *a. b. c. d.* (2. 6. 4. 12.) are such, that the product of two of them, *ad*, (2×12) is equal to the product of the other two, *bc*, (6×4) then are those quantities proportional.†

Theorem 3.

If four quantities *a. b. c. d.* (2. 6. 4. 12.) are proportional, the rectangle of the means, divided by either extreme, will give the other extreme.‡

Theorem 4.

The products of the corresponding terms of two Geometrical Proportions are also proportional.

That is, if $a:b::c:d$ (2 : 6 : : 4 : 12,) and $e:f::g:h$ (2 : 4 : : 5 : 10,) then will $ae:bf::cg:dh$ ($2\times2:6\times4::4\times5:12\times10$.)§

Theorem 5.

* For since the ratio of *a* to *b* (2 to 6) or the part, which *a* is of *b* (2 is of 6) is expressed by $\frac{a}{b}\left(\frac{2}{6}\right)$ and the ratio of *c* to *d* (4 to 12,) in like manner, by $\frac{c}{d}\left(\frac{4}{12}\right)$; and since, by supposition, the two ratios are equal, let them both be multiplied by *bd*, (6×12) and the products $\frac{a}{b}\times bd\left(\frac{2}{6}\times6\times12\right)$ and $\frac{c}{d}\times bd\left(\frac{4}{12}\times6\times12\right)$ will likewise be equal; that is, $\frac{abd}{b}=\frac{cbd}{d}$ or $ad=cb\left(\frac{2\times6\times12}{6}=\frac{4\times6\times12}{12}\right.$, or, $2\times12=6\times4.)$

† For since, by supposition, the products *ad* (2×12) and *bc* (6×4) are equal, let both be divided by *bd* (6×12) and the quotients $\frac{ad}{bd}\left(\frac{a}{b}\right)$ and $\frac{bc}{bd}\left(\frac{c}{d}\right)\ \frac{2\times12}{6\times12}\left(\frac{2}{6}\right)\ \frac{6\times4}{6\times12}\left(\frac{4}{12}\right)$ will also be equal; and therefore $a:b::c:d$.

‡ For by the second Theorem, $ad=bc$ ($2\times12=6\times4$) whence dividing both sides of the equation by *a* (2) we have $d=\frac{bc}{a}\left(12=\frac{6\times4}{2}\right)$ Hence, if the two means and one extreme be given, the other extreme may be found.

§ For $\frac{a}{b}=\frac{c}{d}\left(\frac{2}{6}=\frac{4}{12},\right)$ and $\frac{e}{f}=\frac{g}{h}\left(\frac{2}{4}=\frac{5}{10}\right)$ by supposition; whence, $\frac{a}{b}\times\frac{e}{f}=\frac{c}{d}\times\frac{g}{h}\left(\frac{2}{6}\times\frac{2}{4}=\frac{4}{12}\times\frac{5}{10}\right)$ by equal multiplication; and consequently $\frac{ae}{bf}=\frac{cg}{dh}\left(\frac{2\times2}{6\times4}=\frac{4\times5}{12\times10}\right)$ that is, $ae:bf::cg:dh$ ($2\times2:6\times4::4\times5:12\times10$:) Hence it follows, that if any quantities be proportional, their squares, cubes, &c. will likewise be proportional.

Theorem 5.

If four quantities, *a. b. c. d.* (2, 6. 4. 12.) are directly proportional,

Then,

1. Directly,	$a : b :: c : d$	$(2 : 6 :: 4 : 12)$
2. Inversely,	$b : a :: d : c$	$(6 : 2 :: 12 : 4)$
3. Alternately,	$a : c :: b : d$	$(2 : 4 :: 6 : 12)$
4. Compoundedly,	$a : a{+}b :: c : c{+}d$	$(2 : 8 :: 4 : 16)$
5. Dividedly,	$a : b{-}a :: c : d{-}c$	$(2 : 4 :: 4 : 8)$
6. Mixtly,	$b{+}a : b{-}a :: d{+}c : d{-}c$	$(8 : 4 :: 16 : 8)$
7. By Multiplication,	$ra : rb :: c : d$	$(2r : 6r :: 4 : 12)$
8. By Division,	$\frac{a}{r} : \frac{b}{r} :: c : d$	$\left(\frac{2}{r} : \frac{6}{r} :: 4 : 12\right)$

Because the product of the means, in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 1.

Theorem 6.

If three numbers, *a. b. c.* (2. 4. 8.) be in continued proportion, the square of the first will be to *that* of the second, as the first number to the third; that is, $a^2 : b^2 :: a : c$ $(2\times2 : 4\times4 :: 2 : 8.)$*

Theorem 7.

In any continued Geometrical Proportion (1. 3. 9. 27. 81. &c.) the product of the two extremes, and *that* of every other two terms equally distant from them are equal.†

Theorem 8.

The sum of any number of quantities, in continued Geometrical Proportion, is equal to the difference of the rectangle of the second and last terms, and the square of the first, divided by the difference of the first and second terms.‡ GEOMETRICAL.

* For since $a : b :: b : c$ $(2 : 4 :: 4 : 8)$ thence will $ac=bb$ $(2\times8=4\times4)$ by Theorem 1; and therefore $aac=abb$ $(2\times2\times8=2\times4\times4)$ by equal multiplication; consequently, $a^2 : b^2 :: a : c$ $(2\times2 : 4\times4 :: 2 : 8)$ by Theorem 2.

In like manner it may be proved that, of four quantities continually proportional, the *cube* of the first is to *that* of the second, as the first quantity to the fourth.

† For, the ratio of the first term to the second being the same as *that* of the last but one to the last, these four terms are in proportion; and therefore by Theorem 1, the rectangle of the extremes is equal to *that* of their two adjacent terms; and after the same manner, it will appear that the rectangle of the third and last but two is equal to *that* of their two adjacent terms, the second and last but one, and so of the rest; whence the truth of the proposition is manifest.

‡ For, let the first term of the proportion be denoted by a, the common ratio by r, the number of terms by n, and the sum of the whole series by s, then it is plain that the second term will be expressed by $a\times r$, or, ar; the third by $ar\times r$, or ar^2; the fourth by $ar^2\times r$, or, ar^3; and the nth, or last term, by ar^{n-1}; and therefore the proportion will stand thus, $a+ar+ar^2+ar^3 \cdots +ar^{n-2}+ar^{n-1}=s$; which equation multiplied by r, gives $ar+ar^2+ar^3+ar^4 \ldots\ldots +ar^{n-1}+ar^n=rs$; from the first equation being subtracted, there will remain $-a+ar^n=rs-s$: Whence, $s=\left(\frac{ar^n-a}{r-1}=\frac{r\times ar^{n-1}-a}{r-1}\right)=\frac{ar\times ar^{n-1}-aa}{ar-a}$; (Or, take any series of numbers whatever,

GEOMETRICAL PROGRESSION.

A GEOMETRICAL Progression is, when a *rank* or *series* of numbers increases, or decreases, by the continual multiplication, or division, of some equal number.

Problem I.

Given one of the extremes, the ratio, and the number of the terms of a geometrical series, to find the other extreme.

Rule.—Multiply, or divide, (as the case may require) the given extreme by such power of the ratio, whose exponent* is equal to the *number* of terms lefs 1, and the product or quotient, will be the other extreme.

Examples.

ever, as 2. 6. 18. 54. 162. 486. and their fum will be $2+6+18+54+162+486=728$: This equation multiplied by the ratio, will ftand thus, $6+18+54+162+486+1458=2184$: now it is plain that the fum of the fecond feries will be fo many times the firft, as is expreffed by the ratio; fubtract the firft feries from the fecond, and it will give $1458-2=2184-728$, which is evidently fo many times the fum of the firft feries, as is expreffed by the ratio lefs 1; whence $\frac{1458-2}{3-1}=\frac{2916-4}{6-2}$, as was to be demonftrated.)

* As the *laft term* or any term near the laft, is very tedious to be found, by continual multiplication, it will often be neceffary in order to afcertain them, to have a feries of numbers in Arithmetical Proportion, called *indices*, or *exponents*, beginning with a cypher, or an unit whofe common difference is *one.*

When the *firft* term of the feries and the *ratio* are *equal*, the *indices* muft begin with an unit, and, in this cafe, the product of any two terms is equal to that term, fignified by the *fum* of their *indices.*

Thus, $\begin{cases} 1.\ 2.\ 3.\ 4.\ 5.\ 6, \text{ \&c. indices, or arithmetical feries.} \\ 2.\ 4.\ 8.\ 16.\ 32.\ 64, \text{ \&c. geometrical feries (leading terms.)} \end{cases}$

Now, $6+6=12$=the index of the twelfth term, and

$64\times64=4096$=the twelfth term.

But, when the *firft* term of the feries and the *ratio* are *different*, the *indices* muft begin with a cypher, and the fum of the *indices*, made choice of, muft be *one lefs* than the *number of terms*, given in the queftion; becaufe 1 in the *indices* ftands over the *fecond term*, and 2 in the *indices*, over the *third* term, &c. And, in this cafe, the *product* of any *two* terms divided by the *firft*, is equal to that term *beyond* the firft, fignified by the *sum* of their *indices.*

Thus, $\begin{cases} 0.\ 1.\ 2.\ 3.\ 4.\ 5.\ 6, \text{ \&c. indices.} \\ 1.\ 3.\ 9.\ 27.\ 81.\ 243.\ 729, \text{ \&c. geometrical feries.} \end{cases}$

Here, $6+5=$ 11 the index of the 12th term.

$729\times243=177147$ the 12th term, becaufe the firft term of the feries and the ratio are different, by which mean a cypher ftands over the firft term.

Thus, by the help of thefe indices, and a few of the firft terms in any geometrical feries, any term, whofe diftance from the firft term is affigned, though it were ever fo remote, may be obtained without producing all the terms.

Note. If the *ratio* of any geometrical feries be *double*, the *difference* of the *greateft* and *leaft* terms is equal to the fum of all the terms, except the greateft; if the *ratio* be *triple*, the *difference* is *double* the fum of all but the greateft; if the *ratio* be *quadruple*, the difference is *triple* the fum of all but the greateft, &c.

In any feries in ∺ decreafing to infinity—if the *fquare* of the *firft* term be divided by the *difference* between the *firft* and *fecond*, the quotient will be the fum of the feries.

EXAMPLES.

1. If the first term be 4, the ratio 4, and the number of terms 9: What is the last term?

1. 2. 3. 4+ 4= 8
4. 16. 64. 256.×256=65536=power of the ratio, whose exponent is less by 1, than the number of terms

65536=8th power of the ratio.
Multiply by 4=first term.

262144=last term.

Or, 4×4^8=262144=the Answer.

2. If the last term be 262144, the ratio 4, and the number of terms 9, what is the first term?

Last term.
8th power of the ratio 4^8=65536)262144(4=the first term.

Or, $\frac{262144}{4^8}=4$ the first term.

Again, given the first term, and the ratio, to find any other term assigned.

RULE I.

When the indices begin with an unit.

1. Write down a few of the leading terms of the series, and place their indices over them.

2. Add together such indices, whose sum shall make up the entire index to the term required.

3. Multiply the terms of the geometrical series, belonging to those indices, together, and the product will be the term sought.

EXAMPLES.

1. If the first term be 2, and the ratio 2, what is the 13th term?

1. 2. 3. 4. 5+5×3= 13
2. 4. 8. 16. 32×32×8=8192 Ans. Or, 2×2^{12}=8192.

2. A merchant wanting to purchase a cargo of horses for the West-Indies, a jockey told him he would take all the trouble and expence upon himself, of collecting and purchasing 30 horses for the voyage, if he would give him what the last horse would come to by doubling the whole number by a half penny, that is, two farthings for the first, four for the second, eight for the third, &c. to which, the merchant, thinking he had made a very good bargain, readily agreed: Pray what did the last horse come to, and what did the horses, one with another, cost the merchant?

1. 2. 3. 4. 5. 6+ 6=12th. 12+ 12+ 6=last term.
2. 4. 8. 16. 32. 64×64=4096, and 4096×4096×64=
1073741824 qrs.=£.1118481 1s. 4d. and their average price was £. 37282 14s. $0\frac{1}{2}$d. a piece.

RULE II.

When the indices begin with a cypher.

1. Write down a few of the leading terms of the series, as before, and place their indices over them.

2. Add

2. Add together the most convenient indices to make an index, less by 1, than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power, whose index is one less than the *number* of terms multiplied, and make the result a divisor, by which divide the dividend, and the quotient will be that term *beyond the first*, signified by the sum of those indices, or the term sought.

3. If the first term be 5, and the ratio 3; what is the 7th term?
0. 1. 2. 3.+ 2+ 1= 6=index to 6th. term beyond the 1st. or 7th
5. 15. 45. 135.×45×15=91125=dividend.

The number of terms, multiplied, is 3 (viz. 135×45×15,) and 3—1=2 is the power to which the term 5 is to be raised; but the 2d. power of 5 is 5×5=25, and therefore 91125÷25=3645 the 7th. term required.

PROBLEM II.

Given the first term, the ratio, and number of terms, to find the sum of the series.

RULE.

Raise the ratio to a power, whose index shall be equal to the number of terms, from which subtract 1; divide the remainder by the ratio, less 1, and the quotient, multiplied by the first term, will give the sum of the series.

EXAMPLES.

1. If the first term be 5, the ratio 3, and the number of terms 7; what is the sum of the series?

Ratio=3×3×3×3×3×3×3=2187=7th. power of the ratio.
Subtract 1

Divide by the ratio less 1=3—1=2)2186

Quotient=1093
Multiply by the first term = 5

Sum of the series=5465

Or, $\frac{3^7-1}{3-1}\times 5=5465$ Ans.

2. A shopkeeper sold 13 yards of cloth, on the following terms, viz. 2d. for the first yard, 4d. for the second, 8d. for the third, &c. I demand the price of the cloth?

$\frac{2^{13}-1}{2-1}\times 2=16382$d.=£.68 5s. 2d. Ans.

3. A gentleman, whose daughter was married on a new year's day, gave her a guinea, promising to triple it on the first day of each month in the year; pray what did her portion amount to?

$\frac{3^{12}-1}{3-1}\times 1=265720$ guineas, Ans.

4. What

4. What debt can be discharged in a year, by paying 1 cent the first month, 10c. the second, and so on, each month in a tenfold proportion?

$10^{12} - 1 \div 10 - 1 \times 1 =$ 111111111111c.=D.1111111111 11c. Ans.

5. A man threshed wheat 9 days for a farmer, and agreed to receive but 8 wheat corns for the first day's work, 64 for the second, and so on, in an eightfold proportion; I demand what his 9 days' labour amounted to, rating the wheat at 5s. per bushel?*

$$\frac{8^9-1}{8-1} \times 8 = 153391688$$ corns. Amount = £.78 0s. $5\frac{1}{2}$d. Ans.

6. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz. that he should give him a cent for the first door, 2 cents for the second, 4 cents for the third, and so on doubling at every door, which were 36 in all: It is a bargain, cried the simpleton, and here is a guinea to bind it: Pray what did the house cost him?

$$\frac{2^{36}-1}{2-1} \times 1 = 68719476735$$ c. = D.687194767 35c. Ans.

7. A young fellow, well skilled in numbers, agreed with a rich farmer to serve him 10 years, without any other reward, but the produce of one wheat corn for the first year, and that produce to be sowed from year to year, till the end of the time, allowing the increase but in a tenfold proportion; what is the sum of the whole produce, and what will it amount to at D.1 25c. per bushel?

$$\frac{10^{10}-1}{10-1} \times 10 = 11111111110$$ corns. Amount=D.22605 61c. 3m.+

8. Suppose one farthing had been put out, at 6 per cent. per annum, Compound Interest,† at the commencement of the Christian era; what would it have amounted to in 1784 years; and suppose the amount to be in standard gold, allowing a cubick inch to be worth 53l. 2s. 8d. how large would the mass have been?

Ans. $\frac{2^{150}-1}{2-1} \times 1 =$ £1486716346568748209435714551509890767065361 11 $3\frac{3}{4}$

=279808597221212304159795712329335942107 66 cubick inches of gold.

As 355 : 113 :: $\overline{360 \times 69{\cdot}5}$: 7964 earth's diameter. $360 \times 69{\cdot}5 \times 7964 \times 1327{\cdot}33$
=264482820122 cubick miles in the globe,
=672733373088547413688320 00 cubic inches in the globe. Then,
279808597221212304159795712329335942107 66
÷672733373088547413688320 00=415930899840288·8, which, however incredible it may appear to some, is more than four hundred and fifteen millions of millions, nine hundred and thirty thousand, eight hundred and ninety-nine millions, eight

* *Note*, 7680 wheat or barley corns are supposed to make a pint.

† Any sum at 6 per cent. per annum, compound interest, will double in eleven years and three hundred and twenty five days, or 11·889 years, or 11·89 is near enough, then, if you divide 1784 by 11·89, it will give the number of terms in this case equal to 150; the ratio will be 2, and the first term 1.

eight hundred and forty thousand, two hundred and eighty-eight times larger than the globe we inhabit.*

For the solution of the four following questions, see last part of note under Problem I.

9. A frigate pursues a ship at 8 leagues distance, and sails twice as fast as the ship ; how far must the frigate sail, before she comes up with her?

First, 8. 4. 2. 1. $\frac{1}{2}$. $\frac{1}{4}$. &c. $8\times8=64$, and $64\div\overline{8-4}=16$ leagues, Ans.

10. Suppose a ball to be put in motion by a force which impels it 10 rods the first minute, 8 the second, and so on, decreasing by a ratio of 1·25 each minute to infinity ; what space would it move through? $10\times10\div\overline{10-8}=50$ rods, Ans.

11. Required the value of ·999, to infinity, or $\cdot\dot{9}$†?

The first 9 or ·9,$=\frac{9}{10}$, the second, or ·09$=\frac{9}{100}$; therefore, $\overline{\cdot9\times\cdot9}\div\overline{\cdot9-\cdot09}=1$, Ans.

12. Required the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. to infinity? Ans. 1.

Problem III.

The first term, the last term (or the extremes) and the ratio given, to find the sum of the series.

Rule.

Divide the difference of the extremes by the ratio less by 1 ; add the greater extreme to the quotient, and the result will be the sum of all the terms.

Or, Multiply the greatest term by the ratio, from the product subtract the least term ; then divide the remainder by the ratio, less by 1, and the quotient will be the sum of all the terms.

Or, When all the terms are given, then, from the product of the *second* and *last* terms, subtract the *square* of the *first* term ; this remainder being divided by the *second* term *less* the *first*, will give the sum of the series.

Examples.

1. If the series be 2. 6. 18. 54. 162. 486. 1458. 4374. what is its sum total?

First Method

From the greatest term = 4374
Subtract the least = 2

Divide by the ratio, less 1=3—1=2)4372 diff. of extremes.

Quotient= 2186
Add the greater extreme= 4374

6560

Or,

* To find the solid content of a globe. See Art. 34th. of Mensuration of Solids. Note, that ·523598 is two thirds of ·785398 the area of a circle, whose diameter is 1.

† It will be seen, when we come to *circulating decimals*, that $\cdot\dot{9}$ is the manner of expressing ·999, &c. to infinity.

Or, $\frac{4374-2}{3-1}+4374=6560$ Ans.

Second Method.

Greatest term=	4374
Multiply by the ratio =	3
Product =	13122
Subtract the least term =	2
Divide by the ratio, less by 1=3—1=2)	13120
	6560 Ans.

Or, $\frac{4374\times3-2}{3-1}=6560$

Third Method.

Greatest term =	4374
Multiply by the second term =	6
Product =	26244
Subtract the square of the first term =2×2=	4
Divide the remainder by the 2d. term less the first=6—2=4)	26240
Ans.	6560

Or, $\frac{4374\times6-4}{6-2}=6560.$

2. A man travelled 6 days, the first day he went 4 miles, and each day doubling his day's travel, his last day's ride was 128 miles; how far did he go in the whole? $\frac{128-4}{2-1}+128=252$ miles, Ans.

3. A gentleman, dying, left 5 sons, to whom he bequeathed his estate as follows, viz. to his youngest son £.1000; to the eldest £.5062 10s. and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$; what did the several legacies amount to?

$\frac{5062{\cdot}5-1000}{1{\cdot}5-1}+5062{\cdot}5=$£.13187 10s. Ans.

Problem IV.

Given the extremes and ratio, to find the number of terms.

Rule.

Divide the greatest term by the least; find what power of the ratio is equal to the quotient, then, add one to the index of that power, and the sum will be the number of terms.

Or,

Or, Subtract the logarithm* of the least term from that of the greatest; divide the remainder by the logarithm of the ratio, and add 1 to the quotient.

EXAMPLES.

1. If the least term be 2, the greatest term 4374, and the ratio 3; what is the number of terms?

Divide by the least term=2)4374=greatest term.

3×3×3×3×3×3×3=quotient, 2187=7th. power, then 7+1=8 Ans.

Or, From the logarithm of the greatest term=3·64088
Subtract the logarithm of the least term =0·30103

Divide the remainder by the logarithm of the ratio } =.47712)3·33985(7+1=8, Ans.
3·33984

1

2. A gentleman travelled 252 miles; the first day he rode 4 miles; the last 128, and each day's journey was double to the preceding one: How many days was he in performing the journey? Ans. 6 days.

PROBLEM V.

Given the least term, the ratio, and the sum of the series, to find the last term.

RULE.—Multiply the sum of the series by the ratio less 1, to that product add the first term, and the result, divided by the ratio, will give the last term.

EXAMPLES.

1. If the first term be 2, the ratio 3, and the sum of the series 6560: What is the last term?

Sum of the series=6560
Multiply by the ratio less 1= 2

Product=13120
Add the least term= 2

Divide their sum by the ratio=3)13122

4374 Ans.

Or, $\frac{\overline{3-1}\times6560+2}{3}=4374$ Ans.

2. A gentleman performed a journey of 252 miles; the first day he rode 4 miles, and each day after the first, twice so far as the day before: How far did he ride the last day?

$\frac{\overline{2-1}\times252+4}{2}=128$ miles, Ans.

PROBLEM

* Logarithms are artificial numbers, the addition of which answers to multiplication of whole numbers, and subtraction, to division.

Problem VI.

Given the least term, the ratio, and the sum of the series, to find the number of terms.

Rule.—To the product of the sum of the series, and the ratio minus 1, add the first term; which sum, divided by the first term, will give that power of the ratio signified by the number of terms.

Or, from the logarithm of the sum of the series plus the first term, multiplied by the ratio minus unity, take the logarithm of the first term; the remainder, divided by the logarithm of the ratio, will give the number of terms.

Example.

If the first term be 2, the ratio 3, and the sum of the series 80: What is the number of terms?

Sum=80
Multiply by the ratio less 1=3—1= 2

160
Add the first term= 2

Divide by the first term=2)162

81 which, found in the Table of Powers, is the fourth power of the ratio, therefore the number of terms is 4.

By Logarithms.

Sum=80
Add the first term= 2

82
Multiply by the ratio less 1=3—1= 2

Logarithm of 164=2·21484
Subtract the logarithm of the first term= ·30103

Divide the logarithm of the ratio=·47712)1·91381(4 Ans.
1 90848

533

Problem VII.

Given the extremes, and the sum of the series, to find the ratio.

Rule.—From the sum of the series subtract the least term; divide the remainder by the sum of the series minus the greatest term, and the quotient will be the ratio.

Examples.

1. If the least term be 2, the greatest term 4374, and the sum of the series 6560: What is the ratio?

Sum of the series=6560
Subtract the least term= 2

Divide the rem. by the sum of the series, minus greatest term } =6560—4374=2186)6558(3 Ans.
6558

2. A debt of D.252 was paid in Geometrical Progression, the first payment was D.4 and the last D.128: In what ratio did the payments exceed each other?

Ans. ---- $\frac{252-4}{252-128}=2$, viz. a double ratio.

Problem VIII.

Given the extremes, and the sum of the series, to find the number of terms.

Rule.—1. From the logarithm of the last term subtract the logarithm of the first, and make the remainder a dividend.

2. Subtract the logarithm of the sum minus the last term from the logarithm of the sum minus the first term, and make the remainder a divisor.

3 Divide the dividend by the divisor, and the quotient plus 1, will be equal to the number of terms.

Example.

If the least term be 2, the greatest term 4374, and the sum of the series 6560: What is the number of terms.

From the logarithm of the greatest term=3·64088
Take the logarithm of the least term=0·30103

Dividend=3·33985

From the logarithm of the sum minus the first term=3.81677
Take the logarithm of the sum minus the last term=3·33965

Divisor= ·47712

Then, ·47712)3·33985(7+1=8 Ans.
333984

1

Or, $\frac{L.4374-L.2}{L.6558-L.2186}+1=8.$

Problem IX.

The first term, the number of terms, and the last term given, to find the ratio.

Rule —Divide the greater extreme by the less, and extract such root of the quotient, whose index is equal to the number of terms, less 1. Or, find the quotient in the Table of Powers, the root of which is the answer.

Examples.

1. Given the extremes 2 and 4374, and the number of terms 8: What is the ratio?

Divide by the least term=2)4374=greatest term.

$\sqrt[7]{2187}=3$

Or, $\left|\frac{4374}{2}\right|^{\frac{1}{8-1}}=3$, Ans.

Problem

Problem X.

The extremes and number of terms given, to find the sum of the series.

Rule.—1. Subtract the least term from the greatest, and make the difference a dividend.

2. Divide the greatest term by the least, and extract such root of the quotient, whose index is equal to the number of terms less 1; take 1 from the said root, and make the remainder a divisor. (Or find the quotient in the table of powers, which will shew the root, from which subtract 1.)

3. Divide the dividend by the divisor, and the greatest term, added to the quotient, will give the sum of the series.

Example.

Given the extremes 2 and 4374, and the number of terms 8: What is the sum of the series?

From the greatest term=4374
Take the least= 2

Make this remainder a dividend 4372

Divide the greatest term by the least 2)4374

And extract the 7th root of the quotient, $\sqrt[7]{2187}=3$: Then,

3—1=2)4372

Quotient=2186
Add the greatest term=4374

6560 Ans.

$$\text{Or, } 4374+\frac{4374-2}{1}=6560$$

$$\left|\frac{4374}{2}\right|^{\frac{1}{8-1}}-1$$

Problem XI.

Given the ratio, the number of terms, and the greatest term, to find the least term.

Rule.—Divide the greatest term by such power of the ratio, whose index is equal to the number of terms less 1, and the quotient will be the least term.

Example.

If the ratio be 2, the number of terms 6, and the greatest term 128: What is the least?

Divide the last term by 2×2×2×2×2=5th power of the ratio } =32)128(4 Ans.
128

$$\text{Or, } \frac{128}{2^{6-1}}=4$$

Problem

PROBLEM XII.

Given the ratio, the number of terms, and the greatest term, to find the sum of the series.

RULE.—1. Divide the greatest term by such power of the ratio, whose index is equal to the number of terms less 1: take the quotient from the last term, and make the remainder a dividend.

2. Divide the dividend by the ratio less 1, and the quotient, added to the greatest term, will give the sum of the series.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the greatest term 3072: What is the sum of the series?

Divide the last term by the 5th power of the ratio $\Big\} = 4\times4\times4\times4\times4 = 1024)3072(3$
3072

From the last term=3672
Take the quotient= 3

Divide by the ratio less 1=4—1=3)3069

Quotient=1023
Add the greatest term=3072

Ans.=4095.

$$\text{Or, } 3072 + \frac{3072 - \dfrac{3072}{4^{6-1}}}{4-1} = 4095.$$

PROBLEM XIII.

Given the ratio, the number of terms, and the sum of the series, to find the least term.

RULE.—Divide the ratio, less 1, by such power, less 1, of the ratio, whose index is equal to the number of terms, and the quotient, multiplied by the sum of the series, will give the least term.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the sum of the series 4095: What is the least term?

$4\times4\times4\times4\times4\times4=4096$, and $4096-1=4095$, then, the ratio less 1, divided by 4095, is $\frac{3}{4095}$, and $\frac{3}{4095}\times\frac{4095}{1}=\frac{12285}{4095}=3$ Answer.

$$\text{Or, } \frac{4-1}{4^6-1}\times 4095 = 3.$$

PROBLEM XIV.

Given the ratio, the number of terms, and the sum of the series, to find the greatest term

RULE.—1. Subtract that power of the ratio, which is equal to the number of terms less 1, from that power of it, which is equal to the whole number of terms.

2. Divide

2. Divide the remainder by that power of the ratio minus unity which is equal to the number of terms, and the quotient, multiplied by the sum of the series, will give the greatest term.

EXAMPLE.

If the ratio be 4, the number of terms 6, and the sum of the series 4095 : What is the greatest term ?

From $4\times4\times4\times4\times4\times4=4^6=4096$
Subtract $4\times4\times4\times4\times4 \quad =4^5=1024$

3072

Divide by $4^6-1=4095)3072=\frac{3072}{4095}$ which multiplied by the sum, is $\frac{3072}{4095}\times\frac{4095}{1}=\frac{12579840}{4095}=3072$ Ans.

Or, $\frac{4^6-4^{6-1}}{4^6-1}\times4095=3072$

The two last problems may be solved by one short operation, thus : Divide the sum by the ratio, and the remainder after the operation will be the least term ; then take the quotient from the sum of the series, and the remainder will be the greatest term.

For the least term

```
4)4095(1023 quotient.
  4
  --
  09
   8
  --
  15
  12
  --
   3 Ans.
```

For the greatest term.

```
From the sum=4095
Subtract the quotient=1023
                      ----
Ans. =3072
```

PROBLEM XV.

Given the ratio, the last term, and the sum of the series, to find the first term.

RULE.

From the sum of the series take the last term, and multiply the remainder by the ratio ; then take this product from the sum of the series, and the remainder will be the first term.

EXAMPLE.

If the ratio be 4, the last term 3072, and the sum of the series 4095 ; what is the first term ?

```
            From the sum=4095
        Take the last term=3072
                           ----
            Remainder=1023
   Multiply by the ratio=   4
                         ----
              Subtract 4092 from the sum.
                       ----
And the remainder         3 is the Answer.
```

PROBLEM

PROBLEM XVI.

Given the ratio, the last term, and the sum of the series, to find the number of terms.

RULE.

1. Multiply the difference between the sum and the last term by the ratio, and note the product.
2. Subtract this product from the sum, and note the remainder.
3. From the logarithm of the last term subtract the logarithm of the remainder.
4. Divide this last remainder by the logarithm of the ratio, and the quotient, plus unity, will give the number of terms.

EXAMPLE.

If the ratio be 3, the last term 54, and the sum of the series 80; what is the number of terms?

From the sum=80
Take the last term=54

Remainder=26
Multiply by the ratio= 3

Product=78
From the sum=80
Take the product=78

Remainder=2
From the logarithm of 54=1·73239
Take the logarithm of the remainder= ·30103

Divide by the logarithm of the ratio=·47712)1·43136(3+1=4 Ans.

PROBLEM XVII. and XVIII.

Given the number of terms, the last term, and the sum of the series, to find the first term and the ratio.

The solution of these two Problems being very tedious by the Theorems, they may be solved by a very short operation; thus, Divide the sum of the series by the difference between the sum and the last term; the quotient will give the ratio, and the remainder, after the operation, the first term.

EXAMPLE.

If the number of terms be 4, the last term 54, and the sum of the series 80; required the first term and the ratio?

From the sum=80
Take the last term=54

Divide by the difference=26)80(3 the ratio.
78

The first term= 2

The

The following Table exhibits a summary view of the doctrine of Geometrical Progression.

CASES OF GEOMETRICAL PROGRESSION.

Case	Given	Required	Solution.
1.	*arn*	*l*	ar^{n-1}
		s	$\frac{r^{n}-1}{r-1}\times a$
2.	*arl*	*s*	$l+\frac{l-a}{r-1}$
		n	$\frac{L.l-L.a}{L.r}+1$
3.	*ars*	*l*	$\frac{\overline{r-1}\times s\times a}{r}$
		n	$\frac{L.\overline{\overline{r-1}\times s+a}-L.a}{L.r}$
4.	*als*	*r*	$\frac{s-a}{s-l}$
		n	$\frac{L.l-L.a}{L.\overline{s-a}-L.\overline{s-l}}+1$
5.	*ans*	*r*	$\frac{rs}{a}\,{}^{n-1}=\frac{s-a}{a}$
		l	$l\times\overline{s-l}^{\,n-1}=a\times\overline{s-a}^{\,n-1}$
6.	*anl*	*r*	$\left.\frac{l}{a}\right\vert^{\frac{1}{n-1}}$
		s	$l+\frac{l-a}{\left.\frac{l}{a}\right\vert^{\frac{1}{n-1}}-1}$

Case	Given	Required	Solution.
7.	rnl	a	$\frac{l}{r^{n-1}}$
		s	$\frac{l-\frac{l}{r^{n-1}}}{r-1}+l$
8.	rns	a	$\frac{r-1}{r^{n}-1}\times s$
		l	$\frac{r^{n}-r^{n-1}}{r^{n}-1}+s$
9.	rls	a	$s-\overline{r\times\overline{s-l}}$
		n	$\frac{L.l-L.s-\overline{r\times\overline{s-l}}}{L.r}+1$
10.	nls	a	$a\times\overline{s-a}\,\rvert^{n-1}=l\times\overline{s-l}\,\rvert^{n-1}$
		r	$r^{n}+\frac{s}{l-s}r^{n-1}=\frac{l}{l-s}$

Here $\begin{cases} a=\text{first or least term.} \\ l=\text{last or greatest term.} \\ s=\text{sum of all the terms.} \\ n=\text{number of terms.} \\ r=\text{ratio.} \\ L=\text{logarithm.} \end{cases}$

SIMPLE INTEREST.

INTEREST is a Premium allowed by the Borrower to the Lender, according to a certain Rate per cent. agreed on; which by law is stated at 6 per cent. per annum *Principal* is the money lent. *Rate* is the sum per cent. agreed on. *Amount* is the Sum of Principal and Interest.

Simple

Simple Interest is that which is allowed on the Principal only.

Note.—By this Rule, Commission, Brokerage, Insurance, purchasing Stocks, or any thing else, rated at so much per cent. are calculated.

General Rule.

1. Multiply the Principal by the Rate, and divide by 100 (or cut off the two right hand figures in the Pounds) and the quotient, or left hand figures, will be the answer in Pounds, &c. the right hand figures being reduced and cut off as at first. If the principal be dollars, the right hand figures will be cents.

2. For more years than one, multiply the Interest of one year by the number of years.

3. For any number of months take the aliquot parts a of year ; and for days, the aliquot parts of 30.

Note. When the rate per cent. per annum is $\left\{\begin{matrix}9\\8\\6\\4\\3\\2\end{matrix}\right\}$ multiply the principal by $\left\{\begin{matrix}\frac{3}{4}\\\frac{2}{3}\\\frac{1}{2}\\\frac{1}{3}\\\frac{1}{4}\\\frac{1}{6}\end{matrix}\right\}$ of the given number of months, and you will have the interest for the given time.

Examples.

1. What is the interest of 573l. 13s. 9½d. at 6 per cent. per annum ? Answer, £.34 8s. 5d.

£.573	13	9½
		6
£.34·42	2	9
20		
8·42		
12		
5·13		
4		
·52		

2. What is the interest of 329l. 17s. 6½d. for 3 years, 7 months, and 12 days, at 5 per cent. per annum. Ans. £.59 13s.

£.329	17	6½d.						Then,	
		5	6 mons.	½	16	9	10½	interest of 1 year	
							3		
16·49	7	8½							
20					49	9	7½	do. of 3 years	
			1 mon.	$\frac{1}{6}$	8	4	11¼	do. of	6 months
9·87			10 days	$\frac{1}{3}$	1	7	5¾	do. of	1 month
12			2 days	$\frac{1}{5}$		9	1¾	do. of	10 days
						1	9¾	do. of	2 days
10·52									
4				Ans.	£.59	13		do. of 3y. 7m. 12d.	
2·10									

Or

Or thus:

		£.	s.	d.
£.5	$\frac{1}{20}$	329	17	$6\frac{1}{2}$
6 months	$\frac{1}{2}$	16	9	$10\frac{1}{2}$
				3
		49	9	$7\frac{1}{2}$
1 month	$\frac{1}{6}$	8	4	$11\frac{1}{4}$
10 days	$\frac{1}{3}$	1	7	$5\frac{3}{4}$
2 days	$\frac{1}{5}$		9	$1\frac{1}{4}$
			1	$9\frac{3}{4}$
		£.59	13 Ans.	

3. What is the interest of 347 dollars 50 cents, at 6 per cent. per annum for a year? D.347·50
6
20·8500 Ans. D.20 85c.

4. What is the interest of D.797 13c. at 6 per cent. per annum, for 8 months? D.797·13
4
31·8852 Ans. D.31 88c. $5\frac{2}{10}$ m.

5. What is the interest of D.649 17c. at 6 per cent. per annum, for 15 months? D.649·17
$7\frac{1}{2}$
454419
324585
48·68775 Ans. D.48 68c. $7\frac{3}{4}$m.

6. Required the amount of £.725 12s. 6d. at 5 per cent. per ann. for a year? $5=\frac{1}{20}$|725 12 6
36 5 $7\frac{1}{2}$
Ans. £.761 18 $1\frac{1}{2}$

7. What is the amount of D.560 50c. at 6 per cent. for 16 months?
D.560 50
8
44·84
560·50
Ans. D.605·34c.

8. What is the interest of D.150 75c. for 1 month, at 6 per cent. per annum? $\frac{1}{2}$|150·75
·75375 Ans. 75cts. $3\frac{3}{4}$ mills.

So that any number of dollars, considered as so many cents, is the interest for 2 months, at 6 per cent.

COMMISSION, OR FACTORAGE,

Is an allowance of so much per cent. to a Factor or Correspondent, for bying and selling goods.

9. Required the commission on £.436 9s. 6d. at $3\frac{1}{2}$ per cent.

2...F £.436

```
£.436   9   6
            3½
-------------
 1309   8   6
  218   4   9
-------------
15·27  13   3
   20
 ----
 5·53
   12
 ----
 6·32
    4
 ----
 1·56   Ans. £.15 5 6¼.
```

10. Required the commission on D.649 75c. at 1¾ per cent.

```
| ½ | 649·75
| ¼ | 324·875
      162·4375
    ----------
     1137·0625   Ans. D.11 37c. 0⅝m.
```

BROKERAGE

Is an allowance of so much per cent. to a person called a Broker, for assisting merchants in purchasing or selling goods.

11. Required the Brokerage on £.911 12s. at 5s. or ¼ per cent.

```
5s.=¼ | 911   12
       -------
        2·27  18
          20
        ----
        5·58   Ans. 2l. 5s. 6¾d.
          12
        ----
        6·96
           4
        ----
        3·84
```

12. Required the Brokerage on D.876 21c. at 33⅓ cents, or at ⅓ per cent.

```
⅓ | 876·21
    ------
    292 07   Ans. D.2 92c. 0 7/10 m.
```

BUYING AND SELLING STOCKS.

Stock is a general name for the capitals of trading companies.

13. Required the amount of £.375 15s. bank stock, at £.75 per cent?

```
| 50 | ½ | 375  15
|    |   | -------
| 25 | ½ | 187  17   6
            93  18   9
      ----------------
Ans. £.281  16   3
```

Or thus:

```
| 25 | ¼ | 375  15
           --------
Subtract    93  18   9
Asbefore, £.281  16   3
```

14. Required the amount of D.2195 50c. bank stock, at 125 per cent.

```
| 25 | ¼ | 2195·50
      Add  548·875
Ans. D.2744·375
```

TO

TO CALCULATE INTEREST FOR DAYS.

RULE 1.—Multiply the principal by the days, and that product by the rate on the pound, and divide the last product by 365.

15. Required the interest of £.360 10s. for 175 days, at 6 per cent.

$$\frac{360{\cdot}5\times175\times{\cdot}06}{365}=\text{£}.10{\cdot}17=\text{£}.10\ 7s.\ 4\tfrac{3}{4}d.$$

Rule for making a Divisor for any Rate.

Multiply 365 by 100, and divide by the rate. Thus, for 6 per cent. $\frac{365\times100}{6}=6083$ divisor.

For 5 per cent. $\frac{365\times100}{5}=7300$ divisor, and so for any other rate.

Therefore,

RULE 2. Multiply the principal by the days; divide by 6083 for 6 per cent, and 7300 for 5 per cent. (the days in which any sum will double at those rates) and the quotient is the interest. For months, multiply the principal by them, and divide by 200 for 6 per cent. or 240 for 5 per cent. (the months in which any sum will double at those rates) and the quotient is the answer.

Hence, when interest is to be calculated on cash accounts, or accounts current, where partial payments are made, or partial debts contracted; multiply the several balances into the days they are at interest, which should be done at every transaction, and the sum of these products divided by 6083 and 7300 will give the interest at 6 and 5 per cent. For any other rate, make the proper addition or deduction, or find a divisor as before directed.

When partial payments are made at short periods, subtract the several payments from the original sum in their order, placing their dates in the margin.

16. Suppose a bill of D.359 was due January 1, 1807; that D.75 was paid February 3d. D.50 March 5th, D.80 April 9th, and June 7th, D.145: What interest is due?

Dates.	Bill.	Days	Products.
January 1	D.350	33	11550
Feb. 3, paid	75		
Balance,	275	30	8250
March 5, paid	50		
Balance,	225	35	7875
April 9, paid	80		
Balance,	145	59	8555
June 7, paid	145		

6083)36230(5·955

Ans. D.5 $95\frac{1}{2}$c. at 6 per cent.

7300)36230(4·963

Ans. D.4 96c. 3m. at 5 per cent.

After

After the dates are placed in the margin, the number of days in each of those periods is to be computed and marked against its respective sum: lastly, divide the sum of the products by 6083, &c.

Interest on acconnts current is calculated nearly in the same manner.

17. Compute the interest at 6 per cent. on the following account, to August 10th.

Dr. Mr. A. Jones, his account current, with B. Carr, Cr.

1807.		D.	1807.		D.
Jan. 1,	To Cash, - - -	560	March 10,	By Cash, - -	120
Feb. 10,	To do. - - -	300	April 25,	By do. - -	130
May 15,	To do. - - -	140	June 16,	By do. - - -	450
July 25,	To do. - - -	100	July 21,	By do. - -	150

1807.		Ds.	Days.	Products.	Dr.	Cr.
Jan. 1,	Dr.	560	40	22400	D.560	120
Feb. 10,	Dr.	300			300	130
		——			140	450
	Dr.	860	28	24080	100	150
March 10,	Cr.	120			——	——
		——			1100	850
	Dr.	740	46	34040	850	
April 25,	Cr.	130			——	
		——			250 Balance.	
	Dr.	610	20	12200		
May 15,	Dr.	140			January	30
		——			February	28
	Dr.	750	32	24000	March	31
June 16,	Cr.	450			April	30
		——			May	31
	Dr.	300	35	10500	June	30
July 21,	Cr.	150			July	31
		——			August	10
	Dr.	150	4	600		——
July 25,	Dr.	100			Days	221
		——				
Aug. 10,	Dr.	250	16	4000		
6083)131820(21.672			——	——		
Ans. D.21 67c. 2m.			221	131820		

Here the sums on either side are introduced according to the order of their dates; those on the Dr. side are added to the former balance, and those on the Cr. side subtracted. Before we calculate the days, we try if the last sum D.250 be equal to the balance of the account, which proves the additions and subtractions. And before multiplying we try if the sum of the column of days be equal to the number of days from January 1 to August 10.

17. Required the interest on the following account, from December 31, 1806, to Dec. 31, 1807; allowing 5 per cent. when the balance is due to A. and 6 per cent. when due to B.?

Dr.

Dr. Mr. B. his account current with A. Cr.

1806.		D.	1807.		D.
Dec. 31,	To Balance,	150	April 9,	By Cash,	70
1807.			May 12,	By do.	300
March 12,	To Cash	120	June 3,	By do.	240
June 17,	To do.	165	Aug. 2,	By do.	10
Sept. 24,	To do.	242			
Oct. 9,	To do.	178			

1806.	C.D.	D.	Ds.	Dr. Products.	Cr. Products.
Dec. 31,	Dr.	150	71	10650	
1807.					
March 12,	Dr.	120			
	Dr.	270	28	7560	
April 9,	Cr.	70			
	Dr.	200	33	6600	
May 12,	Cr.	300			
	Cr.	100	22	———	2200
June 3,	Cr.	240			
	Cr.	340	14	———	4760
17,	Dr.	165			
	Cr.	175	46	———	8050
Aug. 2,	Cr.	10			
	Cr.	185	53	———	9805
Sept. 24,	Dr.	242			
	Dr.	57	15	855	
Oct. 9,	Dr.	178			
Dec. 31,	Dr.	235	83	19505	
7300)45170(6·187					
6083)24815(4·079			365	45170	24815

Interest due A. at 5 per cent. D.6 18c. 7m.
Interest due B. at 6 per cent. 4 7 9

Balance due A. D.2 10c. 8m.

In this account the balance is sometimes to one party, and sometimes to the other. These charges are distinguished by Dr. and Cr.

When payments are made on bonds, notes, &c. at considerably distant periods, it is usual to calculate the interest to the date of each payment, and add it to the principal, and then subtract the payment from the amount.

18. A note was given for D.540 the 18th August, 1804, and there was paid the 19th of March, 1805, D.50, and the 19th of December,

ber, 1805, D.25; and the 23d of September, 1806, D.25; and the 18th of August, 1807, D.110: Required the interest, and balance due on the 11th of November, 1807?

A note given 18th August, 1804, for		D 540
Interest to 19th March, 1805, 218 days, D.19·352		19·352
		559·352
Paid 19th March, 1805,		50
Balance due 19th March, 1805,		509·352
Interest to 19th Dec. 1805, 275 days,	23·022	23·022
Balance due 19th Dec. 1805,		532·374
Paid 19th Dec. 1805,		25·00
Balance due 19th Dec 1805,		507·374
Interest to 23d. Sept. 1806, 278 days,	23·197	23·197
Balance due 23d. Sept. 1806,		530·571
Paid 23d. Sept. 1806,		25·000
Balance due 23d. Sept. 1806,		505·571
Interest to 18th Aug. 1807, 329 days,	27·343	27·343
Balance due 18th Aug. 1807,		532·914
Interest to 11th Nov. 1807, 85 days,	7·448	7·448
Balance due 11th Nov. 1807,		540·362
Amount of interest,	D.100·362	

19. A. owes B. the following sums, with interest at 6 per cent. per annum: D.60 for 7 months, D.150 for 9 months, D.75·50 for 3 months, D.365·25 for 8 months, and 510·20 for 5 months: Required the amount?

D. 60 × 7 = 420
150 × 9 = 1350
75·50 × 3 = 226·50
365·25 × 8 = 2922
510·20 × 5 = 2551

1160·95   200)7469·50(37·347 Interest.
1160·95 Principal.

Ans. D. 1198·297 Amount.

20. A note for D.1000 is given January 1, 1803, with interest at 6 per cent. per annum; February 19, 1803, D.100 are paid; June 7, 1803, D.150; April 14, 1804, D.37·50; July 11, 1804, D.75; Sept. 29, 1804, D.250; Dec. 17, 1805, D39; March 4, 1806, D.175; Aug. 7, 1806, D.105; Oct. 30, 1806, D.50; May 12, 1807, D.40, and Nov. 17, 1807, D.72: How much is due, January 1, 1808?

SIMPLE

SIMPLE INTEREST BY DECIMALS.

A Table of Ratios, from one *pound, &c. to* ten *pounds.*

Rate per cent.	ratios.	rate per cent.	ratios.	rate per cent	ratios
1	·01	4	·04	7	·07
$1\frac{1}{4}$	·0125	$4\frac{1}{4}$	·0425	$7\frac{1}{4}$	·0725
$1\frac{1}{2}$	·015	$4\frac{1}{2}$	·045	$7\frac{1}{2}$	·075
$1\frac{3}{4}$	·0175	$4\frac{3}{4}$	·0475	$7\frac{3}{4}$	·0775
2	·02	5	·05	8	·08
$2\frac{1}{4}$	·0225	$5\frac{1}{4}$	·0525	$8\frac{1}{4}$	·0825
$2\frac{1}{2}$	·025	$5\frac{1}{2}$	·055	$8\frac{1}{2}$	·085
$2\frac{3}{4}$	·0275	$5\frac{3}{4}$	·0575	$8\frac{3}{4}$	·0875
3	·03	6	·06	9	·09
$3\frac{1}{4}$	·0325	$6\frac{1}{4}$	·0625	$9\frac{1}{4}$	·0925
$3\frac{1}{2}$	·035	$6\frac{1}{2}$	·065	$9\frac{1}{2}$	·095
$3\frac{3}{4}$	·0375	$6\frac{3}{4}$	·0675	$9\frac{3}{4}$	·0975
				10	1

Ratio is the Simple Interest of £.1 or D. for 1 year, at the rate per cent. agreed on.

A Table for the ready finding of the decimal parts of a year, equal to any number of days, *or* quarters *of a* year.

Days.	decimal parts.	days.	decimal parts.	days	decimal parts.
1	·00274	10	·027397	100	·273973
2	·005479	20	·054794	200	·547945
3	·008219	30	·082192	300	·821918
4	·010959	40	·109589	365	1·000000
5	·013699	50	·136986	$\frac{1}{4}$ of a year = ·25	
6	·016438	60	·164383	$\frac{1}{2}$ of a year = ·5	
7	·019178	70	·191781	$\frac{3}{4}$ of a year = ·75	
8	·021918	80	·219178		
9	·024657	90	·246575		

CASE I*

The principal, time, and ratio given, to find the interest and amount.

Rule. Multiply the principal, time and ratio continually together, and the last product will be the interest, commission, brokerage, &c. to which add the principal, and the sum will be the amount.

Examples.

1. Required the amount of £.537 10s. at £.6 per cent. per annum, for 5 years?

Principal

* The following Theorems will show all the possible cases of Simple Interest Where p=principal, t=time, r=ratio, and a=amount.

I. $ptr+p=a$. II. $\frac{a}{tr+1}=p$. III. $\frac{a-p}{tp}=r$. IV. $\frac{a-p}{rp}=t$.

Principal	537·5
Multiply by the ratio =	·06
Product	32·250
Multiply by the time =	5
Interest =	161·250
Add the principal =	537·5
Amount =	£.698·75
	20
	15·00 Ans. £.698 15s.

Or, 537·5×·06×5+537·5=£.698 15s.

2. What is the simple interest of £.917 16s. at £.5 per cent. per annum, for 7 years? Ans. £.321 4 7.

3. What is the amount of £.391 17s. at £.4½ per cent. per annum, for 3¼ years? Ans. £.449 3 1¾.

4. What is the amount of £.235 3s. 9d. at £.5¼ per cent. per annum, from March 5th. 1784, to November 23d. 1784? Ans. £.244 0 8½.

5. If my correspondent is to have £.2½ per cent; what will his commission on £.785 15s. amount to? Ans. £.19 12 10½

6 What will be the interest and amount of £.445 10s. in 3 years and 129 days, at £.8½ per cent. per annum?

Ans. Interest, £.126 19 8½, and the amount=£.572 9 8½.

7. If a broker disposes of a cargo for me, to the amount of £.637 10s. on commission at £.1¼ per cent. and procures me another cargo of the value of £.817 15s. on commission at £.1¾ per cent.; what will his commission, on both cargoes, amount to? Ans. £.22 5 7.

CASE II.

The amount, time, and ratio given, to find the principal.

Rule. Multiply the ratio by the time; add unity to the product for a divisor, by which sum divide the amount, and the quotient will be the principal.

Examples.

1. What principal will amount to £.1045 14s. in 7 years, at £.6 per cent. per annum?

Ratio=	·06
Multiply by the time=	7
Product=	·42
Add	1·

Divisor=1·42)1045·7(736·4084 + = £.736 8 2.

Or, $\frac{1045{\cdot}7}{{\cdot}06\times7+1}$ =£.736 8 2 Ans.

2. What

2. What principal will amount to £.3810, in 6 years, at £.4½ per cent. per annum? Ans. £.3000.

3. What principal will amount to £.666 9s. 0¼ in 3½ years, at £.5¼ per cent. per annum? Ans. £.563.

4. What principal will amount to £.335 7s. 3d. in 3 years and 97 days, at £.9½ per cent. per annum? Ans. £.255 19 0¾.

CASE III.

The amount, principal, and time given, to find the ratio.

Rule. Subtract the principal from the amount; divide the remainder by the product of the time and principal, and the quotient will be the ratio.

Examples.

1. At what rate per cent. will £.543 amount to £.705 18s. in 5 years?

From the amount=705·9
Take the principal=543

Divide by 543×5=2715)162·90(·06
162 90

Or, $\frac{705{\cdot}9-543}{543\times5}$=·06=£.6 Ans.

2. At what rate per cent. will £.391 17s. amount to £.449 3s. 1¾d. ·74qr. in 3¼ years? Ans. £.4½.

3. At what rate per cent. will £.413 12s. 6d. amount to £.546 4s. 10¼d. in 4¾ years? Ans. £.6¼.

4. At what rate per cent. will £.3000 amount to £.3810 in 6 years? Ans. £.4½.

CASE IV.

The amount, principal, and rate per cent. given, to find the time.

Rule. Subtract the principal from the amount; divide the remainder by the product of the ratio and principal; and the quotient will be the time.

Examples.

1. In what time will £.543 amount to £.705 18s. at £.6 per cent. per annum?

From the amount=705·9
Take the principal=543

Divide by 543×·06=32·58)162·9(5 years, Ans.
162 9

2. In what time will £.3000 amount to £.3810, at 4½ per cent. per annum? Ans. 6 years.

3. In what time will £.391 17s. amount to £.449 3s. 1¾d. at £.4½ per cent. per annum? Ans. 3¼ years.

To find the Interest of any Sum, at 6 per cent. per annum, for any number of months.

Rule. If the months be an even number, multiply the pricipal by half that number; and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus the half of 19 is 9·5; and multiply

 the

the principal as before, cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first.

4. What is the interest of £.345 16s. 6d. for 9 years and 11 months, at 6 per cent. per annum?

```
                              Y. m.
                               9 11
                              12
                              ----
         345·825            2)119 months.
            59·5              ---
         --------             59·5=½ No. of months.
          1729125
         3112425
        1729125
        --------
    £.205·765875=£.205 15 3¾ Ans.
           Principal=£.345 16 6
                     ---------
             Amount=£.551 11 9¾
```

A Table of decimal parts for every day in the twelfth part of a year, which consists of 365¼ days.

days	dec. pts.	days	dec. pts.	days	dec. pts.	days	dec. pts.	days	dec. pts.
1	·033	7	·230	13	·427	19	·624	25	·821
2	·066	8	·263	14	·460	20	·657	26	·854
3	·098	9	·296	15	·493	21	·690	27	·887
4	·131	10	·328	16	·526	22	·723	28	·920
5	·164	11	·361	17	·558	23	·756	29	·953
6	·197	12	·394	18	·591	24	·788	30	·986

To find the Interest of any Sum, either for Months, or Months and Days, at 6 per cent. per annum.

RULE.

Multiply the principal by the number of months, (or months and parts, answering to the given number of days in the table) and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the interest for the given time, in shillings and decimal parts of a shilling.

EXAMPLES.

1. What is the interest of 100l. for a year?

```
        Principal=100
Mult. by the months=12
            ---------
     Ans. s.120|0=£.6
```

Note. This Table may also be used for the *parts of a year*, in Compound Interest, after having worked for whole years.

2. What is the interest of 250l. 10s. for 19 months and 7 days?

```
Principal=£.250·5
   Time=   19·23
           -----
            7515
           5010
         22545
         2505
        --------
Ans. s.481·7115
      =£.24 1 8½
```

Another

Another Method of calulating Interest *for Months, at* 6l. *per cent. per annum.*

RULE.

If the principal consist of pounds only, cut off the unit figure, and, as it then stands, it will be the interest for one month in shillings and decimal parts :—If it consist of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a shilling.

EXAMPLES.

1. What is the interest of 175l. for 5 months ?

£.175=17·5 shill.=interest for 1 month. 17·5
Multiply by the time= 5

2|0)87·5

Ans.=£.4 7 6

2. What is the interest of 255l. 16s. for 7 months ?

Shill.
£.255 16=25·58 int. for 1 mo.
7

2|0)179·06

£.8 19 0½ Ans.

SIMPLE INTEREST IN FEDERAL MONEY.

PROBLEM I.

When the principal is given in Massachusetts pounds, shillings, &c. and the interest is required in federal money, at 6 *per cent. per annum.*

RULE.

Reduce the shillings, &c. to their equivalent decimal, by inspection, divide the whole by 5, and the quotient is the annual interest : Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off) will be the interest as before.

EXAMPLES.

1. Required the annual interest of 517l. 3s. 7½d. at 6 per cent. ?

3s. = ·15
7½d. = ·030
Excess of 12 = ·001

·181

5)517·181
D. c. m.
103·436 = 103 43 6 Ans.

Or, 517·181
2
D. c. m.
103·4362 = 103 43 $6\frac{2}{10}$.

2. Required the annual interest of 1l. in cents ?

5)1·00

20 cents, Ans.

PROBLEM II.

When the principal is given in Massachusetts old currency, and the interest and amount are required in federal money at 6 *per cent.*

RULE.—Reduce the Massachusetts money to federal, then divide the principal by 20 and that quotient by 5 ; add those quotients together, and they are the interest ; or add them to the principal, and their sum is the amount.

EXAMPLES.

EXAMPLES.

1. Required the amount of 425l. 16s. 8½d. for 1 year, at 6 per cent.?

```
 ·8           3)425·835
 ·034        20)1419·450
 ·001           5)70·9725
 ----             14·1945
 ·835          ----------   D.    c. m.
              1504·6170 = 1504·61 7. Ans.
```

2. Required the amount of 112l. 4s. 6d. for one year?

```
 ·2           3)112·225
 ·024        20)374·083
 ·001          5)18·7041
 ----             3·7408
 ·225          ---------   D.   c. m. dec.
              396·52 79 = 396·52 7 9, Ans.
```

PROBLEM III.

When the principal is Massachusetts old currency, and the monthly interest is required in federal money.

RULE.—Reduce the shillings, &c. to decimals, by inspection, then separate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the answer in dollars, cents, &c.

EXAMPLE.

Required the monthly interest of 425l. 16s. 8½d. in federal money?

```
     ·8
     ·034          6)42·5835
     ·001          ---------  D. c. m.
     ----          7·09725 = 7·09 7¼ Ans.
  £.·835
```

PROBLEM IV.

When the principal is federal money, and the interest is required in the same.

RULE.—Work according to the general rule in simple interest, that is, multiply by the rate of interest, separate the two right hand figures of the dollars in the product, and it will give the interest in dollars, cents, &c.

N. B. The figures, which are more than three places to the right hand of the point, are of no account, unless the fourth place exceed 5, in which case increase the mills 1.

EXAMPLES.

1. What is the annual interest of D.537 24c. 6m. at 6 per cent.?

```
D.  c. m.
537·24 6
       6
---------  D. c. m.
32·23476 = 32·23 5 Ans.
```

2. What is the interest of D.1465 46c. 6m. for 16 months, at 6 per cent. per annum?

```
D.   c. m.
1465·46 6
        8=half the number of months.
---------  D. c. m.
117·23728=117·23 7 Ans.
```

3. What

3. What is the interest of D.537 34c. 7m. for 19 months, at 6 per cent. per annum ?

D. c. m.
537·34 7
9·5=half the number of months.

2686735
4836123

D. c. m.
51.047965=51.04 8 Ans.

N. B. Because there are 4 decimals in the multiplicand and multiplier, I cut off 4 figures for them, and two more according to the rule.

Problem V.

When the principal is federal money, and the monthly interest is required in the same, at 6 per cent. per annum.

Rule.—Separate the two right hand figures of the dollars, and you then have the interest for two months; half of which is the monthly interest in dollars, cents, &c. If there be but one place, or figure of dollars, a cypher must be prefixed to the left hand.

Examples.

1. What is the monthly interest of D.9 59c. 7m. at 6 per cent per annum ?

2)·09597 c.m.
·047985=4 8 nearly, Ans.

2. What is the monthly interest of D.100 50c. 5m. ?

2)1·00505 c. m.
·50252=50 2½, Ans.

Rules for calculating interest for days.

Rule I.

Multiply the given principal by the given number of days, and that product by the rate on the pound: divide the last product by 365 (the number of days in a year) and it will give the interest.

Example.

What is the interest of 360l. 10s. for 175 days, at 6 per cent. ?

$$\frac{360{\cdot}5\times175\times{\cdot}06}{365}=£.10{\cdot}37=£.10\ 7s.\ 4\tfrac{3}{4}d.\ \text{Ans.}$$

Rule II.

Multiply the given principal by the given number of days, and divide the product by 6083, for 6l. per cent.; (the number of days in which any sum will double, at that rate) the quotient will give the answer.

Example.

What is the interest of 327l. 10s. at 6 per cent. per annum, for 210 days ?

$$\frac{327{\cdot}5\times210}{6084}=£.11{\cdot}306=£.11\ 6\ 1\tfrac{1}{4}d.\ \text{Ans.}$$

Rule

Rule for making a divisor for any rate per cent.

Multiply 365 by 100, and divide the product by the rate.

Thus, for 6 per cent. $\frac{365\times100}{6}=6083$ divisor.

For 5 per cent. $\frac{365\times100}{5}=7300$ divisor, &c.

Perhaps the most convenient way to calculate at 6 per cent. is first, to do it for 5, and then add one fifth of the quotient to itself; because, by cutting off the two cyphers in the divisor, you have to divide only by 73.

Hence, when interest is to be calculated on cash accounts, accounts current, or any other accounts, where partial payments are made, or partial debts contracted; multiply the several balances into the days they are at interest, and the sum of these products, divided as above, will give the interest at 5l. or 6l. per cent. and for any other rate, make the proper addition or deduction; or find a divisor as before directed.

EXAMPLES.

1. On the 1st of January I lent 450l. 10s. 6d. which I received back in the following partial payments, viz. on the 14th of January 57l. 11s. 9d.; on the 7th of February, 39l. 3s. 10d.; on the 19th of March, 63l. 5s. 2d.; on the 4th of April, 45l.; on the 26th of April, 19l. 12s. 6d.; on the 12th of May, 100l.; on the 10th of June, 60l. 7s. 3d.; and on the 1st of August, 65l. 10s.: What interest is due at 6 per cent.?

Dates.			£.	s.	d.	Ds.	Products.		
January 1	Lent on demand	- - -	450	10	6	13	5856	16	6
14	Received in part	- - -	57	11	9				
		Balance	392	18	9	24	9430	10	0
February 7	Received in part	- - -	39	3	10				
		Balance	353	14	11	40	14149	16	8
March 19	Received in part	- - -	63	5	2				
		Balance	290	9	9	16	4647	16	0
April 4	Received in part	- - -	45	0	0				
		Balance	245	9	9	22	5400	14	6
April 26	Received in part	- - -	19	12	6				
		Balance	225	17	3	16	3613	16	0
May 12	Received in part	- - -	100	0	0				
		Balance	125	17	3	29	3650	0	3
June 10	Received in part	- - -	60	7	3				
		Balance	65	10	0	52	3406	0	0
August 1	Received in full of the principal		65	10	0		50155	9	11

```
                    (5)
73|00)501|55  9  11(6   17   4¾ interest at 5 per cent.
      438            1    7   5¾
      ----          -----------
      6355       £.8    4  10½ interest at 6 per cent.
        20
      ----
     )1271|09
      73
      ----
      541
      511
      ----
      3009
        12
      ----
     )361|19
      292
      ----
      6919
         4
      ----
     )276|76
      219
      ----
      5776
```

2. I have given Peter Trusty a cash credit for 1000l in consequence of which, on the 12th of May, I paid his bill for 250l.; May 27th, paid his draught for 280l.; June 1st, he gave me a bill on the Massachusetts bank at sight for 290l.; July 17th, he paid me per receipt 70l.; August 20th, he drew for 750l. at sight; August 31, he paid me per receipt, 500l.; Sept. 15th, he drew at sight for 135l.; and 3d of October, for 175l.; Oct. 29th, he paid me per receipt, 250l.; and November 3d, 125l.; Nov. 12th, he drew at sight for 375l.; and Nov. 18th, for 125l.; January 1st, he paid me per receipt 290l.; and January 20th, 210l. On the 1st of March following he demands a settlement: What is then due to me, interest at 6 per cent.?

Dates.			£.	Ds.	Products.
May	12	Paid his bill - - - -	250	15	3750
May	27	Paid his draught - - -	280		
		Balance	530	5	2650
June	1	Received in part - - -	290		
		Balance	240	46	11040
July	17	Received in part - - -	70		
		Balance	170	34	5780
August	20	Paid - - - - - - - -	750		
		Balance	920	11	10120
August	31	Received in part - - -	500		
		Balance	420	15	6300
September	15	Paid - - - - - - - -	135		
		Balance	555	18	9990
October	3	Paid - - - - - - - -	175		
		Carried over.] Balance	730	26	18980

Dates.			£.	Ds.	Products.
	Brought over]	Balance	730	26	18980
October 29	Received in part - - -		250		
		Balance	480	5	2400
November 3	Received in part - - -		125		
		Balance	355	9	3195
November 12	Paid - - - - - - - -		375		
		Balance	730	6	4380
November 18	Paid - - - - - - - -		125		
		Balance	855	44	37620
January 1	Received in part - - -		290		
		Balance	565	19	10735
January 20	Received in part - - -		210		
March 1		Balance	355	40	14200
					141140

(5)
Then, 73|00)141140(19 6 8 interest at 5 per cent.
3 17 4

£.23 4 0 interest at 6 per cent.
355 0 0

£.378 4 0 balance in my favour.

When cash credits are given, a balance should be made upon every transaction, which should be multiplied into the days the first leisure minute; then, when the time of settlement comes, you will only have to add up the products, and divide as above, and the account will be finished.

3. A owes B the following sums, with the interest on them, at 6 per cent. per annum, as follows; viz. D.60 for 7 months, D.150 for 15 months, D.75 50c. for 9 months, D.145 75c. for 27 months, and D.397 60c. for $45\frac{1}{2}$ months: What is the amount of principal and interest?

D. c.	Months.	
60	× 7 =	420
150	× 15 =	2250
75·5	× 9 =	679·5
145·75	× 27 =	3935·25
397·60	× 45·5 =	18090·8

828·85 200)25375·55(126·877 interest. (D.)
828·85 principal.

D.955·727 amount, Answer.

Note.

Note. I divide by 200, the number of months, in which any sum will double at 6 per cent. per annum, and it gives the interest.

When partial payments are made upon bonds, notes, &c. at any interval *greater* than a year, the interest is calculated in a progressive manner, by adding the interest to the principal at the time of the first payment, and from the sum deducting the payment, &c.

DISCOUNT

IS an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The *present worth* of any sum or debt, due some time hence, is such a sum, as if put to interest, would in that time and at the rate per cent. for which the discount is to be made, amount to the sum or debt then due.

Rule I.*

As the amount of 100l. for the given rate and time is to 100l.; so is the given sum or debt to the present worth.

Subtract the present worth from the given sum, and the remainder will be the discount required.

Or,

* That an allowance ought to be made for paying money before it becomes due, which is fuppofed to bear no intereft till after it is due, is very reafonable; for if I keep the money in my own hands till the debt fhall become due, it is plain I may make an advantage of it by putting it out to intereft for that time; but if I pay it before it is due, I give that benefit to another; therefore we have only to inquire what difcount ought to be allowed. And here, many fuppofe that, fince by not paying till it becomes due they may employ it at interest; therefore, by paying it before due, they fhall lofe that intereft, and for that reafon all fuch intereft ought to be difcounted; but the fuppofition is falfe, for they cannot be faid to lofe that intereft till the time arrives, when the debt becomes due; whereas we are to confider what would probably be loft, at prefent, by paying the debt before it becomes due; this can, in point of equity, be no other than fuch a fum, which being put out at intereft till the debt fhall become due, would amount to the intereft of the debt for the fame time. It is befides plain, that the advantage arifing from difcharging a debt due fome time hence, by a prefent payment, according to the principles above mentioned, is exactly the fame as employing the whole fum at intereft till the time when the debt becomes due, arrives: for, if the difcount allowed for prefent payment be put out at intereft for that time, its *amount* will be the fame as the intereft of the whole debt for the fame time; thus the difcount of 106l. due one year hence, reckoning intereft at 6l. per cent. will be 6l. and 6l. put out to intereft at 6l. per cent. for one year, will amount to 6l. 7s. 2¼d. which is exactly equal to the intereft of 106l. for one year at 6l. per cent.

The truth of the rule for working is evident from the nature of Simple Intereft; for fince the debt may be confidered as the *amount* of fome principal (called here the prefent worth) at a certain rate per cent. and for the given time, that amount muft be in the fame proportion either to its principal or intereft, as the amount of any other fum, at the fame rate, and for the fame time, to its principal or intereft.

Or,

As the amount of 100l. for the given rate and time, is to the interest of 100l. for that time: so is the given sum or debt to the discount required.

Or,

In *federal money,* divide the given sum by the amount of D.100 for the given time and rate; point off from the right of the quotient two places less than in division of decimals for the present worth

EXAMPLES.

1. What is the discount of $\left\{\begin{array}{l}\text{£. 635 17s.}\\ \text{D.2119 50c.}\end{array}\right\}$ due 2 years hence, at $5\frac{1}{2}$ per cent. per annum?

Interest of 100l. per annum=5 10
2 years.

11
Add 100

As £.111 : 11 :: £.635 17s. : £.63 0s. 2d. disc. Ans.

Or,

As £.111 : £.100 :: £.635 17s. : £.572 16s. $9\frac{1}{4}$d. present worth.

And £.635 17s.—£.572 16s. $9\frac{1}{4}$d.=£.63 0s. $2\frac{1}{4}$d. discount.

In federal money.

As D.111 : D.11 :: D.2119 50c. : D.210 04c. $0\frac{1}{2}$m.=discount. *Or,*

As 111 : 100 :: 2119 50 : $\frac{2119{\cdot}5\times100}{111}$=D.1909 45c. $9\frac{1}{2}$m.

=present worth; and 2119·5−1909·4595=210·0405=discount as before.

Or, $\frac{2119{\cdot}5}{111}$=19·094595; and 1909·4595=present worth, as before.

2. What is the present worth of D.350 payable in half a year, discounting at 6 per cent. per annum? Ans. D.339 80c. 5m.

3. What is the present worth of 65l. due 15 months hence, at 6l. per cent. per annum? Ans. £.60 9s. $3\frac{1}{2}$d.

4. What is the discount on £.97 10s. due January 22, this being September 7th. reckoning interest at 5l. per cent? Ans. £.1 15 11.

5. What ready money will discharge a debt of D.1595 due 5 months and twenty days hence, at 6 per cent? Ans. D.1541 32c. 6m.

6. Bought a quantity of goods for D.250, ready money, and sold them for D.300 payable 9 months hence: What was the gain, in ready money, supposing discount to be made at 6 per cent?

Ans. D.37 8c. $1\frac{1}{4}$m.

7. What

7. What is the present worth of D.960, payable as follows; viz. $\frac{1}{2}$ at 3 months, $\frac{1}{3}$ at 6 months, and the rest at 9 months, supposing the discount to be made at 6 per cent? Ans. D.936 70c.

Rule II.

As any sum of money, at 6 per cent. per annum, will double, at simple interest, in 200 months; therefore,

To 200 add the number of months for which the discount is required, by which sum divide the product of the money and time, (in months,) and the quotient will be the discount.

Examples.

1. What is the discount of D.150 75c. for a year?

```
 200    150·75
+ 12     × 12
 ———   ———————       D. c. m.
212)1809·00(8·533=8·53 3 discount, Ans.
    1696
    ————
     1130
     1060            150·75
     ————              8·533
      700            ———————
      636 Present worth 142·217
      ———
       640
       636
       ———
         4
```

2. What is the present worth of D.426 55c. at 6 per cent. to be paid 8 months hence? Ans. D.410 14c. 5m.

3. What is the discount of 361l. 15s. 6d. to be paid 1 year and 8 months hence? Ans. £.32 17s. $9\frac{1}{4}$d.

ABBREVIATIONS IN DISCOUNT.

Any principal to be discounted for one year, at any of the following rates, (or for any rate and time, whose product is equal to any of the following rates) being (multiplied by the multiplier, if any, and) divided by the corresponding divisor, the quotient will be the discount.

count.	Rates.	
At	$1\frac{1}{4}$	÷81 (or by 9 and 9)
	2	÷51
	$2\frac{1}{2}$	÷41
	4	÷26
	5	÷21 (or by 7 and 3)
	6	÷53 and × 3
	$7\frac{1}{2}$	÷43 and × 3
	8	÷27 and × 2 (or × 2 and ÷9 and 3)
	$8\frac{1}{3}$	÷13
	10	÷11
	12	÷28 and × 3 (or × 3, and ÷7 and 4)
	$12\frac{1}{2}$	÷ 9

Examples.

EXAMPLES.

1. How much must I abate of 5394l. 10s. due 3 years hence, at $2\frac{2}{3}$ per cent. per annum?

$$
\begin{array}{ll}
2\frac{2}{3} & \pounds5394\ \ 10s. \\
\times 3 & \quad\quad 2 \\
\hline
 & 27 \div 3 = 9)10789\ \ 0 \\
8,\ \text{therefore}, \times 2,\ \text{and} \div 27 & 3)1198\ \ 15\ \ 6\frac{1}{2} \\
 & \pounds.399\ \ 11\ \ 10\ \text{Ans.}
\end{array}
$$

2. What is the discount of D.546 62c. 5m. for $8\frac{1}{3}$ years, at 1 per cent. per annum, (or for 1 year, at $8\frac{1}{3}$ per cent. per annum?)

$$
\begin{array}{l}
\text{D. c. m.} \\
13)546{\cdot}62\ 5 \\
\hline
\text{Ans. D.}42{\cdot}04\ 8
\end{array}
$$

3. What is the discount of D.125 at $1\frac{1}{4}$ per cent. per annum, for four years, (or, at 4 per cent. per annum, for $1\frac{1}{4}$ year?)

$$
\begin{array}{l}
1\frac{1}{4} \\
\times 4 \\
\hline
5,\ \text{therefore}, \div 21)125 \\
\qquad\qquad \text{D.5 95c. 2m. Ans.}
\end{array}
$$

*DISCOUNT BY DECIMALS.**

The sum to be discounted, the time and the ratio given, to find the present worth.

RULE.

Multiply the ratio by the time, add unity to the product for a divisor; by which sum divide the sum to be discounted, and the quotient will be the present worth.

Subtract the present worth from the principal, or sum to be discounted, and the remainder will be the discount.

Or, as the amount of 1l. for the given time, is to 1l. so is the interest of the debt for the said time, to the discount required.

Subtract the discount from the principal, and the remainder will be the present worth.

EXAMPLES.

* As in Simple Interest, let a=amount of any debt, p=prefent worth, t=time, and r=ratio; then will the following Theorems exhibit all the cafes in Difcount at Simple Interest.

I. $\frac{a}{tr+1}=p$. II. $ptr.+p=a$. III. $\frac{a-p}{tp}=r$. IV. $\frac{a-p}{rp}=t$.

Note. When the ratio is ·06 per cent. per annum, and the given time is expreffed in months, whether lefs or more than a year, if the debt be divided by 1 plus half as many hundredths of an unit, as there are months in the given time, the quotient will be the prefent worth.—Thus, for 1 month $\frac{a}{1{\cdot}005}$—2, months $\frac{a}{1{\cdot}01}$—3, months $\frac{a}{{\cdot}015}$, 36 months $\frac{a}{1{\cdot}18}$, 42 months, $\frac{a}{1{\cdot}21}$, &c. &c.

EXAMPLES.

1. What is the present worth of 600l. due 3 years hence, at 6l. per cent. per annum?

First Method.

Ratio=·06
Multiply by the time= 3

Product=·18
Add 1·

Divisor=1·18)600(508·4745 present worth.

Or, $\frac{600}{\cdot06\times3+1}$=£.508 9s. $5\frac{3}{4}$d. Ans.

Present worth=508·4745=£.508 9s. $5\frac{3}{4}$d. which, subtracted from the principal, will give the discount =£.91 10s. $6\frac{1}{4}$d.

Second Method.

Ratio=·06
Multiply by 3   As 1·18 : 1 :: 108
                                    1

·18
Add 1·   1·18)108·00(91·5254

Amount of 1l. for the given time =1·18

And 600×06×3=108=interest of the debt for the given time.—Discount=91·5254=£.91 10s. 6d. which taken from the principal will leave the present worth=£.508 9s. 6d.

2. What is the present worth of D.558 62c. 5m. due 2 years hence, at $4\frac{1}{2}$ per cent. per annum?

First Method.

Ratio=·045
×Time= 2·

·09
+ 1·

Divisor=1·09)558·625(512·5=present worth.
545

136
109

272
218

545
545

...

Or, $\frac{558\cdot625}{\cdot045\times2+1}$=D.512·5 Ans.

And

And D·558·625—D.512·5=D.46 12c. 5m.=discount. Or, As D.1·09 (=amount of D.1 for the given time) : D.1 :: D.50.27625 (= interest of the debt for the given time) : D.46 125=discount, as above. And, D.558·625—D.46·125=D 512·5=present worth, as above.

3. Required the present worth and discount of D.4125 50c. at $6\frac{3}{4}$ per cent. per annum, due 18 months hence? D. c. m.

Ans. { present worth 3746 19 $7\frac{1}{2}$
{ discount 379 30 $2\frac{1}{2}$

4. What ready money will discharge a debt of 1354l. 8s. due 3 years, 3 months, and 12 days hence, at $5\frac{7}{8}$l. per cent. per annum? Ans. £.1135 7s. 9d.

BARTER

IS the exchanging of one commodity for another, and teaches traders to proportion their quantities without loss.

CASE I.

When the quantity of one commodity is given, with its value, or that of its integer, that is, of 1lb. 1cwt. 1yd. &c. as also the value of the integer of some other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of selling it.

Rule.—Find the value of the given quantity by the concisest method, then find what quantity of the other, at the rate proposed, you may have for the same money: *Or,* if the quantity be given, find from thence the rate of selling it. *Or,* As the quantity of one article is to its price, so, *inversely,* is the quantity of the other to its price. *Or,* as the price of one article is to its quantity; so, *inversely,* is the price of the other to its quantity.

Examples.

1 How much tea at 9s. 6d per lb. must be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon?

		Galls.
3d.	$\frac{1}{4}$	156
$\frac{1}{2}$	$\frac{1}{6}$	12
		1872
		39
		6 6
9s. 6d.=114d.		1917 6
		12
		23010

d. lb. d. lb. oz.
As 114 : 1 :: 23010 : 201 $13\frac{54}{114}$ Ans.
price. quan. price. quan.

s. d. gals. s. d. lb. oz.
Or, As 12 $3\frac{1}{2}$: 156 : 9 6 : 201 $13\frac{54}{114}$ Ans. as before.

2. How

2. How much cloth, at 15s. 8d. per yard, must be given for 5cwt. 3qrs. 19lbs. of steel, at 5 guineas per cwt.? Ans. 52yds. 3qrs. 2n.

3 Suppose A has 350 yards of linen, at 25c per yard, which he would truck with B for sugar, at D.5 per cwt. How much sugar will the linen come to? Ans. 17cwt. 2qrs.

4. A has broadcloths at D.44 per piece, and B. has mace, at D.1 42c. per lb.: How many pounds of mace must B give A for 35 pieces of cloth? Ans. 1084½lbs.

5. A has 7½ cwt. of sugar at 12 cents per lb. for which B gave him 12½ cwt. of flour: What was the flour rated at per lb.?

Ans. 7c. 2m.

CASE II.

If the quantities of two commodities be given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.

Rule.—Find the separate values of the two given commodities; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quality and rate may be easily found.

Examples.

1. Two merchants barter; A has 30cwt. of cheese, at 23s. 6d. per cwt. and B has 9 pieces of broadcloth, at 3l. 15s. per piece: Which must receive money, and how much?

Ans. B. must pay A. £1 10s.

2. A and B would barter; A has 150 bushels of wheat, at D.1 25c. per bushel, for which B gives 65 bushels of barley, worth 62½c. per bushel, and the balance in oats at 37½c. per bushel: What quantity of oats must A receive from B? Ans. 391⅔ bushels.

CASE III.

Sometimes, in bartering, one commodity is rated above the ready money price; then, to find the bartering price of the other, say,

As the ready money price of the one, is to its bartering price; so is *that* of the other, to *its* bartering price: Next, find the quantity required, according to either the bartering or ready money price.

Examples.

1. A has ribbands at 2s. per yard ready money; but in barter he will have 2s. 3d. B has broadcloth at 32s. 6d. per yard ready money; at what rate must B value his cloth per yard, to be equivalent to A's bartering price, and how many yards of ribband, at 2s. 3d. per yard, must then be given by A for 488 yards of B's broadcloth?

Ans. B's broadcloth, at £.1 16s. 6½d. per yd. 7930 yds. ribband.

2. A and B barter; A has 150 gallons of brandy, at D 1 37½c. per gallon ready money, but in barter he will have D.1 50c.; B has linen at 44c. per yard ready money; how must B sell his linen per yard in proportion to A's bartering price, and how many yards are equal to A's brandy?

Ans. barter price is 48c. and he must give A 468 yds. 3qrs.

3. P and Q barter; P has Irish linen, at 60c. per yard, but in barter he will have 64c Q delivers him broadcloth at D.6 per yard, worth only D 5 50c per yard: Pray which has the advantage in barter, and how much linen does P give Q for 148 yards of broadcloth?

c. c. D. c. D. c.
As 60 : 64 :: 5 50 : 5 $86\frac{2}{3}$; therefore, Q by selling at D.6 has the advantage. Then,

D. yds. c. yds. qrs.
As 6 : 148 :: 64 : 1387 2 linen, Ans.

4. A has 200 yards of linen, at 1s. 6d. ready money per yard, which he barters with B, at 1s. 9d. per yard, taking buttons at $7\frac{1}{2}$d. per gross, which are worth but 6d.: How many gross of buttons will pay for the linen, who gets the best bargain, and by how much, both in the whole, and per cent.?

Yd. d. Yds. d. d. Gross. d. Gross. Yd. d. Yds. £.
As 1 : 21 :: 200 : 4200. As $7\frac{1}{2}$: 1 :: 4200 : 560. As 1 : 18 :: 200 : 15
gr. d. gr. £. [value of A's linen.
As 1 : 6 :: 560 : 14 value of B's goods. So that B gains 1l. of A.
£. £. £. £. s. d.
As 14 : 1 :: 100 : 7 2 10 per cent.

5. A has linen cloth, at 30c. per yard, ready money, in barter 36c. B has 3610 yards of ribband. at 22c. per yard ready money, and would have of A D.200 in ready money, and the rest in linen cloth; what rate does the ribband bear in barter per yard, and how much linen must A give B?

Ans. The rate of ribband is 26c. 4m. per yard, and B must receive $1980\frac{1}{3}$ yards of linen, and D.200 in cash.

LOSS AND GAIN

IS an excellent rule, by which merchants and traders discover their profit, or loss per cent. or by the gross: It also instructs them to raise or fall the price of their goods, so as to gain or lose so much per cent. &c.

CASE I.

To know what is gained or lost per cent.

RULE.

First see what the gain or loss is, by subtraction; then, as the price it cost, is to the gain or loss: so is 100l. to the gain or loss per cent.

Or, in *federal money*, annex two cyphers to the gain or loss, and divide by the cost for the gain or loss per cent.

EXAMPLE.

1. If I buy serge at 90c. per yard, and sell it again at D.1 2c. per yard: What do I gain per cent. or in laying out D.100?

Sold

c. c. D. D.

Sold for D.1·02 As 90 : 12 :: 100 : $13\frac{1}{3}$ per cent. gain, Ans.
Cost ·90

Gain ·12 per yard.

Or, 1·02—·90=·12=gain per yard ; and $\frac{12·00}{·9}$ =$13\frac{1}{3}$ per cent. gain, [as before.

N. B. The first questions in the several cases, serve to elucidate each other.

2. If I buy serge at D.1 2c. per yard, and sell it again at 90c. per yard : What do I lose per cent. or in laying out D.100?

D. c. D. c c. D D. c. m.

Cost 1·02 As 1 02 : 12 :: 100 : 11 76 5 per cent. loss, Ans.
Sold for ·90

Loss ·12 Or, $\frac{12·00}{1·02}$ =11·765 per cent. loss, Ans. as before.

3. If I buy a cwt. of tobacco for 9l. 6s. 8d. and sell it again at 1s. 10d. per lb. do I gain or lose, and what per cent.?

lb.
112

		£. s. d.
Sold for	10 5 4	
Cost	9 6 8	
	0 18 8	gained in the gross.

£.
| 2d. | $\frac{1}{12}$ | 11 4 value at 2s. per lb.

0 18 8 value at 2d. per lb.

10 5 4 value at 1s. 10d. per lb.

£. s. d. s. d. £. £.
As 9 6 8 : 18 8 :: 100 : 10 Ans. 10 per cent. gain.

4. A draper bought 60 yards of cloth at D.4 50c. per yard, and 38 yards of cloth at D.2 50c. per yard, and sold them, one with another, at D 4 25c per yard : Did he gain or lose, and what per cent.

60	yards at	D.4 50c.	per yard =	D.270
38	yards at	2 50	per yard =	95
98	yards cost - - - - - - - - - - -			365
	which subtract from 98 yds. at D.4 25c.=			416·50
	gain in the gross		=	51·50

D. D c. D. 5150·00 D. c.
Then, as 365 : 51·50 :: 100 : $\frac{5150·00}{365}$ = 14·11 gain per cent. Ans.

5. Bought sugar at $6\frac{1}{2}$d per lb. and sold it at 2l. 3s. 9d. per cwt. What was the gain or loss, per cent.?

lb. d lb. £. s. d.
As 1 : $6\frac{1}{2}$ :: 112 : 3 0 8

Prime cost £.3 0 8 per cwt. £. s. d. s. d. £. £. s. d.
Sold at 2 3 9 per cwt as 3 0 8 : 16 11 :: 100 : 27 17 $8\frac{1}{4}$ [loss per cent. Ans.

Lost £.0 16 11 in the whole.

6. At

2...I

6. At 4s. 6d. in the pound profit: How much per cent.?

£. s. d. £. £. s.
As 1 : 4 6 :: 100 : 22 10 Ans.

7. If I buy candles at 1s. 6d. per lb. and sell them again at 2s. per lb. and allow 3 months for payment: What do I gain per cent.?

d. d. £ £. s. d. Mo. £. Mo. £. s.
As 18 : 24 :: 100 : 133 6 8; then by discount, As 12 : 6 :: 3 : 1 10

£. s. £. s. £. s. d. £. s. d.
Then, as 101 10 : 1 10 :: 133 6 8 : 1 19 $4\frac{3}{4}$, which taken from 133l. 6s. 8d. leaves 131l. 7s. $3\frac{1}{4}$d therefore, Ans. £.31 7s. $3\frac{1}{4}$d.

8. If I buy cloth at 13s per yard, on 8 months credit, and sell it again at 12s. ready money, do I gain, or lose, and what per cent.?

Mo. £. Mo. £. £. s. £. s. d.
As 12 : 6 :: 8 : 4 As 104 : 13 :: 100 : 12 6 : So that 13s. on 8 months credit at 6l. per cent. is equal to 12s. 6d. ready money; then,

	s. d.		s d. d. £. £.
Prime cost	12 6	ready money,	As 12 6 : 6 :: 100 : 4
Sold at	12 0	ready money,	Ans. lost £.4 per cent.
Lost	6	in the yard.	

9. If I buy gloves at D.1 25c. per pair: How long credit must I have, to gain D.13 per cent. when I sell them at D.1 36c. per pair?

	D.c.	
Sold at	1·36	
Prime cost	1·25	
Gained	·11	per pair.

D.c. c. D. D.c.
As 1·25 : ·11 : 100 : 8·80 gain per cent. rdy. mo.

D. D.c. D.c.
Then, 13—8·80=4·20 Now,

D. Mo. D.c. Mo. days.
As 6 : 12 :: 4·20 : 8 12 Ans.

In casting up the amount of goods bought, imported or exported; to the prime cost of such goods we must add all the charges upon them, in order to fix the price they stand us in.

10. Suppose I import from France, 12 bales of cloth, containing 10 pieces each, which, with the charges there, amounted to D.360: I pay duty here 92c. per piece, for freight D.12 and portage D.1 25c.; What does it stand me in per piece, and how must I sell it per piece to gain D.10 per cent.

12 bales.
10
———
120 pieces.

	D. c.
First cost	360·
Duty	110·40
Freight	12·
Porterage	1·25
Whole cost	483·65

Pieces. D. c. Piece. D.c.
As 120 : 483·65 :: 1 : 4·03 cost per piece

D. Pieces. D.c. c. m.
Again, as 100 : 10 :: 4·03 : 40 3 gain.
40 3
———
D.4·43 3 the price at which it must be sold per piece.

CASE

CASE II.

To know how a commodity must be sold, to gain or lose so much per cent.

RULE.—As 100l. is to the price; so is 100l. with the profit added, or loss subtracted, to the gaining or losing price. Or,

In *federal money*, multiply 100 dollars added to the gain, or less by the loss per cent. by the cost; and pointing off the two right hand figures of the product gives the answer.

EXAMPLES.

1. If I buy a quantity of serge, at 90c. per yard: How must I sell it per yard to gain $13\frac{1}{3}$ per cent.?

D. D. c c. D. c.
As 100 : 113 $33\frac{1}{3}$ :: 90 : 1 2 Ans.

D. c. c. D.
Or, 113 $33\frac{1}{3} \times 90 = 102$; and pointing off two right hand places, D.1·02, Ans. as before.

2. If a barrel of powder cost 4l. how must it be sold to lose 10l. per cent

£. £. £.
As 100 : 4 :: 90
4
100)360(3
300
60
20
100)1200(12 Ans. £.3 12s.
1200

Or thus:
90
4
£ 3|60
20
s.12|00

3. Bought cloth, at D.2 50c. per yard, which not proving so good as I expected, I am content to lose $17\frac{1}{2}$ per cent. by it: How must I sell it per yard? Ans. D.2 6c. $2\frac{1}{4}$m.

4. If 120lb. of steel cost 7l. How must I sell it per lb. to gain $15\frac{1}{2}$l. per cent.?

lb. £. lb. s. d £. s. d. £. s. d.
As 120 : 7 :: 1 : 1 2 As 100 : 1 2 :: $115\frac{1}{2}$: 1 4 per lb. Ans.

5. A gentleman bought 10 tons of iron for 200l. the freight and duties came to 25l. and his own charges to 8l. 6s. 8d.; How must he sell it per lb to gain 20l. per cent by it?

£. £. £. s d. £. s. d. £. s. d. £. s. d. £.
As 100 : 20 :: 233 6 8 : 46 13 4 Then, 233 6 8+46 13 4=280

Tons. £. lb. d.
As 10 : 280 :: 1 : 3 per cent. Ans.

6. If a bag of cotton, weighing 8 cwt. 0qrs. 20lb. cost D.45 55c. How must it be sold per cwt. to lose D.8 per cent.?

cwt.qrs.lb. D. c. cwt. D.c.m. D.c.m. D c.m.
As 8 0 20 : 45·55 :: 1 : 5·56 9 As 100 : 92 :: 5·56 9 : 5·12 3 Ans.

7. Bought fish in Newburyport, at 10s. per quintal, and sold it at Philadelphia, at 17s. 6d. per quintal; now, allowing the charges at

an

an average, or one with another, to be 2s. 6d. per quintal, and considering I must lose 20l. per cent. by remitting my money home; what do I gain per cent. ?

Selling price 17 6 Philadelphia currency, per quintal.
Charges 2 6 ditto.

15 0 ditto.

£. s. £. s.
As 100 : 15 :: 80 : 12 New England currency.
Sold at 12s. per quintal.
Bought at 10s. per quintal.

Gained 2s. per quintal.

s. s. £. £.
As 10 : 2 :: 100 : 20 per cent. gained, Ans.

8. Bought 50 gallons of brandy, at 75c. per gallon, but, by accident, 10 gallons leaked out: At what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of 10 per cent.? Ans. D.1 3c. $1\frac{1}{4}$m.

CASE III.

When there is gain or loss per cent. to know what the commodity cost.

Rule. As 100l. with the gain per cent. added, or loss per cent. subtracted, is to the price; so is 100l. to the prime cost. Or,

In *federal money*, divide the price with two cyphers annexed by D.100 added to the gain, or less by the loss, per cent. for the answer.

Examples.

1. If 1 yard of cloth be sold, at D.1 2c. and there is gained $13\frac{1}{3}$ per cent. What did the yard cost ?

D. D. c. D. c.
As $\overline{100+13\frac{1}{3}}$: 1 2 :: 100 : 90 prime cost, Ans.

Or, $\frac{102\cdot00}{113\cdot33\frac{1}{3}} = \cdot9$, Ans. as before.

2. If 12 yards of cloth are sold at 15s. per yard, and there is 7l. 10s. loss per cent. in the sale : What is the prime cost of the whole ?

Yd. s. Yds. £. £. s. £. £. £. s. d.
As 1 : 15 :: 12 : 9 As 92 10 : 9 :: 100 : 9 14 7 Ans.

3. If 40lb of chocolate be sold at 25c. per lb and I gain 9 per cent. What did the whole cost me ? Ans. D.9 17c. 4m.+

4. If $19\frac{1}{2}$ cwt. sugar be sold at D.14 50c. per cwt. and I gain D.15 per cent. : What did it cost per cwt

D. D c. D. D. c. m.
As 115 ; 14·50 :: 100 12·60 8 Ans.

CASE IV.

If by wares sold at such a rate, there is so much gained or lost per cent. to know what would be gained or lost per cent. if sold at another rate.

Rule.—As the first price is to 100l. with the profit per cent. added,

ed, or loss per cent. subtracted; so is the other price, to the gain or loss per cent. at the other rate.

N. B. If your answer exceed 100, the excess is your gain per cent. but if it be less than 100, the deficiency is your loss per cent.

EXAMPLES.

1. If cloth, sold at D.1 2c. per yard, be $13\frac{1}{3}$ profit per cent. What gain or loss per cent shall I have, if I sell the same at 90c. per yard?

D. c. D. c. D.
As 1 2 : $113\frac{1}{3}$ :: 90 : 100

And, 100−100=0, Ans. I neither gain, nor lose.

2. If cloth, sold at 4s. per yard, be 10l. per cent. profit: What shall I gain or lose per cent. if sold at 3s. 6d. per yard?

```
  s.   £.     s. d.
As 4 : 110 :: 3 6
  12          12
  --          --
  48          42
             110
            ----
    48)4620(96¼
       432
       ---
        300
        288
        ---
         12
```

£. £. £.
Then, $100-96\frac{1}{4}=3\frac{3}{4}$

Ans. I lost £.$3\frac{3}{4}$ per cent. by the last sale.

3. If I sell a gallon of wine for D.1 50c. and thereby lose 12 per cent.: What shall I gain or lose per cent. if I sell 4 gallons of the same wine for D.6 75c.?

D. D. D.c. D.
As 6 : 88 :: 6·75 : 99 And 100—99=1 per cent loss.

4. I sold a watch for 50l. and by so doing, lost 17l. per cent. whereas in trading I ought to have cleared 20l. per cent. How much was it sold under its real value?

£. £. £. £. s. d. — £. £. s. d. £. £. s. d.
As 83 : 50 :: 100 : 60 4 $9\frac{3}{4}$ As 100 : 60 4 $9\frac{3}{4}$ :: 120 : 72 5 $9\frac{1}{4}$

£. s. d. £. £. s. d.
Then, 72 5 $9\frac{1}{4}$—50=22 5 $9\frac{1}{4}$ Ans.

EQUATION OF PAYMENTS

IS the finding a time to pay, at once, several debts, due at different times, so that no loss shall be sustained by either party.

RULE

Rule I.*

Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time, or that required.

Examples.

1. A owes B 380D. to be paid as follows, viz. 100D. in 6 months, 120D. in 7 months, and 160D. in 10 months: What is the equated time for the payment of the whole debt?

$$\begin{array}{r} 100\times 6= 600 \\ 120\times 7= 840 \\ 160\times 10=1600 \\ \hline \end{array}$$

$$100+120+160=380)3040(8 \text{ months, Ans.}$$
$$3040$$

2. A owes B 104l. 15s. to be paid in $4\frac{1}{2}$ months, 161l. to be paid in $3\frac{1}{2}$ months, and 152l. 5s. to be paid in 5 months: What is the equated time for the payment of the whole? Ans. 4 months and 8 days.

3. There is owing to a merchant 998l. to be paid, 178l. ready money, 200l. at 3 months, and 320l. in 8 months; I demand the indifferent time for the payment of the whole? Ans. $4\frac{1}{2}$ months.

4. The sum of 164D. 16c. 6m. is to be paid, $\frac{1}{2}$ in 6 months, $\frac{1}{3}$ in 8 months, and $\frac{1}{6}$ in 12 months: what is the mean time for the payment of the whole? Ans. $7\frac{2}{3}$ months.

Rule II.

See, by rule 1st, at what time the first man, mentioned, ought to pay in his whole money; then as his money is to his time, so is the other's money, to his time, inversely, which, when found, must be added to, or subtracted from, the time at which the second ought to have paid in his money, as the case may require, and the sum, or remainder, will be the true time of the second's payment.

Examples.

1. A is indebted to B 150D. to be paid, 50D. at 4 months, and 100D. at 8 months: B owes A 250D. to be paid at 10 months: It is agreed between them that A shall make present pay of his whole debt,

* This rule is founded upon a supposition that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Some, who defend this principle, have endeavoured to prove it to be right by this argument; that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due; but this cannot be the case; for though, by keeping a debt after it is due, there is gained the interest of it for that time; yet, by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest, and therefore the rule is not accurately true; however, in most questions, which occur in business, the errour is so trifling, that it will always be made use of as the most eligible method.
The true rule will be given in Equation of Payments by Decimals.

debt, and that B shall pay his so much the sooner, as to balance that favour; I demand the time at which B must pay the 250D. reckoning simple interest.

$$50\times4=200$$
$$100\times8=800$$

$50+100=15|0)100|0(6\frac{2}{3}$ months, A's equated time.
$$90$$
$$10$$

D. mo. D. mo. mo. mo. mo.
As 150 : $6\frac{2}{3}$ :: 250 : 4 Then, $10-4=6$ time of B's payment.

2. A merchant has 120l. due to him, to be paid at 7 months; but the debtor agrees to pay $\frac{1}{2}$ ready money, and $\frac{1}{3}$ at 4 months; I demand the time he must have to pay in the rest, at simple interest, so that neither party may have the advantage of the other?

Debt £.120

$\frac{1}{2}=$ 60 must be paid down.
$\frac{1}{3}=$ 40 must be paid at 4 months.
$\frac{1}{6}=$ 20 unpaid.

Now, as he pays 60l. 7 months, and 40l. 3 months before they are respectively due, say, as the interest of 20l. for 1 month, is to 1 month, so is the sum of the interest of 60l. for 7 months, and of 40l. for 3 months, to a fourth number, which, added to the 7 months, will give the time for which the 20l. ought to be retained.

Ans. 2 years and 10 months.

3. A merchant has 1200D. due to him, to be paid $\frac{1}{6}$ at 2 months, $\frac{1}{3}$ at 3 months, and the rest at 6 months; but the debtor agrees to pay $\frac{1}{2}$ down: How long may the debtor detain the other half, so that neither party may sustain loss?

mo. mo.
$\frac{1}{6}\times2=0\frac{1}{3}$
$\frac{1}{3}\times3=1$
$\frac{1}{2}\times6=3$

Equated time$=4\frac{1}{3}$

Now, as $\frac{1}{2}$ was paid $4\frac{1}{3}$ months before it was due, it is reasonable that he should detain the other $\frac{1}{2}$ $4\frac{1}{3}$ months after it became due, which, added, gives $8\frac{2}{3}$ months, the true time for the second payment.

EQUATION OF PAYMENTS BY DECIMALS.

Rule.*

1. To the sum of both payments add the continual product of the first payment, the ratio, and the time between the payments, and call this the first number. 2. Multiply

* Suppose a sum of money be due immediately, and another at the expiration of a certain given time forward, and it is proposed to find a time, so that neither party shall sustain loss.

Now, it is plain that the equated time must fall between the two payments; and that what is gotten by keeping the first debt after it is due, should be equal to what is lost by paying the second debt before it is due; but the gain arising from the keeping

2. Multiply twice the first payment by the ratio, and call this the second number.

3. Divide the first number by the second, and call the quotient the third number.

4. Call the square of the third number the fourth number.

5. Divide the product of the second payment and time between the payments by the product of the first payment and the ratio, and call the quotient the fifth number.

6. From the fourth number take the fifth, and call the square root of the difference the sixth number.

7. Then the difference of the third and sixth numbers is the equated time, after the first payment.

EXAMPLE.

There are 100D. payable in 2 years, and 106D at 6 years hence; what is the equated time, allowing simple interest, at 6 per cent. per annum?

1st. payment=100 — 1st. payment 100
Ratio=·06 — Multiply by 2

6·00 — 200

Time between the payments=4 years. Mult. by the ratio=·06

24 — 12.00=2d. num.

Add both payments= {100, 106}

Div. by the 2d. num.=12)230=1st. number.

19·166+=3d. number.
19·166+

3d. number squared=367·335556=4th. number.
2d. payment=106
Multiplied by the time= 4

1st. payment mult. by the ratio=6)424= {product of the 2d. payment and time between the payments.}

70·666+=5th number.

From the 4th. number=367·335556
Take the 5th. number= 70·666666

Carried over.

keeping of a sum of money after it is due, is evidently equal to the *interest* of the debt for that time: And the loss, which is sustained by the paying of a sum of money before it is due, is evidently equal to the *discount* of the debt for that time: Therefore it is obvious that the debtor must retain the sum immediately due, or the first payment, till its *interest* shall be equal to the *discount* of the second sum for the time it is paid before due; because in that case the gain and loss will be equal, and consequently neither party can be a loser.

Brought over. . . . . . .

296·668890(17·224 sqr. root=6th. num.

From the 3d. number=19·166

Take the 6th. number=17·224

1·942=equated time from the first payment; therefore 3·942 years=3y. 11m. 9d.=whole equated time.

Or, $$\frac{100+106+\overline{100\times\cdot06\times4}}{100\times2\times\cdot06}-\left|\left(\frac{100+106+\overline{100\times\cdot06\times4}}{100\times2\times\cdot06}\right)^2-\frac{106\times4}{100\times\cdot06}\right|^{\frac{1}{2}}=1\cdot942.$$

POLICIES OF INSURANCE.

INSURANCE is a security, or assurance, by mean of a writ called a *Policy*, to indemnify the insured of such losses as shall be specified in the policy subscribed by the insurer, or insurers, by which the under writers oblige themselves to make good and effectual the property insured, in consideration of a certain premium at a stipulated rate per cent. (which varies according to the risk) to be immediately paid down, or otherwise secured according to the tenor of the agreement.

The average loss is 10 per cent.; that is, if the insured suffer any damage or loss, not exceeding 10 per cent. he bears it himself, and the insurers are free.

A policy should be taken out for a sum sufficient to cover the principal and premium, and the business of this rule is, in general, to calculate what that sum should be.

CASE I.

When the premium, at a certain rate per cent. for insuring a sum, is required, the operation is the same as in interest, or commission.

EXAMPLES.

1. What is the premium upon 537l. 15s. 9d. at $6\frac{1}{2}$ per cent.?

	£.	s.	d.
	537	15	9
			$6\frac{1}{2}$
	3226	14	6
$\frac{1}{2}=$	268	17	$10\frac{1}{2}$
	34|95	12	$4\frac{1}{2}$
	20		
	19|12		
	12		
	1|48		
	4		
	1|94		

Ans. £.54 19s. $1\frac{1}{2}$d. nearly.

2. What

2. What is the premium upon D.375, at $7\frac{1}{2}$ per cent.?

$$\begin{array}{r} \text{D.}375 \\ \cdot 075 \\ \hline 1875 \\ 2625 \\ \hline \text{Ans. D.}28\cdot 125 \end{array}$$

CASE II.

To find the sum for which a policy should be taken out to cover a given sum.

Rule.—Take the premium from 100l (or in *federal money* D.100) and say, As the remainder is to 100, so is the sum adventured to the policy.* Or,

In *decimals*, take the premium from 100, annex two cyphers to the sum to be covered, and divide by the remainder for the policy.

Examples.

1. It is required to cover 759l. premium 8 per cent.: For what sum must the policy be taken?

$$\begin{array}{l} 100 \\ 8 \\ \hline 92 : 100 :: 759 \\ 100 \end{array}$$

$$\begin{array}{r} \text{£.} \\ 92)75900(825 \text{ Ans.} \\ 736 \\ \hline 230 \\ 184 \\ \hline 460 \\ 460 \end{array}$$

Or, $\frac{75900}{92}$ = £.825, Ans. as before.

2. A merchant sent a vessel and cargo to sea, valued at D.5760: What sum must the policy be taken out for, to cover this property, premium $19\frac{1}{2}$ per cent.?

100

* Now it is plain, that if I want to recover 92l. I must in this case, insure upon 100l.; therefore, to recover 759l. I must insure upon 825l.; for when 8 per cent. for premium is deducted, I shall have 759l. remaining nett.

$$\begin{array}{rl} \text{For £.}825 = & \text{sum insured upon at 8 per cent.} \\ 66 = & \text{premium to be deducted.} \\ \hline 759 = & \text{the first outset.} \end{array}$$

In this and the following cases, let $x=100$, p=premium, a=amount to be insured upon, and s=sum to be covered; then, $x-p : x :: s : a$, or $\frac{xs}{x-p}=a$.

100
19·5
——   D.   D.   D. c.
80·5 : 100 :: 5760 : 7155·28—Ans.

Or, $\frac{576000}{80\cdot5}$ = D.7155·28c.—Ans. as before.

CASE III.

When a policy is taken out for a certain sum in order to cover a given sum.

To find the premium, say, as the policy is to the covered sum; so is £.100 or D.100, to a fourth number, which, being taken from 100, will leave the premium.* Or,

In *decimals*, divide the sum covered, with two cyphers annexed, by the policy; subtract the quotient from 100, the remainder is the premium.

EXAMPLES

1. If a policy be taken out for 1250l. to cover 500l. What is the premium per cent.? 1250 : 500 :: 100

100
——
1250)50000(40 and £.100—40=£.60 Ans.

Or, $\frac{50000}{1250}$ = 40, &c. as before.

2. If a policy be taken out for D.781·25, to cover D.625: Required the premium per cent?

D. c.   D.   D.   D. c.
As 781·25 : 625 :: 100 : 87·50 And, 100—87·5=12·5, or $12\frac{1}{2}$ [per cent. premium, Ans.

Or, $\frac{62500}{781\cdot25}$ = 87·5, &c. as before.

CASE IV.

When the policy for covering any sum and the premium per cent. are given, to find the sum to be covered.

RULE.—Deduct the premium per cent. from 100, and say, As 100 is to the remainder, so is the policy to the sum required to be covered. Or,

In *decimals*, Multiply the policy by the remainder found as before, and point off two right hand places in the product for the answer.

EXAMPLES

1. If a policy be taken out for 1250l. at 60 per cent.: What is the adventure, or sum to be covered?†

100
60
——
100 : 40 :: 1250
40
—— £.
100)50000(500 Ans.

Or, $1250 \times \overline{100-60} = 50000$, and, pointing off two places, 500·00 Ans. as before.

2. If

* $a : s :: x : \overline{x-p}$, or $x - \frac{sx}{a} = p$.

† $x : \overline{x-p} :: a : s$, or $\frac{a \times \overline{x-p}}{x} = s$.

2. If a policy be taken out for D.781 25c. at $12\frac{1}{2}$ per cent. required the sum covered?

As $100 : 100-12\frac{1}{2} :: 781{\cdot}25 : \dfrac{781{\cdot}25\times\overline{100-12\frac{1}{2}}}{100} =$ D.625, Ans.

Or, $781{\cdot}25\times\overline{100-12{\cdot}5}=62500$; and $625{\cdot}00$, Ans. as before.

CASE V.

When a given sum is adventured several voyages round from one place to another, either at the same, or different risks, from place to place, and it is required to take out a policy for such a sum as will cover the adventure all round, supposing the risk out and home to be equal and tantamount to the several given risks.

Rule.

1. Raise 100l. or D. to that power denoted by the number of risks, and multiply the said power by the sum adventured, (or to be covered) for a dividend.

2. Subtract the several premiums, each, from 100l. and multiply the several remainders continually together for a divisor, and the quotient, arising from this division, will give the policy to cover the adventure the voyage round.*

Example.

A merchant adventured D.1500 from Boston to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4, from thence to Nantz, at 5, and from thence home at 6 per cent.: For what sum must he take out a policy to cover his adventure the voyage round, supposing the

* For the first Voyage.	Second Voyage.	Third Voyage.
$\overline{x-p} : x :: s : a.$	$\overline{x-p} : x :: \frac{xs}{x-p} : a.$	$\overline{x-p} : x :: \frac{xxs}{\overline{x-p}^2} : a.$
$\frac{xs}{x-p}=a.$	$\frac{xxs}{\overline{x-p}^2}=a.$	$\frac{xxxs}{\overline{x-p}^3}=a.$
$\frac{a\times\overline{x-p}}{x}=s.$	$\frac{a\times\overline{x-p}^2}{x^2}=s.$	$\frac{a\times\overline{x-p}^3}{x^3}=s.$
$x-\frac{sx}{a}=p.$	$x-\sqrt{\frac{xxs}{a}}=p.$	$x-\sqrt[3]{\frac{x^3s}{a}}=p.$ and so on

for as many voyages as may be required. Hence, making m=exponent of any given power, $\dfrac{x^m s}{\overline{x-p}\times\overline{x-p}\times\overline{x-p}, \&c.}$=sum to be insured upon, all round:—And

$x-\sqrt[m]{\dfrac{x^m\times s}{a}}$= the premium all round, tantamount to the several given premiums; s, in this Theorem being equal to the first adventure, and a=amount which will cover that adventure the voyage round.

the risk to be equal out and home, and tantamount to the several given risks?

$$\frac{100}{100-3}\times\frac{100}{100-4}\times\frac{100}{100-5}\times\frac{100\times1500}{100-6}=\text{D.}1803{\cdot}835,\ \text{Ans.}$$

CASE VI.

When a given sum is adventured several voyages round, as in the last case, either at the same, or different risks, from port to port, and the premium for the voyage round is required, tantamount to the several given rates per cent.

Rule.

1. Find the sum for which the policy must be taken, by the last case.

2. Multiply the sum adventured by 100, and divide that product by the policy.

3. Take the quotient from 100, and the remainder will be the premium per cent. on the policy, tantamount to the several premiums given in the question.

Example.

A merchant adventured D.1500 from Boston to Philadelphia, at 3 per cent.: from thence to Guadaloupe, at 4; from thence to Nantz, at 5; and from thence, home, at 6 per cent.: What will be the premium, tantamount to those given in the question, on a policy for covering the first adventure, the voyage, supposing the risks out and home equal?

In case 5, we found the policy, which would cover the adventure the voyage round, to be D.1803·835. Then, $100-\dfrac{1500\times100}{1803{\cdot}835}=$ 16·844=the premium per cent. on the policy the voyage round, and tantamount to the several given premiums.

CASE VII.

If a policy be taken out for a given sum, to cover a certain adventure, from one port to another, on to several ports, at equal premiums from one place to the other, to find what that equal premium is.

Rule.

1. Involve 100 to that power denoted by the number of risks, and multiply this power by the sum adventured, (or covered.)

2. Divide the last product by the policy.

3. Extract that root of the quotient denoted by the number of risks.

4. Take this root from 100, and the remainder will be the equal premium from one port to the other.

Example.

A merchant adventured 1500D. from Boston to Philadelphia, thence to Guadaloupe, thence to Nantz, and thence home; to cover which

which all round he took out a policy for 1803·835D. ; and the premium was equal from one place to the other : what was the premium per cent ?

$$100-\sqrt[4]{\frac{100\times100\times100\times100\times1500}{1803\cdot835}}=4\cdot507 \text{ per cent. Answer}$$

CASE VIII.

When an adventure is insured out and home at one risk, at a given rate per cent. and the voyage terminates short of what was at first intended : To find what the underwriter must receive per cent.

Rule.—1. If just half the voyage is performed, it must be considered as *two* equal risks : If one third, then, as *three* equal risks ; if but one fourth, then, as *four* risks, and so on ; and by Case 2d must be found the amount which will cover the adventure the voyage round.

2. Involve 100 to that power denoted by the number of risks, and multiply this power by the sum adventured.

3. Divide this product by the aforesaid amount.

4. Extract that root of the quotient denoted by the number of risks.

5. Take this root from 100, and the remainder will be the sum per cent. which the underwriter must receive.

Example.

A merchant covers 200D. at 6 per cent. from Newbury-Port to the West-Indies and home again ; but the voyage terminating in the West-Indies, what must the insurer receive per cent. ?

100
 6
—
94 : 100 :: 200 : 212·765957=amount to cover 200D. voyage round.

$$100\times100\times200=2000000 \text{ and } \frac{2000000}{212\cdot765957}=9400$$

and $100-\sqrt{9400}=3\cdot0465$ to be paid the insurer per cent. upon the above amount.

COMPOUND INTEREST

IS that which arises from the interest being added to the principal, and (continuing in the hands of the borrower) becoming part of the principal, at the end of each stated time of payment.

Method I.

Rule.*

Find the *amount* of the given principal, for the time of the first payment, by Simple Interest : next, find the interest of that sum, or principal,

* It may be obferved that all the computations, relating to Compound Intereft, are founded upon a feries of terms, increafing in Geometrical Progreffion, wherein the number of years affigns the index of the laft and higheft term : Therefore, as one pound is to the amount of one pound, for any given time, fo is any propofed principal, or fum, to its amount for the fame time.

cipaal, and add it as before, and thus proceed for any number of years, still accounting the last amount as the principal for the next payment. The given principal being subtracted from the last amount, the remainder will be the compound interest.

In *federal money*, multiply the principal by the rate for the first time of payment, setting the product two places more to the right than the multiplicand, and the decimal point in the product under that in the multiplicand; then find the amount, and proceed as above.

Note. It is not usually necessary to carry the work beyond mills; therefore, when the figure next beyond mills, at the right, exceeds 5, increase the number of mills 1; when it does not exceed 5, it may be omitted. The result will be exact enough for common purposes.

EXAMPLES.

1. What will { £.480 / D.1600 } amount to in 5 years, at 6 per cent per annum?

Principal 480
Rate of interest 6
28|80
20
16|00

	£.	
Principal for the 1st. year	480	0
Interest of ditto	28	16
Principal for the 2d. year	508	16
		6
	30|52	16
	20	
	10|56	
	12	
	6|72	
	4	
	2|88	

	£.	s.	d.
Prin. for the 2d. year	508	16	0
Interest for ditto	30	10	6½
Prin. for the 3d. year	539	6	6½
			6
	32|35	19	3
	20		
	7|19		
	12		
	2|31		
	4		
	1|24		

Principal for the 3d. year	£.539	6	6½
Interest for ditto	32	7	2¼
Principal for the 4th. year	571	13	8¾

	£.	s.	d.
Prin. for the 4th. year	571	13	8¾
			6
	34|30	2	4½
	20		
	6|02		
	12		
	0|28		
	4		
	1|14		

	£.	s.	d.
Prin. for the 4th. year.	571	13	8¼
Interest for ditto.	34	6	0¼
Prin. for the 5th. year.	605	19	9
			6
	36|35	18	6
	20		
	7|18		
	12		
	2|22		

Prin.

	£.	s.	d.
Principal for the 5th year	605	19	9
Interest for ditto.	36	7	2
Amount for 5 years	642	6	11
Subtract the first principal	480	0	0
Compound interest for 5 years	162	6	11

In *federal money*, thus :

	D.
Principal for the 1st. year	1600·
Rate of interest	6
Interest 1st year	96·00
Amount 1st. and prin. 2d. year	1696·
	6
Interest 2d. year	101·76
Amount 2d. year, prin. 3d.	1797·76
	6
Interest 3d. year	107·8656
Amount 3d. principal 4th.	1905·6256
	6
Interest 4th. year	114·337536
Amount 4th. principal 5th. year	2019·963136
	6
Interest 5th. year	121·19778816
Amount for 5 years	2141·16092416
Subtract 1st. principal	1600·
Compound Interest for 5 years =	541·16092416

Or, thus :

	D.
1st. principal	1600·
	6
Interest	96·00
2d. principal	1696·
	6
Interest	101·76

Interest.

	D.
Interest	101·76
3d. principal	1797·76
	6
Interest	107·866
4th. principal	1905·626
	6
Interest	114·338
5th. principal	2019·964
	6
Interest	121·198
Amount	2141·162
1st. principal	1600·
Compound Interest	541·162 nearly, as before.

2. What is the compound interest of D.740 for 6 years, at 4 per cent. per annum? Ans. D.196 33c. 6m.

3. What will 400l. amount to in 5 years, at 4l. per cent. per annum? Ans. £.486 13s. $2\frac{1}{2}$d.

4. What will $\left\{\begin{array}{l}\text{£. 150}\\ \text{D.500}\end{array}\right\}$ amount to in a year, at 2 per cent. per month? Ans. $\left\{\begin{array}{lll}\text{£. 190} & \text{4s.} & \text{5d.}\\ \text{D.634} & \text{12c.} & \text{1m.}\end{array}\right.$

Method II.

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner find the amount for every succeeding year.

When the rate is at 6 per cent. per annum.

1. Divide the principal by 20, and that quotient by 5: these quotients, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner obtain the amount for every succeeding year.

Examples.

EXAMPLES.

1. What is the amount of 480l. at 6 per cent. per annum, for 5 years?

20)480			
5) 24			
4	16		
20)508	16		amount of 1st year.
5) 25	8	$9\frac{1}{2}$	
5	1	9	
20)539	6	$6\frac{1}{2}$	ditto of 2d.
5) 26	19	$3\frac{3}{4}$	
5	7	$10\frac{1}{4}$	
20)571	13	$8\frac{1}{2}$	ditto of 3d.
5) 28	11	$8\frac{3}{4}$	
5	14	4	
20)605	19	$8\frac{3}{4}$	ditto of 4th.
5) 30	5	$11\frac{3}{4}$	
6	1	$2\frac{1}{4}$	
£.642	6	$10\frac{3}{4}$	do. of 5th. Ans.

2. Of the same sum at 5 per cent. per annum, for 5 years.

£.			
20)480			
24			
20)504			amount of 1st year.
25	4		
20)529	4		ditto of 2d.
26	9	$2\frac{1}{4}$	
20)555	13	$2\frac{1}{4}$	
27	15	$7\frac{3}{4}$	
2)583	8	10	ditto of 4th.
29	3	$5\frac{1}{4}$	
£.612	12	$3\frac{1}{4}$	do. of 5th. Ans.

Note. The same may be done in *federal money*, but the first method is generally more easy.

COMPOUND INTEREST BY DECIMALS.

A Table of the Amount of £.1 or 1D. at $\frac{1}{2}$ per cent. per month, as practised at the Banks.

Months.	£. or D. Dec. pts.	Months.	£. or D. Dec. pts.	Months.	£. or D. Dec. pts.
1	1·005	5	1·025	9	1·045
2	1·01	6	1·03	10	1·05
3	1·015	7	1·035	11	1·055
4	1·02	8	1·04	12	1·06

A Table of the Amount of £.1 or D.1 from 1 Day to 31 Days, at 6 per cent. per annum.

Days.	£. or D Dec. parts.	Days.	£. or D. Dec. parts.	Days.	£. or D. Dec. parts.
1	1·00016	12	1·00197	22	1·00361
2	1·00032	13	1·00213	23	1·00378
3	1·00049	14	1·0023	24	1·00394
4	1·00065	15	1·00246	25	1·0041
5	1·00082	16	1·00263	26	1·00427
6	1·00098	17	1·00279	27	1·00443
7	1·00115	18	1·00295	28	1·0046
8	1·00131	19	1·00312	29	1·00476
9	1·00147	20	1·00328	30	1·00493
10	1·00164	21	1·00345	31	1·00509
11	1·00180				

CASE I.*

When the principal, the rate of interest, and time, are given, to find either the amount or interest.

RULE.

1. Find the amount of £.1 or Dl1 for one year at the given rate per cent.

2 Involve the amount, thus found, to such power, as is denoted by the number of years; or, in Table 1, at the end of Annuities, under

* Let $r=$ amount of 1l. for 1 year, and $p=$ principal, or given ſum; then, ſince r is the amount of 1l. for 1 year, r^2 will be its amount for 2 years, r^3 for 3 years, and ſo on; therefore, it will be as $1 : r :: r : r^2 =$ amount for the ſecond year, or principal for the third: Again, as $1 : r :: r^2 : r^3 =$ amount for the third year, or principal for the fourth, &c. to any number of years. And, if the time or number of years be denoted by t, the amount of 1l. for t years, will be r^t; from hence it will appear that the amount of any other principal ſum p for t years, is pr^t; for, as $1 : r^t :: p : pr^t$, the ſame as in the rule.

If the rate of intereſt be determined to any other time than a year, as $\frac{1}{4}$, $\frac{1}{2}$, &c. the rule is the ſame, and then t will repreſent that ſtated time.

Let $\begin{cases} r = \text{amount of 1l. for 1 year, at the given rate per cent.} \\ p = \text{principal, or ſum put out at intereſt.} \\ i = \text{intereſt.} \\ t = \text{time.} \\ m = \text{amount for the time } t. \end{cases}$

Then the following Theorems will exhibit the ſolutions of all the caſes in compound interest.

I. $pr^t = m$. II. $pr^t - p = i$. III. $\dfrac{m}{r^t} = p$. IV. $\left.\dfrac{m}{p}\right|^{\frac{1}{t}} = r$.

The moſt convenient way of giving the Theorems, eſpecially that for the time, will be by Logarithms, as follows:

I. $t \times Log.\, r + Log.\, p = Log.\, m$. II. $Log.\, m - t \times L.\, r = L.\, p$. III. $\dfrac{L.m - L.p}{L.r} = t$.

IV. $\dfrac{L.m - L.p}{t} = L.r$.

If the compound intereſt, or amount of any ſum, be required for the parts of a year, it may be determined as follows:

I. *When the time is an aliquot part of a year.*

RULE.—1. Find the amount of 1l. for 1 year, as before, and that root of it, which is denoted by the aliquot part, will be the amount of 1l. for the time ſought.

2. Multiply the amount, thus found, by the principal, and it will be the amount of the given ſum required.

II. *When the time is not an aliquot part of a year.*

RULE.—1. Reduce the time into days, and the 365th root of the amount of 1l. for 1 year is the amount for 1 day.

2. Raiſe this amount to that power, whoſe index is equal to the number of days, and it will be the amount of 1l. for the given time.

3. Multiply this amount by the principal, and it will be the amount of the given ſum required.

To avoid extracting very high roots, the ſame may be done by logarithms, thus: Divide the logarithm of the rate, or amount of 1l. for 1 year, by the denominator of the given aliquot part, and the quotient will be the logarithm of the root ſought.

under the rate, and against the given number of years, you will find the power.*

3. Multiply this power by the principal, or given sum, and the product will be the amount required, from which, if you subtract the principal, the remainder will be the interest.

EXAMPLES.

1. What is the compound interest of 600l. for 4 years, at 6 per cent. per annum?

1·06= amount of 1l. for 1 year, at 6 per cent. per annum.
Multiply by 1·06

1·1236=2d power.
Multiply by 1·1236

1·26247696=4th power.
Multiply by 600=principal.

757·48617600=amount.
Subtract 600

157·486176=£.157 9s. 8½d.=interest required.

BY TABLE I.

Tabular amount of 1l. for 4 years, at 6 per cent. per ann.=1·2624769
Multiply by the principal= 600

Amount=757·4861400

2. What is the amount of D.1500 for 12 years, at 3½ per cent. per annum?

D.1·035=amount of D.1 for 1 year at 3½ per cent. per annum.

And, $1{\cdot}035^{12} \times 1500 =$ D.2266 60c. nearly, Ans.

Another method of working compound interest for *years*, *months*, and *days*, which is much more concise than the preceding method.

RULE.

To the logarithm of the principal, found in any Table of logarithms, add the several logarithms, answering to the number of *years*, *months* and *days* found in the following tables, and their sum will be the logarithm of the amount for the given time, which being found in any table of logarithms, the natural number corresponding thereto will be the answer.†

Logarithmick

* The amounts of £.1 or D.1 in this table, are so many powers of the amount of £.1 or D.1 for 1 year; whose indices are denoted by the number of years.

Note. When the given time consists of years and months, or years, months, and days; first seek the amount of £.1 or D.1 in the table of years, then in the table of months, &c. multiply these several amounts and the principal continually together, and the last product will be the amount required.

Thus, if the amount of £.480 in 5½ years, at 6 per cent per annum, were required; the amount of £.1 for 5 years=£.1·33822, ditto for 6 months=£.1·02956 Now, 1·33822×1·02956×480=£.661·2341 Answer.

† Although there is a small errour in the logarithm for days, yet they are exact enough for common use. And if after the first month you deduct ½ per cent. for

each

Logarithmick Tables, *at* 6 *per Cent. per Annum, for Years, Months and Days.*

Years.	Dec. pts.	Y.	Dec. pts.	Y.	Dec. pts.	Y.	Dec. pts.	Months.	Dec. pts.
1	·025306	11	·278366	21	·531426	31	·784586	1	·002166
2	·050612	12	·303672	22	·556732	32	·809792	2	·004321
3	·075918	13	·328978	23	·582038	33	·835098	3	·006466
4	·101224	14	·354284	24	·607344	34	·860404	4	·0086
5	·12653	15	·37969	25	·63265	35	·88571	5	·010724
6	·151836	16	·404896	26	·657956	36	·911016	6	·012837
7	·177142	17	·430202	27	·683262	37	·936322	7	·01494
8	·202448	18	·455508	28	·708568	38	·961628	8	·017033
9	·227754	19	·480814	29	·733974	39	·986934	9	·019116
10	·25306	20	·50612	30	·75938	40	1·01224	10	·021189
								11	·023252
Days.		D.		D.		D.		D.	
1	·000071	8	·000571	14	·000999	20	·001426	26	·001852
2	·000143	9	·000642	15	·00107	21	·001497	27	·001923
3	·000215	10	·000713	16	·001142	22	·001568	28	·001994
4	·000287	11	·000785	17	·001213	23	·001639	29	·002065
5	·000358	12	·000857	18	·001284	24	·00171	30	·002136
6	·000429	13	·000928	19	·001355	25	·001781	31	·002207
7	·0005								

What is the amount of 132l. 10s. at 6 per cent. per annum, for 9 years, 8 months, and 15 days?

To the log. of £.132·5 = 2·122216

Add { Log. for 9 years = ·227754
ditto for 8 months = ·017033
ditto for 15 days = ·00107 }

2·368073

Because 8 months are past, deduct 4 per cent. upon the logarithm of 15 days } = ·0000428

Remains 2·3680302, the nearest to which, in the table of logarithms, is 2.368101, and the natural number answering thereto is 233·4=£.233 8s. Ans.

CASE II.

When the amount, rate and time, are given, to find the principal.

Rule.

Divide the amount by the amount of £.1 or D.1 for the given time, and the quotient will be the principal.

Or, If you multiply the present value of £.1 or D1 for the given number of years, at the given rate per cent. by the amount, the product will be the principal, or present worth.* Examples.

each month paſt (that is, ½ per cent. after 1 month, 1½ per cent. after 3 months, &c.) from the logarithm of the number of days, it will give the true anſwer.

Note, That, after 1 month, ½ per cent. on the logarithm of 1 day is ·000000355, on 2 days, is ·000000715: After 2 months, 1 per cent. on the logarithm of 1 day, is ·00000071, on 2 days, ·00000143: After 10 months, 5 per cent. on the logarithm for 1 day, is ·00000355, on 6 days, is, ·00002145, &c.

* See Table II. ſhewing the preſent value of 1l. diſcounting at the rates of 4, 4½, &c. per cent. the conſtruction of which is thus:

Amount. Pres. worth. Amount. Pres. worth.

As 1·06 : 1 :: 1 : ·9433962, and ſo on, for any other rate per cent. and time.

Examples.

1. What is the present worth of 757l. 9s. 8½d. due 4 years hence, discounting at the rate of 6l. per cent. per annum?

By Tale I.

Divide by the tabular amount of 1l. for 4 years, } =1·2624769)757·4861400(£.600 Ans.

By Table II.

Mult. by the present worth of 1l. for 4 years, at 6 per. cent. per ann. } Amount=757·48614
= ·79 20936

Ans. 599·999923582704+=£.600

2. What principal must be put to interest 6 years, at 5½ per cent. per annum, to amount to D.689·4214033809453125? Ans. D.500.

CASE III.

When the principal, rate and amount, are given, to find the time.

Rule.

Divide the amount by the principal; then divide this quotient by the amount of £.1 or D.1 for 1 year, this quotient by the same, till nothing remain, and the number of the divisions will show the time.

Or, Divide the amount by the principal, and the quotient will be the amount of £.1 or D.1 for the given time, which seek under the given rate in table 1, and, in a line with it, you will see the time.

Example.

In what time will D.500 amount to D.689 42c. 1m.+, at 5½ per cent. per annum?

	500	689·421 +
6 divisions.	1·055	1·379—
	1·055	1·307—
	1·055	1·239—
	1·055	1·174 +
	1·055	1·113—
	1·055	1·053 +
		1·—

Ans. 6 years.

CASE IV.

When the principal, amount and time, are given, to find the rate per cent.

Rule.

Divide the amount by the principal, and the quotient will be the amount of 1l. or 1D. for the given time; then, extract such root as the time denotes, and that root will be the amount of 1l. or D. for 1 year, from which subtract unity, and the remainder will be the ratio.

Or, Having found the amount of 1l. or D. for the time as above directed, look for it in Table 1st. even with the given time, and directly over the amount you will find the ratio.

Example.

EXAMPLE.

At what rate per cent. per annum will D.500 amount to D.689·421403+ in 6 years?

$\frac{689{\cdot}421403+}{500} = 1{\cdot}378843-$; and $\sqrt[6]{1{\cdot}378843-} = 1{\cdot}055$. Then $1{\cdot}055 - 1{\cdot} = {\cdot}055$ = ratio. Hence the rate is $5\frac{1}{2}$ per cent. per annum, Answer.

*DISCOUNT BY COMPOUND INTEREST.**

The sum, or debt to be discounted, the time and rate, given, to find the present worth.

RULE. Divide the debt by that power of the amount of 1l. or D. for 1 year, denoted by the time, and the quotient will be the present worth, which, subtracted from the debt, will leave the discount.

EXAMPLES.

1. What is the present worth, and discount, of 600l. due 3 years hence, at 6l. per cent. per annum, compound interest?

Divide by $\overline{1{\cdot}06}|^3 = 1{\cdot}19101)600{\cdot}00000(503{\cdot}7741$ = £.503 15s. $5\frac{3}{4}$d. present worth, and £.600—£.503 15 $5\frac{3}{4}$=£.96 4s. $6\frac{1}{4}$d. = discount.

Or, $\frac{600}{1{\cdot}19101}$ = £.503·7741, and $600 - \frac{600}{1{\cdot}19101}$ = £.96·2259

By TABLE II.

In this Table, corresponding to the time and rate, we have

·839619=present worth of 1l. for the time and rate.

Multiply by 600=debt, or principal.

503·771400=present worth of the debt.

2. What is the present worth of 312l. 10s. due 2 years hence, at $4\frac{1}{2}$ per cent. per annum, compound interest?

Answer, £.286 3s. 3d. 2·97qrs.

3. What

* Let m=sum, or debt, to be discounted, and the other letters as before: Then the following Theorems will show all the cases in Discount by Compound Interest.

I. $\frac{m}{r^t} = p$. II. $pr^t = m$. III. $\frac{m}{p} = r^t$ which being continually divided by r, till nothing remain, the number of these divisions will be equal to t.

IV. $\frac{m}{p} = r^t$ which being extracted, (the time, given in the question, showing the power) will be equal to r.

Note. Case 2d. may be wrought by Table 1, thus: Find that power of 1l. for 1 year, denoted by the time; multiply the present worth by it, and the product will be the answer.

Or, by Table 2d. thus: Find the present worth of 1l. for the given time, by which divide the present worth, and the quotient will be the debt or principal.

Case 3d. thus: Divide the debt by its present worth, and seek the quotient in Table 1, under the given rate, and in the line with it you will see the time.

Case 4th. is wrought in the same manner, only, seek the quotient in a line with the time, it will show the rate atop.

3. What ready money will discharge a debt of 1000D. due 4 years hence, at 5D. per cent. per annum, compound interest?

Answer, 822D. 70c. 2m.

ANNUITIES OR PENSIONS, IN ARREARS, AT COMPOUND INTEREST.

CASE I.

When the annuity, or pension, the time it continues, and the rate per cent are given, to find the amount.

Rule.*—1. Make 1 the first term of a Geometrical Progression, and the amount of 1l. or D. for 1 year at the given rate per cent. the ratio.

2. Carry the series to so many terms as the number of years, and find its sum.

3. Multiply the sum thus found by the given annuity, and the product will be the amount sought.

Or, multiply the amount of £.1 or D.1 for 1 year into itself so many times as there are years less by 1; then multiply this product by the annuity; and subtract the annuity therefrom. Lastly, divide the remainder by the ratio less 1, and the quotient will be the amount.

Examples.

* It is plain that upon the firft year's annuity there will be due fo many years' compound intereft, as the given number of years lefs 1, and gradually one year lefs, upon every fucceeding year, to that preceding the laft, which has but one year's intereft, and the laft bears none.

Let r=rate, or amount of 1l. for 1 year, then the feries of amounts of 1l. annuity for feveral years, from the firft to the laft, is 1, r, r^2, r^3, &c. to $r^{t}-1$; and the fum of this, according to the rule in Geometrical Progreffion, will be $\frac{r^t-1}{r-1}$ = amount of 1l. annuity for t years. And all annuities are proportional to their amounts; therefore, $1 : \frac{r^t-1}{r-1} :: n : \frac{r^t-1}{r-1} \times n$ = amount of any given annuity n.

Let r=rate, or amount of 1l. for 1 year, and the other letters as before, then

$\frac{r^t-1}{r-1} \times n = a$, and $\frac{ar-a}{r^t-1} = n$

And from thefe equations, all the cafes relating to annuities or penfions in arrears, may be conveniently exhibited in logarithmick terms, thus:

I. $L.n + L.r^t-1 + L.\overline{r-1} = L.a.$

II. $L.a - L.r^t-1 + L.\overline{r-1} = L.n.$

III. $\frac{L.\overline{ar-a+n}-L.n}{L.r} = t.$ IV. $r^t - \frac{ar}{n} - 1 = r.$

Or thus, I. $\frac{nr^t-n}{r-1} = a.$ II. $\frac{ar-a}{r^t-1} = n.$ III. $\frac{ar+n-a}{n} = r^t$

which continually divided by r till nothing remain, the number of thofe divifions will be equal to t: Or, being extracted, (the given time fhewing the power) will be equal to r.

EXAMPLES.

1. What will an annuity of 60l. per annum, payable yearly, amount to in 4 years, at 6l. per cent.?

First Method.

$$1+1{\cdot}06+\overline{1{\cdot}06}\big|^{2}+\overline{1{\cdot}06}\big|^{3}=4{\cdot}374616=\text{sum.}$$

Multiply by 60 = annuity.

```
262·476960
        20
----------
   9·53920
        12
----------
    6·4704
         4
----------
    1·8816   Ans. £.262 9s. 6¼d.
```

$$\text{Or, } 1+1{\cdot}06+\overline{1{\cdot}06}\big|^{2}+\overline{1{\cdot}06}\big|^{3}\times 60=\pounds.262\ 9s.\ 6\tfrac{1}{4}d.$$

Second Method.

$$1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06=1{\cdot}26247$$

Multiply by 60 annuity.

```
                      75·74820
             Subtract 60
                      --------
Divide by 1·06—1=·06)15·7482(262·47 = £.262 9s. 4¾d. Ans.
                     12
                     --
                      37
                      36
                      --
                       14
                       12
                       --
                        28
                        24
                        --
                         42
                         42
```

$$\text{Or, } \frac{1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times60-60}{1{\cdot}06-1}=\pounds.262{\cdot}47$$

OR,

OR, BY TABLE III.*

Multiply the tabular number under the rate, and opposite to the time, by the annuity, and the product will be the amount.

2. What will an annuity of 60l. per annum amount to in 20 years, allowing 6l. per cent. compound interest?

Under 6l. per cent. and opposite 20, in table 3d. you will find,

Tabular number = 36·78559
Multiply by 60 = annuity.

2207·13540 = £.2207 2s. 8½d. Ans.

3. What will a pension of D.75 per annum, payable yearly, amount to in 9 years at 5 per cent. compound interest?

Ans. D.826 99c. $2\frac{3}{10}$m.

4. If a salary of 100l. per annum, to be paid yearly, be forborne 5 years, at 6l. per cent. What is the amount? Ans. £.563 14s. 2d.

5. What will wages of D.25 per month, amount to in a year, at ½ per cent. per month? Ans. D.308 38c. 9m.

CASE II.

When the amount, rate per cent. and time are given, to find the annuity, pension, &c.

RULE.—Multiply the whole amount by the amount of 1l. or D.1 for a year, from which subtract the whole amount, divide the remainder by that power of the amount of 1l. or D.1 for a year, signified by the number of years, made less by unity, and the quotient will be the answer

Or, find the amount of an annuity of 1l. or D.1 for the given time and rate (by Case 1;) divide the given sum by this amount; and the quotient will be the annuity required.

EXAMPLES.

1. What annuity, being forborne 4 years, will amount to £.262·47696, at 6l. per cent. compound interest?

262·47696 = amount.
Multiply by 1·06 = amount of 1l. for 1 year.

157486176	1·06
262476960	1·06
278·2255776	636
Subtract 262·47696	1060
·26247696)15·7486176(£. 60 Ans.	1·1236
15 7486176	1·06
0 Carried over.	67416

* Table 3 is calculated thus: Take the first year's amount, which is 1l. multiply it by 1·06+1=2·06=second year's amount, which also multiply by 1·06+1=3·1836 =third year's amount, &c. and in this manner proceed in calculating tables at any other rates.

Brought over.	67416
	112360
	1·191016
	1·06
	7146096
	11910160
	1·26247696
Subtract	1·
Divisor =	·26247696

Or, $\dfrac{262{\cdot}47696 \times 1{\cdot}06 - 262{\cdot}47696}{1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 - 1} = 60.$

Or, thus.

Amount of an annuity of 1l. for 4 years at 6 per cent. per annum $=4{\cdot}374616$ (by Case 1); and $\dfrac{262{\cdot}47696}{4{\cdot}374616} = £.60$ Ans.

Or, by Table III. the amount of 1l. is found to be 4·374616; and the answer is found, as before.

2. What annuity, being forborne 20 years, will amount to D.2207·1354, at 6. per cent. compound interest?

Amount of an annuity of D.1 for 20 years at 6. per cent. per annum = 36·78559. And,

36·78559)2207·13540(D.60, Ans.
2207·1354

0

CASE III.

When the annuity, amount and ratio are given, to find the time.

Rule.—Multiply the amount by the ratio, to this product add the annuity, and from the sum subtract the amount; this remainder being divided by the annuity, the *quotient* will be that power of the ratio signified by the time, which being divided by the amount of 1l. for 1 year, and this *quotient* by the same, till nothing remain, the number of those divisions will be equal to the time. Or, look for this number under the given rate in table 1, and in a line with it, you will see the time. Or,

Divide the amount by the annuity; from the quotient subtract 1; from the remainder subtract the ratio; from successive remainders subtract the square, cube, &c. of the ratio, till nothing remain; and the whole number of the subtractions will be the answer. Or, find the quotient in Table III. under the rate, and in a line with it stands the answer.

Examples.

1. In what time will 60l. per annum, payable yearly, amount to £.262·47696, allowing 6l. per cent. compound interest, for the forbearance of payment?

262·

```
              262·47696 = amount.
Multiply by        1·06 = ratio.
             ----------
              157486176
             262476960
             ----------
             278 2255776
Add          60·          = annuity.
             ----------
             338·2255776
Subtract     262·47696
             ----------
Divide by 60)75·7486176
             ----------
Divide by 1·06)1·26247696
             ----------
Divide by 1·06)1·191016
             ----------
Divide by 1·06)1·1236
             ----------
Divide by 1·06)1·06
             ----------
                  1
```

The number of divisions by 1·06, being 4, gives the number of years = 4, the answer.

Or thus:

```
Annuity = 60)262·47696 = amount.
             ----------
              4·374616
1. Subtract   1·
             ----------
              3·374616
2. Subtract   1·06      = ratio.
             ----------
              2·314616
3. Subtract   1·1236    = ratio|²
             ----------
              1·191016
4. Subtract   1·191016  = ratio|³
             ----------
              ·······
```

Ans. 4 years.

Or, looking into Table III. under the rate, 6, the quotient, 4·374616, stands against 4 years, Ans. as before.

Or, in Table I under the given rate, you will find 1·262476, and in a line, under years, you will find 4.

2. In what time will an annuity of D.60 payable yearly, amount to D.2207·1354, allowing 6 per cent. for the forbearance of payment? Ans. 20 years.

PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

CASE I.

When the annuity, &c. rate and time are given, to find the present worth.

Rule.*—1. Divide the annuity by the ratio, or the amount of D.1 or £.1 for 1 year, and the quotient will be the present worth of 1 year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth for two years.

3. In

* Let p= present worth of the annuity, and the other letters as before: Then,

$$n \times \frac{r^{t}-1}{r^{t+1}-r^{t}} = p. \quad \text{And } p \times \frac{r^{t+1}-r^{t}}{r^{t}-1} = n.$$

And

3. In like manner, find the present worth of each year by itself, and the sum of all these will be the present value of the annuity, sought.

Or, divide the annuity, &c. by that power of the ratio signified by the number of years, and subtract the quotient from the annuity; this remainder being divided by the ratio less 1, the quotient will be the present worth.

EXAMPLES.

1.* What ready money will purchase an annuity of 60l. to continue 4 years, at 6l. per cent. compound interest?

First Method.

Ratio =	1·06)60·00000(56·603 =	present worth	for 1 year.
$\overline{\text{Ratio}}\vert^{2}$ =	1·1236)60·00000(53·399 =	do.	for 2 years.
$\overline{\text{Ratio}}\vert^{3}$ =	1·191016)60·00000(50·377 =	do.	for 3 years.
$\overline{\text{Ratio}}\vert^{4}$ =	1·26247696)60·00000(47·525 =	do.	for 4 years.
	207·904 = £.207 18s. $0\frac{3}{4}$d. Ans.		

Second

And from these Theorems, all the cases, where the purchase of annuities is concerned, may be exhibited in logarithmick terms, as follows:

$$\text{I. } L.n + L.\overline{1 - \frac{1}{r^t}} - L.\overline{r-1} = L.p.$$

$$\text{II. } L.p + L.\overline{r-1} - L.\overline{1 - \frac{1}{r^t}} = n.$$

$$\text{III. } \frac{L.n - L.\overline{n+p-pr}}{L.r} = t.$$

Or, thus, I. $\dfrac{n - \dfrac{n}{r^t}}{r-1} = p.$ II. $\dfrac{pr^t \times r - pr^t}{r^t - 1} = n.$ III. $\dfrac{n}{p+n-pr} = r^t$ which being continually divided by r till nothing remain, the number of those divisions will be equal to t.

Let t express the number of half years, or quarters, n the half years, or quarter's payment, and r the sum of 1l. and $\frac{1}{2}$ or $\frac{1}{4}$ year's interest, then all the preceding rules will be applicable to half yearly, and quarterly payments, the same as to whole years.

The *amount* of an annuity may also be found for years and parts a of year, thus:

1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a year.
3. Add this interest to the former account, and it will give the whole amount required.

The *present worth* of an annuity for years and parts of a year may be found thus:

1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

* Questions in this case may also be answered by first finding the amount of the given annuity by Case I. of annuities in arrears, page 280, and then the present worth, or principal, by Case II. of Compound Interest, page 278.

Second Method.

4th power of the ratio } =1·26247696)60·0000000(47·525

From 60
Subtract 47·525 Or, $\frac{60}{\cdot 06}$ ^{4}=47·525 60—47·525=12·475

Divis. 1·06—1=·06)12·475 And $\frac{12\cdot475}{\cdot06}$=207·916.

2 07·916=£.207 18s. $3\frac{3}{4}$d. Ans.

By Table III.

Under 6l. per cent. and opposite 4, we find
4·37461 = amount of 1l. annuity for 4 years.
Multiply by 60 = anuity.

262·47660 = amount of 60l. for 4 years.

Then, opposite 4 years, and under 6l. per cent. in Table 2d.
We have ·792093
Multiply by 262·7466

```
         4752558
        4752558
       3168372
      5544651
     1584186
    4752558
   1584186
```

208·1197426338 = £.208 2s. $4\frac{1}{2}$d.

Or, opposite 4 years, and under 6l. per cent. in Table 1st. we have 1·26247 = the amount of 1l. for 4 years:

Then, 262·7466 ÷ 1·26247 = 208·1209 = £.208 2s. 5d. Ans.

By Table IV.*

Multiply the tabular number, under the rate, and opposite the time, into the annuity, and the product will be the present worth.

Thus, in Example 1st. What ready money will purchase 60l. annuity, to continue 4 years, at 6l. per cent. compound interest?

Under 6l. per cent. and even with 4 years,
We have 3·4651 = present worth 1l. for 4 years.
Multiply by 60 = annuity.

Ans. = 207·9060=£.207 18s. $1\frac{1}{4}$d.

2. What is the present worth of an annuity of 60D. per annum, to continue 20 years, at 6 per cent. compound interest?

Ans. D.688·65 (nearly.)

CASE

* Table 4th. is thus made: Divide 1l. by 1·06=·94339 the present worth of the first year, which, divided by 1·06, is equal to ·88999, which, added to the first year's present worth, is = 1·83339, the second year's present worth, then ·88999, divided by 1·06, and the quotient added to 1·83339, gives 2·6701 for the third year's present worth, &c.

CASE II.

When the present worth, time and rate are given, to find the annuity, rent, &c.

RULE.—1. From that power of the ratio, denoted by the number of years, plus 1, subtract that power of it denoted by the number of years.

2. Divide the remainder by that power of the ratio, signified by the time made less by unity.

3. Multiply the present worth into this quotient, and the product will be the annuity, pension, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the present worth.

2. Multiply that power of the ratio, denoted by the time, by the present worth, and subtract this product from the former.

3. Divide the remainder by that power of the ratio, denoted by the time made less by unity, and the quotient will be the annuity.

EXAMPLES.

1. What annuity, to continue 4 years, will £.207·904 purchase, compound interest, at 6l. per cent. ?

First Method.

From $1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 = 1{\cdot}3382255776$
Subt. $1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 = 1{\cdot}26247696$

Divide by $\overline{1{\cdot}06}|^4 - 1 = {\cdot}26247696){\cdot}0757486176({\cdot}2885898$
·2885898
Multiply by 207·9 present worth.

25973082
20201286
57717960

Ans. 59·99781942=£.60.

Second Method.

From $1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 207{\cdot}9 = 278{\cdot}217097573$
Take $1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 207{\cdot}9 = 262{\cdot}468959984$

Divide by $\overline{1{\cdot}06}|^4 - 1 = {\cdot}26247696)15{\cdot}748137589(59{\cdot}998 = 60$l.

By TABLE V.*

Multiply the tabular number, corresponding with the rate and time, by the purchase money, and the product will be the annuity.

Under

* Table 5th. is made in this manner: Divide 1l. by the present worth of 1l. for 1 year, and the quotient will be the annuity, which 1l. will purchase for 1 year: divide 1l. by the present worth of 1l. for 2 years, and the quotient will be the annuity, which 1l. will purchase for 2 years, &c.

Under 6l. per cent and opposite 4 years, you will find
·28859=annuity which 1l. will purchase in 4 years.
Multiply by 207·9

259731
202013
577180

59·997861=£.60.

2. What salary, to continue 20 years, will 688D. 65c. purchase, at 6 per cent. compound interest ? Ans. D.60.

CASE III.

When the annuity, present worth and ratio, are given, to find the time.

RULE.

Divide the annuity by the product of the present worth and ratio subtracted from the sum of the present worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which, being divided by the ratio, and this quotient by the same, till nothing remain, the number of divisions will show the time: Or, the above quotient being sought in Table 1st. under the given rate, in a line with it, you will see the time.

EXAMPLES.

1. For how long may an annuity of 60l. per annum be purchased for £.207·906336762, at 6l. per cent. compound interest ?

Multiply 207·906336762
by 1·06

1247438020572
2079063367620

220·38071696772

To 207·906336762=present worth.
Add 60· =annuity.

From 267·906336762
Subt. 220·380716967

47·525619795=divisor.

47·525619795)60·000000000(1·26247696
Divide by 1·06)1·26247696

1·06)1·191016

1·06)1·1236

1·06)1·06

1 { The number of divisions = time = 4 years.

Or, $\frac{60}{207·906336762+60—207·906336762\times1·06}$ =1·26247696, which being sought in Table 1, under the given rate, in a line with it, is 4=4 years.

2. How

2. How long may a lease of D.300 yearly rent, be had for D.2132·341 allowing 5 per cent. compound interest, to the purchaser?
Ans. 9 years.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION AT COMPOUND INTEREST.

CASE I.

When the annuity, time and ratio, are given, to find the present worth of the annuity in reversion.

RULE.*—1. Divide the annuity by that power of the ratio denoted by the time of its continuance.

2. Subtract this quotient from the annuity: divide by the ratio less 1, and the quotient will be the present worth, to commence immediately.

3 Divide this quotient by that power of the ratio denoted by the time of reversion, (or, time to come, before the annuity commences) and the quotient will be the present worth of the annuity in reversion.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend.

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it denoted by the time of reversion, and the ratio less 1, continually together for a divisor, and the quotient arising from the division of these two numbers will be the present worth of the annuity in reversion.

EXAMPLES.

1. What is the present worth of 60l. payable yearly, for 4 years; but not to commence till two years hence, at 6l. per cent.?

First Method.

Ratio=1·06	Or, in Table 4th, find the present
1·06	value of 1l. at the given rate, both for
——	the time in being and the time in re-
636	version added together, and subtract
1060	the present worth of the time in being
———	from the other, multiply the remain-
2d. power=1·1236	der by the annuity, and the product
Carried over. 1·1236	will be the answer. Pres.

* Let v denote the time in reversion, and the other letters as before. Then the two cases under this rule will be expressed by the following Theorems.

I. $n - \frac{n}{r^t} = p$. Then change p into m, and $\frac{m}{r^v} = p$.

II. $pr^v = m$. Change m into p, and $\frac{pr^t \times r - pr^t}{r^t - 1} = n$.

Or, I. $\frac{r^t - 1 \times n}{r - 1 \times r^t \times r^v} = p$. II. $\frac{r - 1 \times r^t \times r \times vp}{r^t - 1} = p$.

Brought over. 1·1236 Pres. worth of the time in being and reversion } = 4·91732

67416 Present worth of the time in being } = 1·8333

33708

22472

11236

11236

3·08402

60

£.185·04120

Div. by 4th pow.=1·26247696)60·000000000000(47·525619794281

Subtract the quotient=47·525619794281

Divide by 1·06—1=·06)12·474380205719

Divide by 1·06×1·06=1·1236)207·9063367619(185·035899=185l. 0s. 8½d. = the present worth of the annuity in reversion.

$$\text{Or, } \frac{60}{1{\cdot}26247696} = 47{\cdot}5256 \qquad \frac{60-47{\cdot}5256}{1{\cdot}06-1} = 207{\cdot}906$$

$$\text{And } \frac{207{\cdot}906}{1{\cdot}1236} = 185{\cdot}035899$$

Second Method.

·26247696 = 4th power—1

Multiply by 60 = annuity.

15·74861760 = dividend. ·08511115)15·74861760(185·036 [Ans.

1·26247696 = 4th power.

1·1236 = 2d power.

757486176

378743088

252495392

126247696

126247696

1·418519112256

·06 = ratio—1

·085111146735 36 = divisor.

$$\text{Or, } \frac{1{\cdot}06|^4 - 1\times 60}{1{\cdot}06|^4 \times 1{\cdot}06|^2 \times 1{\cdot}06 - 1} = 185{\cdot}036$$

2. What is the present worth of a reversion of a lease of D.60 per annum, to continue 20 years, but not to commence till the end of 8 years, allowing 6 per cent. to the purchaser?

Ans. D.431·782 (nearly.)

An annuity, several times in reversion, and rate being given, to find the several present values.

Find the present value of £.1 or D.1 by Table 4, at the given rate, and for the several given times, which, being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders will answer the question.

3. A

3. A has a term of 6 years in an estate at 60l. per annum. B has a term of 14 years in the same estate, in reversion, after the 6 years are expired; and C has a further term of 16 years, after the expiration of 20 years. I demand the present values of the several terms, at 6 per cent.?

	£.	s.	d.
Pres. value of £.1 for 36 years=14·61722×60=	877	0	7¾
Ditto of ditto for 20 years =11·46992×60=	688	3	10¾
Ditto of ditto for 6 years = 4·91732×60=	295	0	9¼=A's term.

Therefore, 877 0 7¾—688 3 10¾=£.188 16 9 C's term, and 688 3 10¾=295 0 9¼=£.393 3 1½=B's term.

4. For a lease of certain profits for 7 years, A offers to pay D.300 gratuity, and D.300 per annum, B offers D.800 gratuity and D.250 per annum, C bids D.1300 gratuity and D.200 per annum, and D bids D.2500 for the whole purchase, without any yearly rent; which is the best offer, computing at 6 per cent.?

	D.
By Table 4, the present worth of D.300 per annum, for 7 years, at 6 per cent. is	1674·714
To which add	300·
Value of A's offer =	1974·714
Present worth of D.250 per annum for 7 years =	1395·595
To which add	800·
Value of B's offer =	2195·595
Present worth of D.200 per annum for 7 years =	1116·476
To which add	1300·
Value of C's offer =	2416·476
D's offer =	2500·

Hence it appears that D's offer is the best.

The above questions may be answered by the 4th. and 2d. Tables.

Take question 1st. for Example.

1. Multiply the tabular number in Table 4, corresponding to the rate and the time of continuance, into the annuity, and the product will be the present worth, to commence immediately.

2. Multiply this present worth by the tabular number in Table 2, corresponding to the rate and the time of reversion, and the product will be the present worth of the annuity in reversion.

In Table 4th we have 3·4651
Multiply by 60 = annuity.

207·9060
In Table 2d. we have ·889996

1247436 Carried over.

```
      1247436   Brought over.
     1871154
    1871154
   1871154
  1663248
 1663248
 ——————————
 185·035508376=present worth of the reversion.
```

CASE II.

When the present worth of the reversion, rate and time are given, to find the annuity.

RULE.—1. Multiply that power of the ratio signified by the time of reversion, by the present worth, and the product will be the amount of the present worth for the time before the annuity commences.

2. Multiply that power of the ratio signified by the time of continuance plus 1 by the last product.

3. Multiply that power of the ratio, signified by the time, by the aforesaid product, and this last product, divided by that power of the ratio denoted by the time, minus unity, will give the annuity.

Or, divide the continual product of the present worth, that power of the ratio denoted by the time of continuance, that power of it denoted by the time of reversion, and the ratio minus 1, by that power of the ratio denoted by the time of continuance minus 1, and the quotient will be the annuity.

EXAMPLES.

1. What annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for D. 185·035899, at 6 per cent.?

First Method.

```
        1·06×1·06=1·1236 = 2d power of the ratio.
  Multiply by 185·036 = present worth.
              ———————
               67416
              33708
             561800
            89888
           11236
           ——————————
           207·9064496 amount for the time of reversion.
Multiply by 1·33822     = 5th power of the ratio.
            ———————
            415812
           415812
          1663248
          623718
         623718
        207906
        ——————————
From 278·22396732
Take 262·47508782
```

```
4th power of the ratio=1·26247
     Multiply by 207·906
                 ———————
                 757482
               11362230
                883729
              2524940
              ———————————
              262·47508782
                     Divide
```

Divide by $\overline{1\cdot06}|^4 - 1 = \cdot26247)15\cdot7488750(60$ the annuity required.

Or, $185\cdot036 \times 1\cdot1236 = 207\cdot906$

Then, $$\frac{\overline{207\cdot906 \times 1\cdot33822 - 207\cdot906 \times 1\cdot26247}}{1\cdot26247 - 1} = \text{D.}60 \text{ Ans.}$$

Second Method.

185·036 = present worth of the reversion.
1·26247 = 4th power of the ratio.

```
    1295252
    740144
   370072
  1110216
  370072
 185036
 ------
 233·6024
```

Or by Table 4th, divide the present worth of the reversion by the difference between the present worth of D.1 for the time both in being and reversion, and the time in being, and the quotient will be the annuity.

1·1236 = 2d. power of the ratio.

```
    14016144
    7008072
   4672048
  2336024
 2336024
 ----------
 262·47565664
```

4·91732 = { pr. wo. of D.1 for the time in being & revrsn.
1·8333 = { pr. wo. of D.1 for the. time in being.

3·08402)185·0412(60 Ans.

·06 = ratio—1.

$\overline{1\cdot06}|^4 - 1 = \cdot26247)15\cdot7485393984(60.$

Or, $$\frac{\overline{185\cdot036 \times 1\cdot26247 \times 1\cdot1236 \times 1\cdot06 - 1}}{1\cdot26247 - 1} = 60.$$

2. The present worth of a lease of an house is 431l. 15s. 7d. 2·7819 qrs taken in reversion for 20 years; but not to commence till the end of 8 years, allowing 6l. per cent. to the purchaser: What is the yearly rent? Ans. £.60.

PURCHASING ANNUITIES FOREVER, OR FREEHOLD ESTATES, AT COMPOUND INTEREST.

CASE I.

When the annuity, or yearly rent, and the rate are given; to find the present worth, or price.

Rule.*—As the rate per cent. is to 100l. or 100D. so is the yearly rent, to the value required. Or,

* The reafon of this rule is obvious; for fince a year's intereft of the price, which is given for it, is the annuity, there can neither more nor lefs be made of that

Or, Divide the yearly rent by the ratio less 1, and the quotient will be the value required.

EXAMPLES.

1. What is the worth of a freehold estate of 60l. per annum, allowing 6l. per cent. to the purchaser?

$$\begin{array}{l} £.\ \ £.\ \ £. \\ 6 : 100 :: 60 \\ \quad\ \ 60 \\ \hline 6)6000 \\ \hline £.1000 \text{ Ans.} \end{array} \qquad \text{Or, } 1{\cdot}06-1={\cdot}06)60{\cdot}00 \atop \overline{\quad 1000}$$

2. An estate brings in yearly D.75: What will it sell for, allowing the purchaser 5 per cent. compound interest? Ans. D.1500.

CASE II.

When the price, or present worth, and rate are given, to find the annuity, or yearly rent.

RULE.—As £.100 or D.100 is to the rate so is the present worth to its rent.

Or, Multiply the present worth by the ratio less 1, and the product will be the yearly rent.

EXAMPLES.

1. If a freehold estate be bought for 1000l. allowing 6l. per cent. to the purchaser: What is the yearly rent?

$$\begin{array}{l} £.\ \ £.\ \ £. \\ 100 : 6 :: 1000 \\ \qquad\qquad\ \ 6 \\ \hline 100)6000(£.\ 60 \text{ Ans.} \\ \qquad\ 600 \\ \hline \qquad\quad 0 \end{array} \qquad \text{Or, } 1000\times{\cdot}06=£.60.$$

2. If an estate be sold for 1500D. and 5 per cent. allowed to the buyer; what is the yearly rent? Ans. D.75.

CASE III.

When the present worth, or price, and yearly rent, are given, to find the rate.

RULE.

As the present worth is to the rent; so is £.100 or D. to the rate.

Or,

that price, than of the annuity, whether it be employed at simple or compound interest.

The following Theorems shew all the varieties of this rule.

I. $\frac{n}{r-1}=p$. II. $\overline{r-1}\times p=n$. III. $\frac{n}{p}+1=r$, or $\frac{n}{p}=r-1$, or $\frac{p+n}{p}=r$

Or, Divide the rent by the present worth; add 1 to the quotient, and the sum will be the ratio of the rate per cent.

Or, Divide the sum of the present worth and rent by the present worth, and the quotient will be the ratio.

EXAMPLES.

1. If an estate of 60l. per annum be bought for 1000l. what rate of interest was allowed the purchaser for his money?

£. £. £.
1000 : 60 :: 100
100

1000)6000(£.6 Ans.

Or, 1000)60·00(·06+1=1·06
60 00

Or, to 1000=present worth.
Add 60=rent.

1000)1060(1·06
1000

6000
6000

2. An estate of 75D. per annum was purchased for 1500D. what rate of interest had the buyer for his money? Ans. 5 per cent.

To find at how many years' purchase an estate may be bought.

CASE I.

When the rate of interest is given, to find the number of years.

RULE.—Divide 100l. or D. by the rate, and the *quotient* will be the years.

EXAMPLES.

1. How many years' purchase should a gentleman offer for the purchase of an estate, to have 6 per cent. for his money?

6)100

16·666+=$16\frac{2}{3}$ years.

2. How many years' purchase is an estate worth, allowing 5 per cent. to the purchaser? Ans. 20 years.

CASE II.

When the number of years' purchase, at which an estate is bought, or sold, is given, to find the rate of interest.

RULE.—Divide £.100 or D. by the number of years, and the quotient will be the rate.

EXAMPLES.

Examples.

1. A gentleman gives $16\frac{2}{3}$ years' purchase for a farm; what interest is he allowed? $16\frac{2}{3}=16{\cdot}666+)100{\cdot}000(6$ per cent. Ans.

2. A gentleman gives 20 years' purchase for an estate; what interest has he? Ans. 5 per cent.

PURCHASING FREEHOLD ESTATES IN REVERSION.

CASE I.

The rate and rent of a freehold estate being given, to find the present worth of reversion.

Rule.*—1. Find the present worth of the annuity or rent, (by Case 1. of purchasing Freehold Estates, page 293,) as though it were to be entered on immediately.

2. Divide the last present worth by that power of the ratio denoted by the time of reversion (by Case 1 of Discount by Compound Interest) and the quotient will be the answer required.

Or, 1. Having found the present value of the estate, supposing it to be immediate: Multiply the annuity, or rent, by the present worth of 1l. or D. corresponding with the time of reversion and rate in Table 4th. and the product will be the present worth of the annuity, or rent, for the time of reversion; or the value of the present possession.

2. Subtract the value of the possession from the value of the estate, and the remainder will be the value of reversion.

Examples.

1. Suppose a freehold estate of 60l. per annum to commence 2 years hence, be put up to sale; what is its value, allowing the purchaser 6l. per cent.?

First Method.

$1{\cdot}06-1={\cdot}06)60{\cdot}00$ = rent per annum.

1000 = present worth, if entered on immediately.

$\overline{1{\cdot}06}|^2=1{\cdot}1236)1000{\cdot}000(889{\cdot}996=$ £.889 19s. 11d. = present worth of 1000l. for 2 years, or the whole present worth required.

Second

* The following Theorems express all the Cases under this rule.

I. $\frac{n}{r-1}=p$; then change p into m, and $\frac{m}{r^v}=p$.

II. $pr^v=m$; then change m into p, and $\frac{prr-pr}{r}=m$.

Second Method.

1·06—1=·06)60·00

1000 = present worth, for immediate possession.

In Table 4th. we have 1·83339=value of 1l. for 2 years.

Multiply by 60=rent.

110·00340=value of possesssion.

From 1000·0000

Subtract 110·0034

889·9966=value required.

2. Suppose an estate of 75D. per annum, to commence 10 years hence, were to be sold, allowing the purchaser 5 per cent; what is its worth? Ans. D.920 87c. 1m. (nearly.)

CASE II.

The value of a Reversion, the Time prior to its Commencement, and rate of Interest given, to find the Annuity or Rent.

RULE.—1. Multiply the price of the reversion by that power of the amount of 1l. or D. for 1 year, denoted by the time of reversion, and the product will be its amount, (by Case 1 of Compound Interest.)

2. Find the interest of the amount (by Case 1st. Simple Interest) and it will be the annuity, or yearly rent.

EXAMPLES.

1. A freehold estate is bought for £.889·9966 which does not commence till the end of 2 years; the buyer being allowed 6l. per cent. for his money; I desire to know the yearly income?

889·9966=price of the reversion.

Multiply by $\overline{1{\cdot}06|}^{2}$ = 1·1236 denoted by the time of reversion.

53399796
26699898
17799932
8899966
8899966

1000·00017976=amount of the reversion.
·06

Ans. £.60·00

2. If a freehold estate, to commence 10 years hence, be sold for D.920 87c. 1m. allowing the purchaser 5. per cent.; what is the yearly income? Ans. D.75.

TABLE

2..O

TABLE I. *Shewing the amount of £.1 or D.1 from 1 year to 50.*

ys.	3 *per cent.*	3½ *per cent.*	4 *per cent.*	4½ *per cent.*	5 *per cent.*	5½ *per cent.*	6 *per cent.*
1	1·0300000	1·0350000	1·0400000	1·0450000	1·0500000	1·0550000	1·0600000
2	1·0609000	1·0712250	1·0816000	1·0920250	1·1025000	1·1130250	1·1236000
3	1·0927270	1·1087178	1·1248640	1·1411661	1·1576250	1·1742413	1·1910160
4	1·1255088	1·1475230	1·1698585	1·1925186	1·2155062	1·2388245	1·2624796
5	1·1592740	1·1876863	1·2166529	1·2461819	1·2762815	1·3069598	1·3382256
6	1·1940523	1·2292553	1·2653190	1·3022601	1·3400956	1·3788426	1·4185191
7	1·2298738	1·2722792	1·3159317	1·3608618	1·4071004	1·4546789	1·5036302
8	1·2667700	1·3168090	1·3685690	1·4221006	1·4774554	1·5346862	1·5938480
9	1·3047731	1·3628973	1·4233118	1·4860951	1·5513282	1·6190939	1·6894789
10	1·3439163	1·4105987	1·4802842	1·5529694	1·6288946	1·7081440	1·7908476
11	1·3842338	1·4599697	1·5394540	1·6228530	1·7103393	1·8020919	1·8982985
12	1·4257608	1·5110686	1·6010322	1·6958814	1·7958563	1·9012069	2·0121964
13	1·1685337	1·5639560	1·6650735	1·7721961	1·8856491	2·0057732	2·1329282
14	1·5125897	1·6185945	1·7316764	1·8519449	1·9799316	2·1160907	2·2609039
15	1·5579674	1·6753488	1·8009435	1·9352824	2·0789281	2·2324756	2·3965581
16	1·6047064	1·733986	1·8729812	2·0223701	2·1828745	2·3552617	2·5472716
17	1·6528476	1·7946755	1·9479005	2·1133768	2·2920183	2·4848011	2·6927727
18	1·7024330	1·8574892	2·0258161	2·2084787	2·4066192	2·6214652	2·8543391
19	1·7535060	1·9225013	2·1068491	2·3078603	2·5269502	2·7656458	3·0255995
20	1·8061112	1·9897888	2·1911231	2·4117140	2·6532977	2·9177563	3·2071355
21	1·8602945	2·0594314	2·2787680	2·5202411	2·7859625	3·0782329	3·3995636
22	1·9161034	2·1315115	2·3699187	2·6336520	2·9252607	3·2475357	3·6035374
23	1·9735865	2·2061144	2·4647155	2·7521663	3·0715237	3·4261502	3·8197496
24	2·0327941	2·2833284	2·5633041	2·8760138	3·2250999	3·6145885	4·0489346
25	2·0937779	2·3632449	2·6658363	3·0054344	3·3863549	3·8133910	4·2918707
26	2·1565912	2·4459585	2·7724697	3·1406790	3·5556726	4·0231279	4·5493829
27	2·2212890	2·5315671	2·8833685	3·2820095	3·7334563	4·2443999	4·8223459
28	2·2879276	2·6201719	2·9987033	3·4296999	3·9201291	4·4778419	5·1116867
29	2·3565655	2·7118779	3·1186514	3·5840364	4·1661356	4·7241232	5·4183879
30	2·4272624	2·8067937	3·2433975	3·7453181	4·3219423	4·9839499	5·7434912
31	2·5000803	2·9050314	3·3731334	3·9138574	4·5380394	5·2580671	6·0881007
32	2·5750827	3·0067075	3·5080587	4·0899810	4·7649414	5·5472608	6·4533867
33	2·6523352	3·1119123	3·6483811	4·2740301	5·0031885	5·8523600	6·8405899
34	2·7319053	3·2208603	3·7943168	4·4663615	5·2533479	6·1742398	7·2510253
35	2·8138624	3·3335904	3·9460889	4·6673478	5·5160152	6·5138230	7·6860868
36	2·8982783	3·4502661	4·1039325	4·8773784	5·7918101	6·8720832	8·147252
37	2·9852266	3·5710254	4·2680898	5·0968604	6·0814069	7·2500478	8·6360871
38	3·0747834	3·6960113	4·4388134	5·3262192	6·3854772	7·6488004	9·1542523
39	3·1670269	3·8253717	2·6163659	5.5658990	6·7047511	8·0694844	9·7035074
40	3·2620377	3·9592597	4·8010206	5·8164645	7·0399887	8·5133060	10·2857178
41	3·3598988	4·0975337	4·9930614	6·0782054	7·3919881	8·9815378	10·9028608
42	3·4606958	4·2412579	5·1927833	6·3517246	7·7615875	9·4755224	11·5570325
43	3·5645167	4·3897020	5·4004952	6·6375522	8·1496669	9·9966761	12·2504547
44	3·6714522	4·5433415	5·616515	6·9362421	8·5571502	10·5464933	12·9854817
45	3·7815957	4·7023585	5·8411756	7·248373	8·9850077	11·1265504	13·7626109
46	3·8950436	4·8669411	6·0748236	7·5745497	9·4342581	11·7385217	14·5883673
47	4·0118949	5·0372840	6·3168166	7·9154045	9·9059710	12·3841404	15·4636693
48	4·1322518	5·2135889	6·5694892	8·2715977	10·4012696	13·0652681	16·3914894
49	4·2562193	5·3960645	6·8322688	8·6438196	10·9213331	13·7838579	17·3749788
50	4·3830059	5·5849268	7·1055596	9·0327915	11·4673697	14·5410000	18·2174775

TABLE

TABLE II. *Shewing the present value of £.1 or D.1, due at the end of any number of years, from 1 to 40.*

yrs.	*4 per cent.*	*4½ per cent.*	*5 per cent.*	*5½ per cent.*	*6 per cent.*	*yrs.*
1	·961538	·956938	·952381	·947867	·943396	1
2	·924556	·91573	·90703	·898513	·889996	2
3	·888996	·876297	·863838	·851728	·839619	3
4	·854804	·838561	·822702	·807397	·792093	4
5	·821927	·802451	·783526	·765392	·747258	5
6	·790314	·767896	·746215	·725587	·70496	6
7	·759918	·734828	·710681	·687869	·665057	7
8	·730690	·703185	·676839	·652125	·627412	8
9	·702587	·672904	·644609	·618253	·591898	9
10	·675564	·643928	·613913	·586153	·558394	10
11	·649581	·616199	·584679	·573733	·562787	11
12	·624597	·589664	·556837	·526903	·496969	12
13	·600574	·564271	·530321	·49958	·468839	13
14	·577475	·539973	·505068	·473684	·442301	14
15	·555264	·516720	·481017	·449141	·417265	15
16	·533908	·494469	·458311	·425979	·393647	16
17	·513373	·473176	·436297	·40383	·371364	17
18	·493628	·4528	·415521	·382932	·350343	18
19	·474642	·433302	·395734	·363123	·330513	19
20	·456387	·414643	·376889	·344346	·311804	20
21	·438833	·396787	·358942	·326568	·294155	21
22	·421955	·379701	·34185	·309677	·277505	22
23	·405726	·36335	·325571	·293684	·261797	23
24	·390121	·347703	·310068	·278523	·246978	24
25	·375117	·332731	·305303	·26915	·232998	25
26	·360689	·318402	·281241	·250525	·21981	26
27	·340816	·304691	·267848	·237608	·207368	27
28	·333477	·291571	·255094	·225362	·19563	28
29	·320651	·279015	·242946	·213715	·184556	29
30	·308309	·267	·231377	·202743	·17411	30
31	·290460	·255502	·220359	·192307	·164255	31
32	·285058	·2445	209866	·182411	·154957	32
33	·274094	·233971	·199872	·173029	·146186	33
34	·263552	·223896	·190355	·164133	·137912	34
35	·254415	·214251	·18129	·155692	·130105	35
36	·243669	·205028	·172057	·147399	·122741	36
37	·234297	·196299	·164436	·140114	·115793	37
38	·225285	·18775	·156605	·132893	·109182	38
39	·216671	·179665	·149148	·126075	·103002	39
40	·208289	·171929	·142016	·119608	·09717	40

TABLE

TABLE III. *Shewing the amount of £.1 or D.1 annuity for any number of years, from* 1 *to* 40.

ys.	4 *per cent.*	4½ *per cent.*	5 *per cent.*	5½ *per cent.*	6 *per cent.*	*ys.*
1	1·	1·	1·	1·	1·	1
2	2·04	2·045	2·05	2·055	2·06	2
3	3·1216	3·137025	3·1525	3·16802	3·1836	3
4	4·246464	4·278191	4·310125	4·34226	4·374616	4
5	5·416322	5·47071	5·525631	5·58109	5·637093	5
6	6·632975	6·716892	6·801913	6·888051	6·975318	6
7	7·898294	8·019152	8·142008	8·266894	8·393837	7
8	9·214266	9·380014	9·549109	9·721573	9·897467	8
9	10·582795	10·802114	11·026564	11·256259	11·491315	9
10	12·006107	12·2882	12·577892	12·875354	13·180794	10
11	13·486351	13·841179	14·206787	14·583498	14·971642	11
12	15·025805	15·464032	15·917126	16·38559	16·86994	12
13	16·626838	17·159913	17·712983	18·286798	18·882132	13
14	18·291911	18·932109	19·598632	20·292572	21·015064	14
15	20·023588	20·784054	21·578563	22·408663	23·275968	15
16	21·824531	22·719337	23·657492	24·64114	25·672527	16
17	23·697512	24·741707	25·840366	26·996402	28·212879	17
18	25·645413	26·855084	28·132385	29·481205	30·905652	18
19	27·671229	29·063562	30·529004	32·102671	33·759991	19
20	29·778078	31·371423	33·065954	34·868318	36·78559	20
21	31·969202	33·783137	35·719252	37·786075	39·992725	21
22	34·24797	36·303378	38·505214	40·864309	43·392289	22
23	36·617888	38·93703	41·430475	44·111846	46·995826	23
24	39·082604	41·689196	44·501999	47·537998	50·815576	24
25	41·645908	44·56521	47·727099	51·152588	54·86451	25
26	44·311745	47·570645	51·113454	54·96598	59·156381	26
27	47·084214	50·711324	54·669126	58·989109	63·705763	27
28	49·967583	53·993333	58·402583	63·23351	68·528109	28
29	52·966286	57·423033	62·322712	67·711353	73·639796	29
30	56·084938	61·007067	66·438847	72·435478	79·058183	30
31	59·328335	64·752388	70·76079	77·419429	84·801674	31
32	62·701469	68·666245	75·298829	82·677498	90·889775	32
33	66·209527	72·756226	80·063771	88·22476	97·343161	33
34	69·857908	77·030256	85·066959	94·077122	104·183751	34
35	73·652225	81·496618	90·320307	100·251363	111·434776	35
36	77·598314	86·163966	95.836323	106·765188	119·120863	36
37	81·702246	91·041344	107·628139	113·637274	127·268114	37
38	85·970336	96·138205	101·709546	120·887324	135·904201	38
39	90·40915	101·464424	114·095025	128·536127	145·058453	39
40	95·025516	107·030323	120·799774	136·605614	154·761961	30

TABLE IV. *Shewing the present worth of £.1 or D.1 annuity, for any number of years, from 1 to 40.*

ys.	4 per cent.	4½ per cent.	5 per cent.	5½ per cent.	6 per cent.
1	0·96154	0·95694	0·95238	0·94786	0·94339
2	1·88609	1·87267	1·85941	1·8463	1·83339
3	2·77509	2·74896	2·72325	2·6979	2·67301
4	3·62989	3·58752	3·54595	3·49862	3·4651
5	4·45182	4·38997	4·32948	4·25759	4·21236
6	5·24214	5·15787	5·07569	4·97699	4·92732
7	6·00205	5·8927	5·78637	5·65888	5·58238
8	6·73274	6·59589	6·46321	6·30522	6·20979
9	7·43533	7·26879	7·10782	6·91786	6·80169
10	8·11089	7·91272	7·72173	7·49856	7·36008
11	8·76048	8·52892	8·30641	8·04898	7·88687
12	9·38507	9·11858	8·86325	8·5707	8·38384
13	9·98565	9·68285	9·39357	9·06522	8·85268
14	10·56312	10·22282	9·89864	9·53395	9·29498
15	11·11839	10·73954	10·37966	9·97824	9·71225
16	11·65229	11·23401	10·83777	10·39936	10·10589
17	12·16567	11·70719	11·27407	10·79852	10·47726
18	12·65929	12·15099	11·68958	11·17687	10·8276
19	13·13394	12·59329	12·08532	11·53549	11·15811
20	13·59032	13·00793	12·46221	11·87541	11·46992
21	14·02916	13·40472	12·82115	12·1976	11·76407
22	14·45111	13·79442	13·163	12·50299	12·04158
23	14·85684	14·14777	13·48807	12·79245	12·30338
24	15·24696	14·49548	13·79864	13·06682	12·55035
25	15·62208	14·82821	14·09394	13·3683	12·78335
26	15·98277	15·14661	14·37518	13·57338	13·00316
27	16·32959	15·4513	14·64303	13·80702	13·21053
28	16·66306	15·74287	14·89813	14·02848	13·40616
29	16·98371	16·02189	15·14107	14·23838	13·59072
30	17·20202	16·28889	15·37245	14·43733	13·76483
31	17·58849	16·54439	15·59281	14·6259	13·92908
32	17·87355	16·78889	15·80268	14·80463	14·08398
33	18·14764	17·02286	16·00255	14·97404	14·22917
34	18·4112	17·24676	16·1929	15·13461	14·36613
35	18·66461	17·46101	16·37419	15·2868	14·49533
36	18·90828	17·66604	16·54685	15·43105	14·61722
37	19·14258	17·86224	16·71129	15·56779	14·73211
38	19·36787	18·04999	16·86789	15·6974	14·84048
39	19·58448	18·22965	17·01704	15·82024	14·9427
40	19·79277	18·40158	17·15909	15·93667	15·03913

TABLE

TABLE V. *The annuity which* 1£. *or* D.1 *will purchase for any number of years to come, from* 1 *to* 40.

ys.	4 *per cent.*	4½ *per cent.*	5 *per cent.*	5½ *per cent.*	6 *per cent.*
1	1·04	1·045	1·05	1·055	1·06
2	·5302	·534	·5378	·54162	·54544
3	·36035	·36377	·36721	·37065	·37411
4	·27549	·27874	·28201	·28582	·28859
5	·22463	·22779	·23097	·23487	·23739
6	·19076	·19388	·19702	·20092	·20336
7	·16661	·1697	·17282	·17671	·17913
8	·14853	·15161	·15473	·15859	·16103
9	·13449	·13757	·14069	·14455	·14702
10	·12329	·12638	·1295	·13334	·13587
11	·11415	·11725	·12039	·12424	·12679
12	·10655	·10967	·11282	·11667	·11927
13	·10014	·10327	·10645	·11031	·11296
14	·09467	·09782	·10102	·10489	·10758
15	·08994	·09311	·09624	·10022	·10296
16	·08582	·08901	·09227	·0962	·09895
17	·0822	·08542	·0887	·0926	·09544
18	·07899	·08224	·08555	·08947	·09235
19	·07614	·07941	·08274	·08699	·08962
20	·07359	·07688	·08024	·08427	·08718
21	·07128	·0746	·078	·08198	·085
22	·0692	·07254	·07597	·07998	·08303
23	·06731	·07068	·07414	·07825	·08128
24	·06559	·06899	·07247	·07653	·07968
25	·06401	·06744	·07095	·07503	·07823
26	·06257	·06602	·06956	·07367	·0769
27	·06124	·06472	·06829	·07242	·0757
28	·06001	·06352	·06712	·07128	·07459
29	·05888	·06241	·06604	·07023	·07358
30	·05783	·06139	·06505	·06926	·07272
31	·05685	·06044	·06413	·06837	·07179
32	·05595	·05956	·06328	·06754	·071
33	·0551	·05874	·06249	·06678	·07027
34	·05431	·05798	·06175	·06607	·06959
35	·05358	·05727	·06107	·06541	·06899
36	·05289	·0566	·06043	·0648	·06839
37	·05224	·05598	·05984	·06423	·06785
38	·05163	·0554	·05928	·0637	·06735
39	·05106	·05485	·05876	·06321	·06689
40	·05052	·05434	·05828	·06274	·06646

CIRCULATING

CIRCULATING DECIMALS

ARE produced from Vulgar Fractions, whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. The circulating figures are called *repetends;* and, if one figure only repeats, it is called a *single repetend:* As ·1111 &c. ·6666 &c.

2. A *compound repetend* has the same figures circulating alternately: As ·010101, &c. ·379379379, &c.

3. If other figures arise before those which circulate, the decimal is called a *mixed repetend;* thus, ·375555 &c. is a *mixed single repetend,* and ·378123123, &c. a *mixed compound repetend.*

4. A single repetend is expressed by writing only the circulating figure with a point over it; thus, ·1111, &c. is denoted by $\cdot\dot{1}$, and ·6666, &c. by $\cdot\dot{6}$.

5. Compound repetends are distinguished by putting a point over the first and last repeating figures; thus, ·010101, &c. is written $\cdot\dot{0}\dot{1}$, and ·379379379, &c. thus, $\cdot\dot{3}7\dot{9}$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after the decimal point; thus, $\cdot\dot{3}$ and $\cdot\dot{5}$ are similar circulates; as are also $\dot{3}\cdot\dot{5}4$ and $\dot{7}\cdot3\dot{6}$, &c.

7. *Dissimilar repetends* consist of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $47\cdot3\dot{4}57\dot{6}$, $9\cdot7\dot{3}52\dot{8}$ and $\cdot0\dot{5}46\dot{3}$, &c.

REDUCTION OF CIRCULATING DECIMALS.

CASE I.

To reduce a simple Repetend to its equivalent Vulgar Fraction.

RULE*.—1. Make the given decimal the numerator, and let the denominator, be a number, consisting of so many nines as there are recurring places in the repetend.

2. If

* If unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; that is, if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate $\cdot\dot{1}$, and fince $\cdot\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $\cdot\dot{2}$ will $= \frac{2}{9}$, $\dot{3} = \frac{3}{9}$, and fo on till $\cdot\dot{9} = \frac{9}{9} = 1$. Therefore every fingle repetend is equal to a vulgar fraction, whofe numerator is the repeating figure and denominator 9.

Again, $\frac{1}{99}$ or $\frac{1}{999}$ being reduced to decimals, make ·010101, &c. and ·001001001, &c. *ad infinitum* $= \cdot\dot{0}\dot{1}$ and $\cdot\dot{0}0\dot{1}$; that is, $\frac{1}{99} = \cdot\dot{0}\dot{1}$, and $\frac{1}{999} = \cdot\dot{0}0\dot{1}$, confequently $\frac{2}{99} = \cdot\dot{0}\dot{2}$, $\frac{3}{99} = \cdot\dot{0}\dot{3}$, &c. and $\frac{2}{999} = \cdot\dot{0}0\dot{2}$, $\frac{3}{999} = \cdot\dot{0}0\dot{3}$, &c. and the fame will hold univerfally.

2. If there be integral figures in the circulate, so many cyphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

Examples.

1. Required the least vulgar fractions equal to $\cdot\dot{3}$ and $\cdot\dot{3}2\dot{4}$.

$\cdot\dot{3}=\frac{3}{9}=\frac{1}{3}$; and $\cdot\dot{3}2\dot{4}=\frac{324}{999}=\frac{12}{37}$ Ans. $\frac{1}{3}$ and $\frac{12}{37}$.

2. Reduce $\cdot\dot{7}$ to its equivalent vulgar fraction. Ans. $\frac{7}{9}$.

3. Reduce $\dot{2}\cdot3\dot{7}$ to its equivalent vulgar fraction, Ans. $\frac{2370}{999}$.

4. Required the least vulgar fraction equal to $\cdot\dot{3}8461\dot{5}$. Ans. $\frac{5}{13}$.

CASE II.

To reduce a mixed Repetend to its equivalent Vulgar Fraction.

Rule.*—1. To so many nines as there are figures in the repetend, annex so many cyphers as there are finite places, (that is, as there are decimal places before the repetend) for a denominator.

2. Multiply the nines in the said denominator by the finite part, and add the repeating decimals to the product for the numerator.

3. If the repetend begins in some integral place, the finite value of the circulating part must be added to the finite part.

Examples.

1. What is the vulgar fraction equivalent to $\cdot15\dot{3}$?

There being 1 figure in the repetend, and 2 finite places, I annex 2 cyphers to 9 for a denominator, viz. 900; then I multiply the 9 in the denominator by the two figures in the finite part, and add the repeating figure for a numerator; thus, $9\times15+3=138$ numerator.

Therefore, $\cdot15\dot{3}=\frac{138}{900}=\frac{23}{150}$ the Ans.

2. What is the least vulgar fraction equal to $\cdot4\dot{1}2\dot{3}$? Ans. $\frac{4079}{9990}$.

3. Required the finite number equivalent to $4\dot{5}\cdot7\dot{8}$? Ans. $45\frac{785}{999}$.

CASE

* In like manner for a mixed circulate; consider it as divisible into its finite and circulating parts, and the same principle will be seen to run through them also; thus the mixed circulate $\cdot1\dot{3}$ is divisible into the finite decimal $\cdot1$, and the repetend $\cdot0\dot{3}$: but $\cdot1=\frac{1}{10}$, and $\cdot0\dot{3}$ would be equal to $\frac{3}{9}$ provided the circulation began immediately after the place of units; but as it begins after the place of tenths, it is $\frac{3}{9}$ of $\frac{1}{10}=\frac{3}{90}$, and so the vulgar fraction $=\cdot1\dot{3}$ is $\frac{1}{10}+\frac{3}{90}=\frac{9}{90}+\frac{3}{90}=\frac{12}{90}$, and is the same as by the rule.

CASE III.

To make any number of dissimilar repetends similar and conterminous; that is, of an equal number of places.

RULE.*

Change them into other repetends, which shall each consist of so many figures, as the least common multiple of the sums of the several numbers of places, found in all the repetends, contains units.

EXAMPLES.

1. Make $6{\cdot}\dot{3}1\dot{7}$; $3{\cdot}4\dot{5}$; $52{\cdot}3$; $191{\cdot}\dot{0}\dot{3}$; $\cdot\dot{0}5\dot{7}$; $5{\cdot}\dot{3}$ and $1{\cdot}35\dot{9}$ similar and conterminous.

Here, in the first repetend, there are three places, in the second, one, in the third, none, in the fourth, two, in the fifth, three, in the sixth, one, and in the seventh, one.

Now find the least common multiple of these several sums, thus:

$$3\,\underline{)\,3,\ 1,\ 2,\ 3,\ 1,\ 1}$$
$$\quad 1,\ 1,\ 2,\ 1,\ 1,\ 1$$

and $2\times3=6$ units; therefore, the similar and conterminous repetends must contain 6 places.†

Dissimilar made similar and conterminous.

$$6{\cdot}\dot{3}1\dot{7} = 6{\cdot}31\dot{7}3173\dot{1}$$
$$3{\cdot}4\dot{5} = 3{\cdot}45\dot{5}5555\dot{5}$$
$$52{\cdot}3 = 52{\cdot}30\dot{0}0000\dot{0}$$
$$191{\cdot}\dot{0}\dot{3} = 191{\cdot}03\dot{0}3030\dot{3}$$
$$\cdot\dot{0}5\dot{7} = \cdot05\dot{7}0570\dot{5}$$
$$5{\cdot}\dot{3} = 5{\cdot}33\dot{3}3333\dot{3}$$
$$1{\cdot}35\dot{9} = 1{\cdot}35\dot{9}9999\dot{9}$$

2. Make $\cdot\dot{5}3\dot{1}$, $\cdot73\dot{1}\dot{8}$, $\cdot\dot{0}\dot{7}$ and $\cdot05\dot{0}\dot{3}$ similar and conterminous.

CASE IV.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will consist of.

RULE.‡—1. Reduce the given fraction to its least terms, and divide the denominator by 2, 5 or 10, as often as possible. 2. Divide

* Any given repetend whatever, whether single, compound, pure, or mixed, may be transformed into another repetend, which shall consist of an equal or greater number of figures at pleasure; thus, $\cdot\dot{3}$ may be transformed into $\cdot\dot{3}\dot{3}$, or $\cdot\dot{3}3\dot{3}$, &c. also $\cdot\dot{7}\dot{9}=\cdot\dot{7}97\dot{9}=\cdot7\dot{9}\dot{7}$, and so on.

† The learner may observe that the similar and conterminous repetends begin just so far from unity, as is the farthest among the dissimilar repetends; and is so in all cases.

‡ In dividing 1·000, &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as soon as the remainder is

 1: and

2. Divide 9999, &c. by the former result, till nothing remain, and the number of 9s used will show the number of places in the repetend; which will begin after so many places of figures as there were 10s, 2s, or 5s, divided by.

If the whole denominator vanish in dividing by 2, 5 or 10, the decimal will be finite, and will consist of so many places as you perform divisions.

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{475}{2800}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

$$\text{First } 25\Big)\frac{475}{2800}=\frac{19}{112} \qquad 2\Big)\,112=\overset{(2)}{56}=\overset{(2)}{28}=\overset{(2)}{14}=7.$$

Then, $\dfrac{7)999999}{142857}$; therefore, because the denominator 112 did not vanish in dividing by 2, the decimal is infinite, and, as six 9s were used, the circulate consists of 6 places, beginning at the fifth place, because four 2s were used in dividing.

2. Let $\frac{2}{11}$ be the fraction proposed.

3. Let $\frac{3}{7}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE.—1. Make the repetends *similar and conterminous*, and find their sum as in common addition.

2. Divide this sum (of the repetends only) by so many nines as there are places in the repetend, and the remainder is the repetend of their sum; which must be set under the figures added, with cyphers on the left hand, when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest as infinite decimals.

EXAMPLES.

1: and ſince 999, &c. is leſs than 1000, &c. by 1, therefore 999, &c. divided by any number whatever, will, when the repeating figures are at their period, leave 0 for a remainder.

Now, whatever number of repeating figures we have, when the dividend is 1, there will be exactly the ſame number, when the dividend is any other number whatever.

Thus, let ·390539053905, &c, be a circulate, whoſe repeating part is 3905. Now, every repetend (3905,) being equally multiplied, muſt give the ſame product: For although theſe products will conſiſt of more places, yet the overplus in each, being alike, will be carried to the next, by which means, each product will be equally increaſed, and conſequently every four places will continue alike. And the ſame will hold for any other number.

Now from hence it appears that the dividend may be altered at pleaſure, and the number of places in the repetend will ſtill be the ſame; thus, $\frac{1}{11}=\cdot\dot{0}\dot{9}$; and $\frac{4}{11}$, or $\frac{1}{11}\times4=\cdot\dot{3}\dot{6}$, whence the number of places in each are alike.

EXAMPLES.

1. Let $5\cdot\dot{3}+59\cdot4\dot{3}5\dot{6}+397\cdot\dot{6}+519+\cdot\dot{3}\dot{9}+217\cdot\dot{5}$ be added together.

$$
\begin{array}{rcr}
5\cdot\dot{3} & = & 5\cdot3\dot{3}33333\dot{3} \\
59\cdot4\dot{3}5\dot{6} & = & 59\cdot4\dot{3}5635\dot{6} \\
397\cdot\dot{6} & = & 397\cdot6\dot{6}66666\dot{6} \\
519\cdot & = & 519\cdot0\dot{0}00000\dot{0} \\
\cdot\dot{3}\dot{9} & = & \cdot3\dot{9}39393\dot{3} \\
217\cdot\dot{5} & = & 217\cdot5\dot{5}55555\dot{5} \\
\hline
 & & 1199\cdot3851303 \\
\hline
 & & 1199\cdot3\dot{8}51305\dot{5}
\end{array}
$$

the sum.

In this question, the sum of the repetends is 2851303, which divided by 999999, gives 2 to carry to the next column 5,3,0, &c. and the remainder is 851305.

2. Let $3275\cdot319+36\cdot\dot{4}5+123\cdot1\dot{9}+5\cdot317\dot{3}+112\cdot351\dot{3}+11\cdot13\dot{1}+\cdot12\dot{5}$ $+29\cdot1005\dot{3}$ be added together. Ans. $3593\cdot0004\dot{2}$.

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.

Make the repetends *similar* and *conterminous*, and subtract as usual, observing, that if the repetend of the number to be subtracted be greater than the repetend of the number it is to be taken from, then the right hand of the remainder must be less by unity than it would be if the expressions were finite.

EXAMPLES.

1. From $57\cdot\dot{0}\dot{3}$ take $29\cdot7\dot{3}58\dot{7}$

$$
\begin{array}{rcr}
57\cdot\dot{0}\dot{3} & = & 57\cdot03\dot{0}30\dot{0} \\
23\cdot7\dot{3}58\dot{7} & = & 29\cdot7\dot{3}58\dot{7} \\
\hline
 & & 27\cdot2\dot{9}44\dot{2}
\end{array}
$$

the difference.

2. From $325\cdot\dot{1}7$ take $137\cdot581\dot{9}$. Ans. $187\cdot595\dot{7}$.

MULTIPLICATION

MULTIPLICATION OF CIRCULATING DECIMALS.

RULE.

1. Turn both the terms into their equivalent vulgar fractions, and find the product of those fractions as usual.

2. Turn the vulgar fraction, expressing the product, into an equivalent decimal one, and it will be the product required.

EXAMPLES.

1. Multiply $\cdot\dot{5}\dot{4}$ by $\cdot 1\dot{5}$. $\cdot\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$ and $\cdot 1\dot{5} = \frac{14}{90} = \frac{7}{45}$

$\frac{6}{11} \times \frac{7}{45} = \frac{42}{495} = \cdot 0\dot{8}\dot{4}$ the product.

2. Multiply $378\cdot\dot{5}$ by $23\cdot\dot{6}$. Ans. $8959\cdot\dot{1}4\dot{8}$.

DIVISION OF CIRCULATING DECIMALS.

RULE.

1. Change both the divisor and dividend into their equivalent vulgar fractions, and find their quotient as usual.

2. Turn the vulgar fraction, expressing the quotient, into its equivalent decimal, and it will be the quotient required.

EXAMPLES.

1. Divide $\cdot\dot{5}\dot{4}$ by $\cdot 1\dot{5}$.

$\cdot\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$ and $\cdot 1\dot{5} = \frac{14}{90} = \frac{7}{45}$

$\frac{6}{11} \div \frac{7}{45} = \frac{6}{11} \times \frac{45}{7} = \frac{270}{77} = 3\frac{39}{77} = 3\cdot\dot{5}0649\dot{3}$ the quotient.

2. Divide $345\cdot\dot{8}$ by $\dot{6}$. Ans. $518\cdot 8\dot{3}$.

ALLIGATION

IS the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality; It consists of two kinds, viz. Alligation Medial, and Alligation Alternate.

ALLIGATION MEDIAL

Is, when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

RULE.

As the sum of the quantities, or the whole composition, is to their total value; so is any part of the composition to its mean price or value.

EXAMPLES.

EXAMPLES.

1. A Tobacconist would mix 60lb. of tobacco, at 6d. per lb. with 50lb. at 1s. 40lb. at 1s. 6d. and 30lb. at 2s. per lb.: What is 1lb. of this mixture worth?

	lb.		s.	d.		£.	s.
	60	at	0	6	is	1	10
	50	—	1	0	—	2	10
	40	—	1	6	—	3	0
	30	—	2	0	—	5	0
Sum of the simples,	180	Total value				10	0

lb. £. lb.
As 180 : 10 : 1
1
—
10
20
—
180)200(1s.
180
—
20
12
—
180)240($1\frac{1}{3}$d. s. d.
180 Ans. 1 $1\frac{1}{3}$pr.lb.
—
60

2. A farmer would mix 20 bushels of wheat at D.1 per bushel, 16 bushels of rye at 75c. per bushel, 12 bushels of barley at 50c. per bushel, and 8 bushels of oats at 40c. per bushel: What is the value of one bushel of this mixture? Ans. 73c. $5\frac{5}{7}$m.

3. A wine merchant mixes 12 gallons of wine, at 75c. per gallon, with 24 gallons, at 90c. and 16 gallons at D.1 10c.: What is a gallon of this composition worth? Ans. 92c. 6m.

4. A goldsmith melted together 8oz. of gold of 22 carats fine, 1lb. 8oz. of 21 carats fine, and 10oz. of 18 carats fine: Pray what is the quality, or fineness of the composition?

$$\frac{\overline{8\times22}+\overline{20\times21}+\overline{10\times18}}{8+20+10}=20\tfrac{8}{19}\text{ carats fine, Ans.}$$

5. A refiner melts 5lb. of gold of 20 carats fine with 8lb. of 18 carats fine: How much alloy must be put to it, to make it 22 carats fine?

$$22-\overline{\overline{5\times20}+\overline{8\times18}\div\overline{5+8}}=3\tfrac{3}{13}$$

Answer. It is not fine enough by $3\frac{3}{13}$ carats, so that no alloy must be added, but more gold.

*ALLIGATION ALTERNATE**

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate: So that it is the reverse of Alligation Medial, and may be proved by it.

CASE

* *Demon.* By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss, upon the whole, are equal, and are exactly the proposed rate. In

CASE I.

The whole work of this case consists in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities sought.

Rule.—1. Place the several prices of the simples, being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase, with a line of connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those, which are greater than the compound, and each greater rate or price with one or any number of the less.

3. Place the difference, between the mean price (or mixture rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

Examples.

1. A merchant has spices, some at 1s. 6d. per lb. some at 2s. some at 4s. and some at 5s. per lb.: How much of each sort must he mix that he may sell the mixture at 3s. 4d. per lb.?

	d.	lb.	s. d.	
Mean rate 40d.	18	20 at	1 6	per lb.
	24	8 —	2 0	
	48	16 —	4 0	
	60	22 —	5 0	

	d.	lb.	s. d.	
40d.	18	8 at	1 6	per lb.
	24	20 —	2 0	
	48	22 —	4 0	
	60	16 —	5 0	

d.	d.		lb.	s. d.	
40	18	20+ 8	28 at	1 6	per lb.
	24	8	8 —	2 0	
	48	16+22	38 —	4 0	
	60	22	22 —	5 0	

d.	d.		lb.	s. d.	
40	18	20	20 at	1 6	per lb.
	24	8+20	28 —	2 0	
	48	16	16 —	4 0	
	60	22+16	38 —	5 0	

40d.	18	8	8 at	1s. 6d.	per lb.
	24	8+20	28	2	
	48	22+16	38	4	
	60	16	16	5	

40d.	18	8+20	28 at	1s. 6d.	per lb.
	24	20	20	2	
	48	22	22	4	
	60	22+16	38	5	

40d.

In like manner, let the number of ſimples be what it may, and with how many ſoever, each one is linked, ſince it is always a leſs with a greater than the mean price, there will be an equal balance of loſs and gain between every two, and conſequently an equal balance on the whole.

It is obvious from the rule, that queſtions of this ſort admit of a great variety of anſwers; for having found one anſwer, we may find as many more as we pleaſe, by only multiplying or dividing each of the quantities, found by 2, 3, 4, &c. the reaſon of which is evident; for if two quantities of two ſimples make a balance of loſs and gain with reſpect to the mean price, ſo muſt alſo the double or triple, the half or third part, or any other ratio of theſe quantities, and ſo on *ad infinitum.*

If any one of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be nothing, as water mixed with wine, and alloy with gold and ſilver.

40d.	18	20+ 8	28 at 1 6	per lb.	
	24	8+20	28 – 2 0		
	48	16+22	38 – 4 0		
	60	22+16	38 – 5 0		

Note. These seven answers arise from as many various ways of linking the rates of the ingredients together.

2. *A merchant has Canary wine, at 3s. per gallon, Sherry, at 2s. 1d. and Claret at 1s. 5d. per gallon : How much of each sort must he take, to sell it at 2s. 4d. per gallon ?

Mean rate 28d.	36	3+11	14 at 3 0	per gallon.
	25	8	8 2 1	
	17	8	8 1 5	

3. How much barley at 40c. rye at 60c. and wheat at 80c. per bushel, must be mixed together, that the compound may be worth $62\frac{1}{2}$c. per bushel ?

Ans. $17\frac{1}{2}$ bushels of barley, $17\frac{1}{2}$ of rye, and 25 of wheat.

4. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine : What quantity of each must he take ?

Ans. 5oz. of 16 carats fine, 5oz. of 18, 5oz. of 19, 10oz. of 23, and 10oz. of 24 carats fine.

5. It is required to mix several sorts of wine, at 60c. 90c. and D.1 15c. per gallon, with water, that the mixture may be worth 75c. per gallon : Of how much of each sort must the composition consist ?

Ans. 40 galls. of water, 15 galls. of wine, at 60c. 15 galls. do. at 90c. and 75 galls. do. at D.1 15c.

CASE II.

When the rates *of* all *the* ingredients, *the* quantity *of but* one *of them, and the* mean rate *of the whole mixture are given, to find the several quantities of the rest, in proportion to the quantity given.*

Rule.

Take the differences between each price, and the mean rate, and place them alternately, as in Case 1. Then, as the difference standing against that simple, whose quantity is given, is to that quantity, so is each of the other differences, severally, to the several quantities required.

Examples.

1. A merchant has 40lb. of tea, at 6s. per lb. which he would mix with some at 5s. 8d. some at 5s. 2d. and some at 4s. 6d. : How much of each sort must he take, to mix with the 40lb. that he may sell the mixture at 5s. 5d. per lb. ?

			lb.	
65	54	7+3	10	
	62	3+7	10	
	68	3+11	14	
	72	11+3	14	stands against the given quantity.

As

* Note, the 2d. and 3d. questions admit but of one way of linking, and so but one answer ; yet all numbers in the same proportion between themselves, as the numbers, which compose the answer, will likewise satisfy the condition of the question.

$$\text{As } 14 : 40 :: \left\{\begin{array}{l} \text{lb.} \quad \text{lb.} \qquad \text{s. d.} \\ 10 : 28\tfrac{8}{14} \text{ at } 4\ 6 \\ 10 : 28\tfrac{8}{14} \text{ — } 5\ 2 \\ 14 : 40 \quad \text{ — } 5\ 8 \end{array}\right\} \text{per lb.}$$

2. A farmer being determined to mix 20 bushels of oats, at 60c. per bushel, with barley, at 75c. rye, at D.1, and wheat, at D.1·25c. per bushel; I demand the quantity of each, which must be mixed with the 20 bushels of oats, that the whole quantity may be worth 90c. per bushel? Ans. 70 of barley, 60 of rye, and 30 of wheat, (or 20 of each.)

3. How much gold of 16, 20 and 24 carats fine, and how much alloy, must be mixed with 10 oz. of 18 carats fine, that the composition may be 22 carats fine.

Ans. 10oz. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

Alternation Total.*

CASE III.

When the rates *of the several ingredients, the* quantity *to be compounded, and the* mean rate *of the whole mixture are given, to find how much of each sort, will make up the quantity.*

Rule.

Place the differences between the mean rate, and the several prices alternately, as in Case 1; then, as the sum of the quantities, or differences thus determined, is to the given quantity, or whole composition; so is the difference of each rate, to the required quantity of each rate.

Examples.

1. Suppose I have 4 sorts of currants, at 8d. 12d. 18d. and 22d. per lb.; the worst will not sell, and the best are too dear; I therefore conclude to mix 120lb. and so much of each sort as to sell them at 16d. per lb.; how much of each sort must I take?

16d.

* To this Case belongs that curious question concerning king Hiero's crown.

Hiero, king of Syracuse, gave orders for a crown to be made, entirely of pure gold; but suspecting the workmen had debased it, by mixing with it silver or copper, he recommended the discovery of the fraud to the famous Archimides, and desired to know the exact quantity of alloy in the crown.

Archimides, in order to detect the imposition, procured two other masses, one of pure gold, and the other of silver, or copper, and each of the same weight with the former; and by putting each separately into a vessel *full* of water, the quantity of water expelled by them, determined their specifick bulks; from which, and their given weights, it is easier to determine the quantities of gold and alloy in the crown by this case of Alligation, than by an Algebraick process.

Suppose the weight of each mass to have been 5lb. the weight of the water expelled by the alloy, 23oz. by the gold, 13oz. and by the crown 16oz. that is, that their specifick bulks were as 23, 13, and 16; then, what were the quantities of gold and alloy respectively in the crown?

Here, the rates of the simples are 23 and 13, and of the compound 16, whence,

$16\begin{cases}13 & 7 \text{ of gold} \\ 23 & 3 \text{ of alloy}\end{cases}$ And the sum of these is $7+3=10$, which should have been but 5, whence, by the rule,

$$10 : 5 :: \begin{cases}7 : 3\tfrac{1}{2}\text{lb. of gold} \\ 3 : 1\tfrac{1}{2}\text{lb. of alloy}\end{cases} \text{the Answer.}$$

$$\begin{array}{cc} & \text{d.} \quad \text{lb.} \\ 16\text{d.}\left\{\begin{array}{l}8\\12\\18\\22\end{array}\right. & \begin{array}{l}6\\2\\4\\8\\ \hline \text{Sum}=20\end{array}\end{array} \quad \begin{array}{c}\text{lb.} \quad \text{lb.}\\ \text{As } 20 : 120 ::\end{array} \left\{\begin{array}{l}\text{lb. lb.}\\ 6 : 36 \text{ at } 8\text{d.}\\ 2 : 12 — 12\text{d.}\\ 4 : 24 — 18\text{d.}\\ 8 : 48 — 22\text{d.}\\ \hline 120\end{array}\right\} \text{per lb.}$$

2. A goldsmith has several sorts of gold; viz. of 15, 17, 20 and 22 carats fine, and would melt together, of all these sorts, so much as may make a mass of 40oz. 18 carats fine; how much of each sort is required?

Ans. 16oz. 15 carats fine, 8oz. 17, 4oz. 20, and 12oz. of 22 carats fine.

3. A merchant would mix 4 sorts of wine, of several prices, viz. at 75c. 1D. 25c. 1D. 50c. and 1D. 62½c. per gallon; of these he would have a mixture of 60 gallons, worth 7s. per gallon; what quantity of each sort must he have?

Ans. 8 at 75c. 16 at 1D. 25c. 40 at 1D. 50c. and 8 at 1D. 62½c. Or, 16 at 75c. 8 at 1D. 25c. 8 at 1D. 50c. and 40 at 1D. 62½c.

4. How many gallons of water, of no value, must be mixed with wine, at 4s. per gallon, so as to fill a vesssl of 80 gallons, that may be afforded at 2s. 9d. per gallon?

$$\begin{array}{cc} & \text{Gal.} \\ 33\left\{\begin{array}{l}0\\48\end{array}\right. & \begin{array}{l}15\\33\\ \hline \text{Sum } 48\end{array}\end{array} \quad \begin{array}{c}\text{Gal. Gal.}\\ \text{As } 48 : 80 ::\end{array} \left\{\begin{array}{l}\text{Gal.}\\ 15 : 25 \text{ gallons of water.}\\ 33 : 55 \text{ gallons of wine.}\end{array}\right\} \text{Ans.}$$

CASE IV.*

When more than one of the simples are limited.

Rule.

Find, by Alligation Medial, what will be the *rate* of a mixture made of the *given quantities* of the *limited simples* only; then, consider this as the *rate* of a *limited simple*, whose *quantity* is the sum of the first given *limited simples*, from which, and the *rates* of the *unlimited simples*, by Case 2d. calculate the quantity.

Examples.

1. How much wine, at 80c. and at 87½c. per gallon, must be mixed with 8 gallons at 75c. and 12 galls. at 90c. per gallon, that the mixture may be worth 82½c. per gallon?

$$\text{Limited simples}\left\{\begin{array}{l}8 \text{ gallons, at } 75\text{c.} = \text{D. } 6\\ 12 \text{ gallons, at } 90 \;\; = \;\; 10 \;\; 80\text{c.}\end{array}\right\}$$

$$\begin{array}{lr} 20 & 16 \;\; 80 \end{array}$$

Gal. D. c. Gal. c.
As 20 : 16 80 :: 1 : 84 per gallon.

Now,

* The three last Cases need no demonstration, as the 2d. and 3d. evidently result from the first, and the last, from Alligation Medial, and the second Case in Alternate,

Now, having found the rate of the limited simples, the question may stand thus: How much wine, at 80c. and $87\frac{1}{2}$c. per gallon, must be mixed with 20 gallons at 84c. per gallon, that the mixture may be worth $82\frac{1}{2}$c. per gallon?

$$82\tfrac{1}{2}\begin{cases}80 \\ 84 \\ 87\tfrac{1}{2}\end{cases} \quad \begin{matrix}1\tfrac{1}{2}+5 \\ 2\tfrac{1}{2} \\ 2\tfrac{1}{2}\end{matrix} \quad \begin{array}{|l}6\tfrac{1}{2} \text{ gallons, at 80c.} \\ 2\tfrac{1}{2} \text{ ——— 84} \\ 2\tfrac{1}{2} \text{ ——— } 87\tfrac{1}{2}\end{array}$$

$$\text{As } 2\tfrac{1}{2} : \begin{Bmatrix}6\tfrac{1}{2} \\ 2\tfrac{1}{2}\end{Bmatrix} :: 20 : \begin{cases}52 \text{ gallons, at 80c. per gallon.} \\ 20 \text{ ——— } 87\tfrac{1}{2} \text{ ———}\end{cases}\Bigg\} \text{Answer.}$$

Proof.

52	gallons at 80c.	=	D.41	60c.
20	——— $87\frac{1}{2}$	=	17	50
8	——— 75	=	6	
12	——— 90	=	10	80
92	——— $82\frac{1}{2}$	=	75	90

2. How much gold, of 14 and 16 carats fine, must be mixed with 6 oz. of 19, and 12 of 22 carats fine, that the composition may be 20 carats fine? Ans. $1\frac{8}{10}$ oz. of each sort.

POSITION.

POSITION is a rule, which, by false or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts; *single* and *double.*

SINGLE POSITION.

Single Position teaches to resolve those questions, whose results are proportional to their suppositions: such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times.

RULE.*—1. Take any number, and perform the same operations with it as are described to be performed in the question.

2. Then say, as the sum of the errours is to the given sum, so is the supposed number, to the true one required.

Proof. Add the several parts of the sum together, and if it agrees with the sum, it is right.

EXAMPLES.

* The reaſon of this rule is obvious, it being evident that the reſults are proportional to the ſuppoſitions,

$$\text{Thus,}\begin{cases} nx : x :: na : a \\ \dfrac{x}{n} : x :: \dfrac{a}{n} : a \\ \dfrac{x}{n} \pm \dfrac{x}{m} \text{\&c.} : x :: \dfrac{a}{n} \pm \dfrac{a}{m} \text{ \&c.} : a, \text{ and ſo on.} \end{cases}$$

EXAMPLES.

1. A school-master, being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I should then have 435: Of what number did his school consist?

Suppose he had 80	As 290 : 435 :: 80	
As many = 80	80	
$\frac{3}{4}$ as many = 60	———	120
$\frac{1}{2}$ as many = 40	29|0)3480|0(120 Ans.	120
$\frac{1}{4}$ as many = 20	29	90
$\frac{1}{8}$ as many = 10	——	60
——	58	30
290	58	15
	——	——
	0	435 Proof.

2. A person lent his friend a sum of money unknown, to receive interest for the same at 6 per cent. per annum, simple interest, and, at the end of 12 years, received for principal and interest 860D.: What was the sum lent? Ans. D.500.

3. A, B and C joined their stocks, and gained D.353 12½c. of which A took up a certain sum, B took up four times so much as A, and C, five times so much as B: What share of the gain had each?

Ans. { D. 14 12½c. A's share. / 56 50 B's share. / 282 50 C's share. }

4. A, B, C and D spent 35s. at a reckoning, and, being a little dipped, they agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$: What did each pay in the above proportion?

	s.	d.
Ans. A,	13	4
B,	10	0
C,	6	8
D,	5	0

5. A certain sum of money is to be divided between 5 men, in such a manner as that A shall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is £.40: What is the sum?

Suppose 200l. then $\frac{1}{4}+\frac{1}{5}+\frac{1}{10}+\frac{1}{20}$ = 120.

200—120=80. As 80 : 40 :: 200 : 100 Ans.

6. A person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had 26⅔l. left: What had he at first? Ans. £.160.

7. A and B, talking of their ages, B said his age was once and an half the age of A; C said his was twice and one tenth the age of both, and that the sum of their ages was 93: What was the age of each? Ans. A's 12, B's 18, and C's 63 years.

8. A vessel has 3 cocks, A, B and C; A can fill it in $\frac{1}{2}$ an hour, B in $\frac{1}{4}$ of an hour, and C in $\frac{1}{3}$ of an hour: In what time will they all fill it together? Ans. $\frac{1}{9}$ hour.

9. A person having about him a certain number of dollars, said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 57: Pray, how many had he? Ans. 60.

10. A

10. A gentleman bought a chaise, horse and harness, for 500D. the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{5}$ more than the horse: What was the price of each?

Ans.	Harness	127D.	65c.	$9\frac{27}{47}$m.
	Horse	159	57	$4\frac{22}{47}$
	Chaise	212	76	$5\frac{45}{47}$

11. A and B, having found a purse of money, disputed who should have it: A said that $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to £.35, and, if B could tell him how much was in it, he should have the whole, otherwise he should have nothing: How much did the purse contain?

Ans. £.100.

12. A gentleman divided his fortune among his sons; to A he gave 9D. as often as to B 5D. and to C but 3D. as often as to B 7D. yet C's portion came to 1059D.: What was the whole estate?

Ans. 7979D. 80c.

13. Seven eighths of a certain number exceeds four fifths by 6: What is that number? Ans. 80.

14. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itself, the sum will be $234\frac{3}{4}$? Ans. 90.

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

Those questions, in which the results are not proportional to their positions, belong to this rule: such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

Rule.*

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the result or errours against their positions or supposed numbers, thus,

Pos.		Err.
30		12
	X	
20		6

and if the errour be too great, mark it with +; and if too small with —.

3. Multiply

* The rule is founded on this ſuppoſition, that the firſt errour is to the ſecond, as the difference between the true and firſt ſuppoſed number is to the difference between the true and ſecond ſuppoſed number: When that is not the caſe, the exact anſwer to the queſtion cannot be found by this rule.

That the rule is true according to the ſuppoſition, may be thus demonſtrated:

Let A and B be any two numbers produced from a and b by ſimilar operations, it is required to find the number from which N is produced by a like operation.

Put x = number required, and let $N-A=r$, and $N-B=s$. Then, according to the ſuppoſition on which the rule is founded, $r:s::x-a:x-b$, whence, by multiplying means and extremes, $rx-rb=sx-sa$; and by tranſpoſition, $rx-sx=rb-sa$; and by diviſion, $x=\frac{rb-sa}{r-s}$ = number ſought; and if r and s be both negative, the Theorem is the ſame, and if r or s be negative, x will be equal to $\frac{rb+sa}{r+s}$ which is the rule.

3. Multiply them crosswise ; that is, the first position by the last errour, and the last position by the first errour.

4. If the errours be alike, that is, both too small or both too great, divide the difference of the products by the difference of the errours, and the quotient will be the answer.

5. If the errours be unlike ; that is, one too small, and the other too great, divide the sum of the products by the sum of the errours, and the quotient will be the answer.

Note. When the errours are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

1. A lady bought damask for a gown, at 8s. per yard, and lining for it at 3s. per yard ; the gown and lining contained 15 yards, and the price of the whole was 3l. 10s. : How many yards were there of each ?

Suppose 6 yards damask, value 48s.
Then she must have 9 yards lining, value 27s.

Sum of their values = 75s.
So that the first errour is 5 too much, or + 5
Again, suppose she had 4 yards, of damask, value 32s.
Then she must have 11 yards of lining, value 33s.

Sum of their values = 65s.
So that the second errour is 5 too little, or — 5s.

Then 6 × 5+ ; 4 × 5—

				£.	s.	d.
20	30	5 yards at 8s. =		2	0	0
	20	10 yards at 3s. =		1	10	0
				3	10	0 proof.

Sum of errours = 5+5 = 10)50

Ans. 5 yds. damask, and 15-5=10 yds. lining.
Or, 6+4÷2=5 as before.

2. A and B have the same income ; A saves $\frac{1}{8}$ of his ; but B, by spending 30l. per annum more than A, at the end of 8 years finds himself 40l. in debt ; what is their income, and what does each spend per annum :

Suppose { 80 × 120+ ; 160 × 40+

Ans. Their income is 200l. per annum. Also, A spends 175l. and B 205l. per annum. Then, 80—10=70 A's expense per annum, and 70+30=100, B's expense per annum. Then $\overline{100\times8}-\overline{80\times8}=160$, which should have been 40 ; therefore, 160—40=120 more than it should be, for the first errour. In like manner proceed for the second errour.

3. A and B laid out equal sums of money, in trade : A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost D.225, then A's money was double that of B : What did each lay out ?

Suppose

Suppose $\begin{cases} 300 \\ 900 \end{cases}$ X $\begin{matrix} 225+ \\ 225- \end{matrix}$ Ans. D.600.

4. A labourer was hired for 60 days, upon this condition, that, for every day he wrought, he should receive 75c.; and for every day he was idle, should forfeit $37\frac{1}{2}$c.; at the expiration of the time he received D.18: How many days did he work, and how many was he idle?

Suppose he worked $\begin{cases} 20 \\ 40 \end{cases}$ X $\begin{matrix} 13- \\ 48+ \end{matrix}$

Ans. He was employed 36 days, and was idle 24.

5. A gentleman has two horses of considerable value, and a carriage worth 100l.; now if the first horse be harnessed in it, he and the carriage together will be triple the value of the second; but if the second be put in they will be 7 times the value of the first: What is the value of each horse?

Suppose $\begin{matrix} 32 \\ 44 \end{matrix}$ X $\begin{matrix} 80- \\ 160- \end{matrix}$ Ans. One 20l. and the other 40l.

6. There is a fish, whose head is 10 feet long; his tail is as long as his head and half the length of his body, and his body as long as the head and tail: What is the whole length of the fish?

First, suppose the body 20 X 10—
2d. suppose it 30 X 5—

Head = 10
Tail = 30
Body = 40
—
Ans. 80 feet.

7. What number is that, which, being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be doubled?

Suppose $\begin{matrix} 8 \\ 16 \end{matrix}$ X $\begin{matrix} 3+ \\ 1+ \end{matrix}$ Ans. 20.

8. A farmer, having driven his cattle to market, received for them all D.320, being paid at the rate of D.24 per ox, D.16 per cow, and D.6 per calf; there were as many oxen as cows, and 4 times as many calves as cows: How many were there of each sort?

Suppose cows $\begin{matrix} 6 \\ 12 \end{matrix}$ X $\begin{matrix} 64+ \\ 448+ \end{matrix}$ Ans. 5 oxen, 5 cows, and 20 calves.

9. A, B and C built a ship, which cost them D.5000, of which A paid a certain sum, B paid D.500 more than A, and C D.500 more than both; having finished her, they fixed her for sea, with a cargo worth twice the value of the ship: The outfits and charges of the voyage, amounted to $\frac{1}{8}$ of the ship; upon the return of which, they found their clear gain to be $\frac{2}{7}$ of $\frac{3}{4}$ of the vessel, cargo and expenses: Please to inform me what the ship cost them, severally; what share each had in her, and what, upon the final adjustment of their accompts, they had severally gained? Suppose

Suppose it cost A $\begin{array}{ccc} 500 & & 1500- \\ & \times & \\ 1000 & & 500+ \end{array}$

Ans. A owned $\frac{7}{40}$ of the ship, which cost him D.875, and his share of the gain, was D.1093 75c. B owned $\frac{11}{40}$, which cost D.1375, and his gain was D.1718 75c. C owned $\frac{11}{20}$, which cost D.2750, and his gain was D.3437 50c.

PERMUTATIONS AND COMBINATIONS.

THE Permutation of Quantities is, the shewing how many different ways any given number of things may be changed.

This is also called *variation*, *alternation* or *changes;* and the only thing to be regarded here is the order they stand in; for no two parcels are to have all their quantities placed in the same situation.

The Combination of Quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

This is sometimes called *election*, or *choice;* and here every parcel must be different from all the rest, and no two are to have precisely the same quantities, or things.

The Composition of Quantities is the taking of a given number of quantities out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places; and it differs from Combination only as that admits but of one row of things

Combinations of the same form are those, in which there are the same number of quantities, and the same repetitions; thus, *abcc*, *bbad*, *deef*, &c. are of the same form; but *abbc*, *abbb*, *aacc* are of different forms.

Problem I.

To find the number of permutations, or changes, that can be made of any given number of things, all different from each other.

Rule.*

Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

Examples.

1. Christ church, in Boston, has 8 bells: How many changes may be rung on them? $1\times2\times3\times4\times5\times6\times7\times8=40320$ Ans.

2. Nine

* The reaſon of this rule may be ſhewn thus, any one thing *a* is capable of one poſition only, as *a*.

Any two things *a* and *b* are capable of two variations only; as *ab*, *ba;* whoſe number is expreſſed by 1×2.

If there be three things *a*, *b* and *c*, then any two of them, leaving out the third, will have 1×2 variations; and conſequently when the third is taken in, there will be $1\times2\times3$ variations; and ſo on, as far as you pleaſe.

2. Nine gentlemen met at an inn, and were so pleased with their host, and with each other, that in a frolick, they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner: Pray how long, had they kept their agreement, would their frolick have lasted? Ans. 9941 $\frac{335}{365}$ years.

3. How many changes, or variations, will the alphabet admit of?

Ans. 620448401733239439360000.

Problem II.

Any number of different things being given, to find how many changes can be made out of them by taking any given number of quantities at a time.

Rule.*

Take a series of numbers, beginning at the number of things given, and decreasing by 1, as many terms as the number of quantities to be taken at a time; the product of all the terms will be the answer required.

Examples.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array}$$

Or, $8\times7\times6\times5$ (= 4 terms) = 1680 Ans.

2. How many words can be made with 6 letters of the alphabet, admitting a number of consonants may make a word?

$24\times23\times22\times21\times20\times19$ (6 terms) = 96909120, Ans.

Problem III.

Any number of things being given, whereof there are several things of one sort, several of another, &c. to find how many changes may be made out of them all.

Rule.†

1. Take the series $1\times2\times3\times4$, &c. up to the number of things given, and find the product of all the terms. 2. Take

* This Rule, expressed in terms, is as follows; $m\times\overline{m-1}\times\overline{m-2}\times\overline{m-3}$, &c. to n terms; whence m = number of things given, and n = quantities to be taken at a time.

† This Rule is expressed in terms thus; $\dfrac{1\times2\times3\times4\times5, \text{ \&c. to } m.}{1\times2\times3, \text{ \&c. to } p.\times1\times2\times3, \text{ \&c. to } q \text{ \&c.}}$ whence m = number of things given, p = number of things of the first sort, q = number of things of the second sort, &c.

Any 2 quantities, a, b, both different, admit of 2 changes; but if the quantities are the same, or ab becomes aa, there will be only one alteration, which may be expressed by $\dfrac{1\times2}{1\times2} = 1$.

Any

2. Take the series 1×2×3×4, &c. up to the number of the given things of the first sort, and the series, 1×2×3×4, &c. up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series by the *joint* product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zaphnathpaaneah*?

1×2×3×4×5×6×7×8×9×10×11×12×13×14×15 (= number of letters in the word) = 1307674368000.

1×2×3×4×5	(= number of *a*s) =	120
1×2	(= number of *p*s) =	2
1	(= number of *t*s) =	1
1×2×3	(= number of *h*s) =	6
1×2	(= number of *n*s) =	2

2×6×1×2×120 = 2880)1307674368000(454053600 Ans.

2. How many different numbers can be made of the following figures, 1223334444? Ans. 12600.

PROBLEM IV.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.*

1. Take the series 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

EXAMPLES.

Any 3 quantities, *a*, *b*, *c*, all different from each other, admit of 6 variations; but if the quantities are all alike, or, *a b c* become *aaa*, then the 6 variations will be reduced to 1, which may be expreffed by $\frac{1\times2\times3}{1\times2\times3} = 1$. Again, if two quantities out of three are alike, or *abc* become *aac*; then the 6 variations will be reduced to thefe 3, *aac*, *caa*, *aca*, which may be expreffed by $\frac{1\times2\times3}{1\times2} = 3$, and fo of any greater number.

* This Rule, expreffed algebraically, is $\frac{m}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}$, &c. to n terms; where m is the number of given quantities, and n thofe to be taken at a time.

Note. In any given number of quantities, the number of Combinations increafes gradually till you come about the mean numbers, and then gradually decreafes. If the number of quantities be *even*, half the number of places will fhew the greateft number of Combinations, that can be made of thofe quantities; but if odd, then thofe two numbers which are the middle, and whofe fum is equal to the given number of quantities, will fhew the greateft number of Combinations.

Examples.

1. How many combinations may be made of 7 letters out of 12?

$1\times2\times3\times4\times5\times6\times7$ (= the number to be taken at a time) = 5040.

$12\times11\times10\times9\times8\times7\times6$ (= same number from 12) = 3991680.

5040)3991680(792 Ans.

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet? Ans. 134596.

3. A general was asked by his king what reward he should confer on him for his services; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men: What did it amount to?

Ans. £.23836022841 7s. $11\frac{65}{1134}$d.

4. A farmer bargained with a gentleman for a dozen sheep, (at 2 dollars per head) which were to be picked out of 2 dozen; but being long in choosing them, the gentleman told him that if he would give him a cent for every different dozen which might be chosen out of the two dozen, he should have the whole, to which the farmer readily agreed: Pray what did they cost him? Ans. D.27041 56c.

5. How many locks, whose wards differ, may be unlocked with a key of 6 several wards?

Ans. 63: 6 of which may have one single ward, 15 double wards, 20 triple wards, 15 four wards, 6 five wards, and 1 lock 6 wards.

Wards.		Locks.	Wards.		Locks.
1		6	1		5
2		15	2		10
3	in 6 =	20	3	in 5 =	10
4		15	4		5
5		6	5		1
6		1			

Problem V.

To find the number of combinations of any given number of things, by taking any given number at a time; in which there are several things of one sort, several of another, &c.

Rule.

Find the number of different *forms*, which the things, to be taken at a time, will admit of, in the following manner:

1. Place the things so that the greatest indices may be first, and the rest in order.

2. Begin with the first letter, and join it to the second, third, fourth, &c. to the last.

3. Join the second letter to the third, fourth, &c. to the last; and so on till they are all done, always rejecting such combinations as have occurred before; and this will give the combination of all the twos.

4. Join the first letter to every one of the twos; then join the second, third, &c. as before; and it will give the combinations of all the threes.

5. Proceed

5. Proceed in the same manner to get the combinations of all the fours, fives, &c. and you will at last get all the several *forms* of combination, and the number in each form.

6. Having found the number of combinations in each *form*, add them all together, and the sum will be the number required.

Example.

Let the things proposed be *aaabbc*: It is required to find the number of combinations of every 2, of every 3, and of every 4 of these quantities.

Combinations at large.	Forms.	Combinations in each form.
aa,aa,ab,ab,ac	a^2,b^2	2
aa,ab,ab,ac	*ab,ac,bc*	3
ab,ab,ac		—
bb,bc		5 = sum of the twos.
bc		
	a^3	1
aaa,aab,aab,aac	a^2b,a^2c,b^2a,b^2c	4
aab,aab,aac	*abc*	1
abb,abc		—
bbc		6 = sum of the threes.
aaab,aaab,aaac	a^3b,a^3c	2
aabb,aabc	a^2b^2	1
abbc	a^2bc,b^2ac	2
		—
		5 = sum of the fours.

Ans. 5 combinations of every 2; 6 of every 3, and 5 of every 4 quantities.

Problem VI.

To find the changes of any given number of things, taken a given number at a time; in which there are several given things of one sort, several of another, &c.

Rule.

1. Find all the different forms of combination of all the given things, taken, as many at a time, as in the question, by Problem 5.

2. Find the number of changes in any form, (by Problem 3,) and multiply it by the number of combinations in that form.

3. Do the same for every distinct form, and the sum of all the products will give the whole number of changes required.

Example.

How many changes can be made of every 4 letters out of these 6, *aaabbc*?

No.

No. of forms.	Comb.	Changes.
a^3b, a^3c	2	$\left\{\dfrac{1\times2\times3\times4=24}{1\times2\times3\quad=6}\right\}=4$
a^2b^2	1	$\left\{\dfrac{1\times2\times3\times4=24}{1\times2\times1\times2=4}\right\}=6$
a^2bc, b^2ac	2	$\left\{\dfrac{1\times2\times3\times4=24}{1\times2\qquad=2}\right\}=12$

$$\text{Therefore,}\left\{\begin{array}{l} 2\times\ 4=\ 8\\ 1\times\ 6=\ 6\\ 2\times12=24 \end{array}\right.$$

$$\overline{38} = \text{number of changes required.}$$

Problem VII.

To find the compositions of any number, in an equal number of sets, the things being all different.

Rule.

Multiply the number of things in every set continually together, and the product will be the answer required.

Examples.

1. Suppose there are five companies, each consisting of 9 men; it is required to find how many ways 5 men may be chosen, one out of each company?

Multiply 9 into itself continually, as many times as there are companies. $9\times9\times9\times9\times9=59049$ different ways, Ans.

2. How many changes are there in throwing 4 dice?

As a die has 6 sides, multiply 6 into itself four times continually. $6\times6\times6\times6=1296$ changes, Ans.

3. Suppose a man undertakes to throw an ace at one throw with 4 dice, what is the probability of his effecting it?

First, $6\times6\times6\times6=1296$ different ways with and without the ace. Then, if we exclude the ace side of the die, there will be 5 sides left; and $5\times5\times5\times5=625$ ways without the ace; therefore there are $1296-625=671$ ways, wherein one or more of them may turn up an ace; and the probability that he will do it, as 671 to 625, Ans.

4. In how many ways may a man, a woman and a child be chosen out of three companies, consisting of 5 men, 7 women and 9 children? Ans. 315.

MISCELLANEOUS

MISCELLANEOUS MATTERS.

A short method of reducing a Vulgar Fraction, into its equivalent Decimal, by Multiplication.

RULE.—Divide unity or 1 by the denominator, till the remainder is a single figure, 10, 100, &c. if convenient, then multiply the whole quotient, including the remainder after division, by the remainder (which is now the numerator, and the divisor, the denominator) and annex the product of the quotient, then multiply the quotient, thus increased by the last numerator, and annex the product to the increased quotient; and thus it may be reduced to what exactness you please. But if the numerator of the given fraction exceed 1, you must finally multiply the last product by the said numerator.

EXAMPLES.

1. Reduce $\frac{1}{26}$ to its equivalent decimal.

26)1·00(·03846$\frac{4}{26}$
78 — This multiplied by 4 (the numerator) is ·15384$\frac{16}{26}$=$\frac{8}{13}$
Which annexed to the quotient ·03846 is ·0384615384$\frac{8}{13}$
220 — And ·0384615384$\frac{8}{13}$ × 8 and annexed to the last product
208 — = ·03846153843076923076$\frac{12}{13}$, &c.

120
104

160
156

4

2. Reduce $\frac{5}{246}$.

246)1·000000(·004065$\frac{10}{246}$ and ·0040650$\frac{10}{246}$ × 10 = ·0040650$\frac{100}{246}$ and this annexed to the quotient is ·004065040650$\frac{100}{246}$, and this multiplied by the given numerator, 5, is ·02032703252$2\frac{8}{246}$.

For any number of pounds, avoirdupois, under 23, multiply the decimal ·00892857 by the given number of pounds, which generally gives the decimal true to the sixth place.

A short method of finding the duplicate, triplicate, &c. Ratio of any two numbers, whose difference is small, compared with the two numbers.

FOR THE DUPLICATE RATIO.

RULE.—Assume two numbers, whose difference is small; subtract *half* their difference from the least, and add it to the greatest, and the two numbers, thus found, will be in the same proportion nearly as the squares of the assumed numbers.

EXAMPLE.

Let the assumed numbers be 10 and 11; Then 11—10=1. 10—·5=9·5 and 11+·5=11·5.

Proof, As $10^2 : 11^2 :: 9·5 : 11·5$ nearly.

FOR A TRIPLICATE RATIO.

RULE.—Subtract the *difference* of the assumed numbers from the least, and add it to the greatest, and the numbers, thus obtained, will be in the same proportion nearly as the cubes of the assumed numbers.

Let

Let the numbers be 164 and 165: Then 165—164=1. 164—1= 163 and 165+1=166. Proof, As $164^3 : 165^3 :: 163 : 166$ nearly.

For a quadruplicate proportion subtract, and add once and a half the difference, and so on, for each higher power, increasing the number to be subtracted and added by ·5.

To reduce a Ratio, consisting of large numbers, to its least terms, and very nearly of the same value.

RULE.

1. Divide the greater of the terms by the less, and the least divisor by the remainder, and so on continually, till nothing remain, in the same manner as we get the greatest common measure for reducing a vulgar fraction: This will give a number of ratios, from which we can choose one, that will suit our purpose.

2. Place the first quotient under unit for the first ratio; multiply that by the next quotient, adding nothing to the numerator, and 1 to the product of the denominator, for a new denominator, and it will give a second ratio, nearer than the first: Then, multiply the last ratio by the next quotient, adding the preceding ratio, and so on, continually till you have gone through.

EXAMPLES.

1. Sir Isaac Newton has demonstrated, in his Principia, that the velocity of a comet, moving in a parabola, is to that of a planet, moving in a circular orb, at the same distance from the sun, as $\sqrt{2}$ to 1. Let this be taken for an example.

$\sqrt{2}=1·4142$; those motions, then, are as 1·4142 to 1; or as 14142 to 10000?

10000)14142(1
10000
4142)10000(2
8284
1716)4142(2
3432
710)1716(2
1420
296)710(2
592
118)296(2
236
60)118(1
60
58)60(1
58
2)58(29
58

Then $\frac{1}{1}$ = first ratio.

$\frac{1\times2+0=2}{1\times2+1=3}$ = second.

$\frac{2\times2+1=5}{3\times2+1=7}$ = third.*

$\frac{5\times2+2=12}{7\times2+3=17}$ = fourth.

$\frac{12\times2+5=29}{17\times2+7=41}$ = fifth, &c.

2. Geometers

* The late Professor Winthrop chose 7 to 5 for a proportion.

2. Geometers have found the proportion of the circumference of a circle to its diameter, to be as 3·1416 to 1: Let this ratio be reduced.

```
10000)31416(3
      30000
      -----
       1416)10000(7
             9912
             ----
               88)1416(16
                   88
                   --
                   536
                   528
                   ---
                     8)88(11
                       88
```

Then $\frac{1}{3}$ = first ratio.

```
1×7+0    =   7
             — = second.
3×7+1    =  22

7×16+1   = 113
             —— = third: this is the
22×16+3  = 355
```

ratio generally made use of, and is sufficiently exact for very nice calculations.

3. The area of a circle is to its circmuscribing square, as ·7854 to 1, very nearly: Let this be reduced.

```
7854)10000(1
     7854
     ----
     2146)7854(3
          6438
          ----
          1416)2146(1
               1416
               ----
                730)1416(1
                     730
                     ---
                     686)730(1
                         686
                         ---
                          44)686(15
                              44
                              --
                              246
                              220
                              ---
                               26 &c.
```

```
Then        1
            — = first ratio.
            1
1×3+0 =     3
            — = second.
1×3+1 =     4

3×1+1 =     4
            — = third.
4×1+1 =     5

4×1+3 =     7
            — = fourth.
5×1×4 =     9

7×1+4 =    11
            — = fifth:
9×1+5 =    14
```

This is very exact, and the proportion generally used.

Therefore, as 14 : 11 :: the square of the diameter of a circle to its area.

To estimate the Distance of Objects on level ground, or at sea, having only the height given.

Rule.—1. To the earth's diameter, (viz. 42056462 feet,) add the height of the eye, and multiply the sum by that height, then the square root of the product is the distance, at which an object on the surface of the earth or water, can be seen by an eye so elevated.

2. As objects are seen in a straight line, and that line is a tangent to the earth's surface; therefore, *To find the distance of two elevated objects, when the right line joining them touches the earth's surface between those objects, (for instance, the line from the eye of the observer to the distance found by the first part of the rule, and from thence to the object;)* work for each object separately, and the sum of the square roots of the products is the distance of the two objects from each other.

Example.

EXAMPLE.

How far may a mountain be seen on level ground, or at sea, which is a mile high, supposing the eye of the observer elevated 5 feet above the surface?

$$\sqrt{42056462+5\times5}=2\cdot746 \text{ miles.}$$
$$\sqrt{42056462+5280\times5280}=89\cdot253 \text{ miles.}$$

Ans. 91·999 miles.

To estimate the Height of Objects on level ground, or at sea, having only the distance given.

RULE.—1. From the given distance, take the distance which the elevation of your eye above the surface will give, found by the last problem.

2. Divide the square of the remainder in feet by 42056462 feet, and the quotient will be the height required.

EXAMPLE.

Being on my return from a foreign voyage, and finding by my reckoning I was about $5\frac{1}{2}$ leagues from Boston light-house, it being in the dusk of the evening, with my telescope I descried the lamp of the light-house in the horizon, at which time, my eye was elevated 6 feet above the surface of the water: Now, supposing my reckoning to be true, what is the height of the light-house above the water?

$5\frac{1}{2}$ leagues $=16\cdot5$ miles; then $16\cdot5-\sqrt{42056462+6\times6}=13\cdot943$ miles $=73619$ feet nearly, and $73619\times73619\div42056462=129$ feet nearly, Ans.

MISCELLANEOUS QUESTIONS, WITH THE METHOD OF SOLUTION.

1. What part of 9d. is $\frac{2}{5}$ of 7d?

$$\frac{2}{5}\text{ of }\frac{7}{1}=\frac{14}{5}\text{, and }\frac{9}{1}\div\frac{14}{5}=\frac{1\times14}{9\times5}=\frac{14}{45}\text{ Ans.}$$

2. What number is that, from which $\frac{3}{7}$ being taken, the remainder will be $\frac{1}{5}$?

$$\frac{1}{5}+\frac{3}{7}=\frac{1\times7+3\times5}{5\times7}=\frac{22}{35}\text{ Ans.}$$

3. What number is that, to which if $\frac{3}{7}$ of $\frac{12}{5}$ of $\frac{129}{313}$ be added, the total will be 1?

$$\frac{3}{7}\text{ of }\frac{12}{5}\text{ of }\frac{129}{313}=\frac{4644}{10955}\text{, and }\frac{1}{1}-\frac{4644}{10955}=\frac{1\times10955-1\times4644}{1\times10955}=\frac{6311}{10955}\text{ Ans.}$$

4. What number is that, of which $19\frac{3}{13}$ is $\frac{5}{7}$?

$19\frac{3}{13}=\frac{250}{13}$; then, As $\frac{5}{7}:\frac{250}{13}::\frac{7}{1}:26\frac{12}{13}$ Ans.

5. In

5. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches, and 40 cherries : How many trees does the orchard contain ?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$, and $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}$; therefore, as $\frac{1}{12} : \frac{60+40}{1} :: \frac{12}{12} : 1200$ Ans.

6. A person, who was possessed of $\frac{2}{5}$ of a vessel, sold $\frac{5}{8}$ of his interest for £.375 : What was the ship worth at that rate ?

$\frac{5}{8}$ of $\frac{2}{5}=\frac{1}{4}$. As $\frac{1}{4} : \frac{375}{1} :: \frac{1}{1}$: £.1500 Ans.

7. If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of the cargo, valued at £.1000 : What did both ship and cargo cost ?

$\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}=\frac{6}{28}$, and $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of $\frac{1000}{1}=\frac{7000}{39}$, then, as $\frac{6}{28} : \frac{7000}{39} :: \frac{28}{28} : \frac{28\times7000\times28}{6\times39\times28}=$ £.837 12s. $1\frac{25}{39}$d. the cost of the ship ; and £.1000 +£.837 12s. $1\frac{25}{39}$d.=£.1837 12s. $1\frac{25}{39}$d. value of the ship and cargo, [Answer.

8. Two ships, A and B, sailed from a certain port at the same time ; A sailed north 8 miles an hour, and B east 6 miles an hour : Required, by an easy method, to find their distance asunder at every hour's end ?

$\sqrt{8\times8+6\times6}=10$ miles distant in 1 hour, and $10\times2=20$ miles in 2 hours, &c. Ans.

9. If a body be weighed in each scale of a balance, whose beam is unequally divided, and those different weights of the body be multiplied together, the square root of the product will be the true weight of that body.

Suppose the weight of a bar of silver, in one scale, to be 10oz. and in the other scale 12oz. ; required the true weight of the bar ?

	oz.	oz.	pwt.	gr.	
$\sqrt{12\times10}$	$= 10{\cdot}95445+$	$= 10$	19	$2{\cdot}1384+$	Ans.

10. A younger brother received D.3125 92c. which was just $\frac{7}{12}$ of his elder brother's fortune ; and $5\frac{3}{8}$ times the elder's money, was $1\frac{2}{3}$ the value of the father's estate : Pray, what was their father worth ?

As 7 : 3125·92 :: 12 : 5358·72 the elder brother's fortune ; then, $5358{\cdot}72\times5\frac{3}{8}\div1\frac{2}{3}=$D.17281 87c. 2m. Ans,

11. A gentleman divided his fortune among his sons, giving A 9l. as often as B 5l. and to C but 3l as often as to B 7l. and yet C's dividend was $1537\frac{5}{6}$l. : What did the whole estate amount to ?

As 7 : 5 :: 3 : $2\frac{1}{7}$; then, as $2\frac{1}{7} : 1537\frac{5}{6} :: 9+5+2\frac{1}{7}$: £.11583 8s. 10d. Ans.

12. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in 3 months, $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only 537l. left : Pray, what did his father bequeath him ?

$\frac{16}{16}$=whole legacy, $\frac{16}{16}-\frac{5}{16}=\frac{11}{16}$ left at three months, then, $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{16}=\frac{165}{384}$, and $\frac{11}{16}-\frac{165}{384}=\frac{1584}{6144}=\frac{33}{128}=$£.537, therefore, as $\frac{33}{128} : \frac{537}{1} :: \frac{1}{1}$: £.2082 18s. $2\frac{2}{11}$d. Ans.

13. A gay young fellow soon got the better of $\frac{2}{7}$ of his fortune; he then gave £.1500 for a commission, and his profusion continued till he had but £.450 left, which he found to be just $\frac{6}{16}$ of his money, after he had purchased his commission: What was his fortune at first?

As 6 : 450 :: 16 : 1200, and 1200+1500=£.2700=$\frac{5}{7}$ of his fortune, and, as 5 : 2700 :: 7 : £.3780 Ans.

14. A merchant begins the world with D.5000, and finds that by his distillery he clears D.5000 in 6 years; by his navigation D.5000 in $7\frac{1}{2}$ years, and that he spends in gaming D.5000 in 3 years: How long will his estate last?

$$\text{As} \left\{ \begin{matrix} 6 \\ 7\frac{1}{2} \\ 3 \end{matrix} \right\} : 5000 :: 1 : \left\{ \begin{matrix} 833\frac{1}{3} \\ 666\frac{2}{8} \\ 1666\frac{2}{3} \end{matrix} \right\}$$

$$\text{As } 1666\tfrac{2}{3} - \overline{133\tfrac{1}{3} + 666\tfrac{2}{3}} : 1 :: 5000 : 30 \text{ years Ans.}$$

15. A has £.100 of B's money in his hands, for the remittance of which B allows him 9 per cent.: What sum must he remit, to discharge himself of the £.100?

As 100+9 : 100 :: 100 : £.$91\frac{81}{109}$; or, $\frac{100\times100}{100+9}$=£.$91\frac{81}{109}$ to be remitted, and $\frac{100\times9}{100+9}$=£.$3\frac{28}{109}$ his commission.

16. Said Harry to Edmund, I can place four 1s, so that, when added, they shall make precisely 12: Can you do so too?

17. A and B are on opposite sides of a circular field 268 poles about; they begin to go round it, both the same way, at the same instant of time; A goes 22 rods in 2 minutes, and B 34 rods in 3 minutes: How many times will they go round the field, before the fwifter overtakes the slower?

$$\begin{matrix} \text{min.} & & \text{po.} & & \text{min.} & & \text{po.} \\ 2 & : & 22 & \multirow{2}{*}{} & & & 11 \quad \text{A goes in a minute.} \\ 3 & : & 34 & & & & 11\frac{1}{3} \quad \text{B do. do.} \end{matrix}$$

$$\left.\begin{matrix} 2 : 22 \\ 3 : 34 \end{matrix}\right\} :: 1 : \left\{\begin{matrix} 11 & \text{A goes in a minute.} \\ 11\frac{1}{3} & \text{B do. do.} \end{matrix}\right.$$

therefore, B gains $11\frac{1}{3}-11=\frac{1}{3}$ of a pole of A every minute. And, as $\frac{1}{3}$ po.: 1 min. :: $\frac{268}{2}$ po. (= half round the field) : 402 min. (= the time in which B will overtake A.) Then,

$$\text{As } 1 : \left\{\begin{matrix} \overset{\text{po.}}{11} \\ 11\frac{1}{3} \end{matrix}\right\} :: \overset{\text{min.}}{402} : \left\{\begin{matrix} \overset{\text{po.}}{4422} & \text{A travels.} \\ 4556 & \text{B travels.} \end{matrix}\right.$$

And, $\frac{4422}{268}=16\frac{1}{2}$ times round the field, A travels;
and $\frac{4556}{268}=17$ times round the field B travels.

18. If 15 men can perform a piece of work in 11 days, how many men will accomplish another piece of work, four times as large in a fifth part of the time?

work.	men.	works.	men.	time	men.	time.	men.
As 1 :	15 ::	4 :	60	As $\frac{1}{1}$:	$\frac{60}{1}$ ::	$\frac{1}{5}$:	300 Ans.

19. If A can do a piece of work alone in 7 days, and B in 12; let them both go about it together: In what time will they finish it

As

Days. work. day works. work work work. work. day. work. day,

$$\text{As}\left\{\begin{array}{l} 7 : 1 :: 1 : \frac{1}{7} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array}\right\} \text{Then } \tfrac{1}{7}+\tfrac{1}{12}=\tfrac{19}{84} \quad \text{As } \tfrac{19}{84} : \tfrac{1}{1} :: \tfrac{1}{1} : 4\tfrac{8}{19} \text{ Ans.}$$

20. A and B together can build a boat in 20 days ; with the assistance of C they can do it in 12 : In what time would C do it by himself ?

D. W. D. W. W. W. W. W. D. W. D.

$$\text{As}\left\{\begin{array}{l} 20 : 1 :: 1 : \frac{1}{20} \\ 12 : 1 :: 1 : \frac{1}{12} \end{array}\right\} \text{Then, } \tfrac{1}{12}-\tfrac{1}{20}=\tfrac{8}{240}, \text{ \& as } 8 : 1 :: 240 : 30 \text{ Ans.}$$

21. A can do a piece of work alone in 13 days, and A and B together in 8 days : In what time can B do it alone ?

D. W. D. W. W. W. W. W. D. W. D.

$$\text{As}\left\{\begin{array}{l} 13 : 1 :: 1 : \frac{1}{13} \\ 8 : 1 :: 1 : \frac{1}{8} \end{array}\right\} \text{Then, } \tfrac{1}{8}-\tfrac{1}{13}=\tfrac{5}{104}, \text{ and, as } 5 : 1 :: 104 : 20\tfrac{4}{5}.$$

22. A, B and C can complete a piece of work in 12 days ; A can do it alone in 23 days, and B in 37 days : In what time can C do it by himself ?

D. W. W.

$$\text{As}\left\{\begin{array}{l} 12 : 1 :: 1 : \frac{1}{12} \\ \hline 23 : 1 :: 1 : \frac{1}{23} \\ 37 : 1 :: 1 : \frac{1}{37} \end{array}\right\} \quad \text{Then, } \underset{\text{W.}}{\tfrac{1}{12}}-\overset{\text{W.}}{\underset{\text{D.}}{\overline{\tfrac{1}{23}+\tfrac{1}{37}}}}=\underset{\text{W.}}{\tfrac{131}{10212}}$$

As 131 : 1 :: 10212 : $77\frac{125}{131}$ days, Ans.

23. A cistern, for water, has 2 cocks to supply it ; by the first, it may be filled in 45 minutes, and by the second, in 55 minutes ; it has likewise a discharging cock, by which it may, when full, be emptied in 30 minutes : Now, if these three cocks be all left open, when the water comes in, in what time will the cistern be filled ?

Min.	Cist.	Min.	Cist.
45 :	1 ::	60 :	1·3333
55 :	1 ::	60 :	1·0909
			2·4242
30 :	1 ::	60 :	2·

Gains in an hour ·4242 of a cistern.

Cist. Hour. Cist. h. m. s.

As ·4242 : 1 :: 1 : 2 21 $26\frac{1}{2}$ Ans.

Or, by vulgar fractions, more accurately, 2h. 21m. $25\frac{5}{7}$s. Ans.

24. A water tub holds 73 gallons ; the pipe, which conveys the water to it, usually admits 7 gallons in 5 minutes ; and the tap discharges 20 gallons in 17 minutes : Now, supposing these both to be carelessly left open, and the water to be turned on at 4 o'clock in the morning ; a servant, at 6, finding the water running, puts in the tap ; in what time, after this accident, will the tub be full ?

min. gal. min. gal. h.

$$\text{As}\left\{\begin{array}{l} 5 : 7 :: 60 : 84 \\ 17 : 20 :: 60 : 70\frac{10}{17} \end{array}\right\} \begin{array}{l} \overline{84-70\frac{10}{17}}\times 2=26\frac{14}{17} \text{ gal. and } 73-26\frac{14}{17}= \\ 46\frac{3}{17} \text{ gal. which now remain to be filled.} \end{array}$$

gal. min. gal. M. s.

Therefore, as 7 : 5 :: $46\frac{3}{17}$: 32 $58\frac{118}{119}$, and therefore the tub will be full at 32 m. $58\frac{118}{119}$ s. after 6.

25. A has a chest of tea, weighing $3\frac{1}{4}$cwt. the prime cost of which is 60l. : Now, allowing interest at 6 per cent. per annum, how must he

he rate it per lb. to B, so that, by taking his note of hand, payable at 6 months, he may clear D.50 by the bargain?

Interest £.2 5s. Then, as $3\frac{1}{2}$cwt. : £.60+£.15+£.2 5s. :: 1lb. : 3s. $11\frac{29}{98}$d. Ans.

26. Suppose the American continental debt to be 18 millions, what annuity, at 6 per cent. per annum, will discharge it in 25 years?

By Table 5, of annuities, page 302, ·07823 is the annuity which 1l. will purchase in 25 years, then, ·07823×18000000=£.1408140 Ans.
The annual interest of the debt=1080000

Therefore, there must be a sinking fund of £.328140 *pr. ann.*

27. The hour and minute hands of a watch are exactly together at 12 o'clock: When are they next together?

The velocities of the two hands of a watch, or clock, are to each other, as 12 to 1; therefore, the difference of velocities is 12—1=11.

$$\text{As } 11 : 1 :: \left\{\begin{array}{llll} 12\times1 & : 1 & 5 & 27\frac{3}{11} \\ 12\times2 & : 2 & 10 & 54\frac{6}{11} \\ 12\times3 & : 3 & 16 & 21\frac{9}{11} \end{array}\right\} \text{\&c. Ans.}$$

28. A hare starts 12 rods before a hound; but is not perceived by him till she has been up 45 seconds; she scuds away at the rate of 10 miles an hour, and the dog, on view, makes after at the rate of 16 miles an hour: How long will the course hold, and what space will be run over, from the spot where the dog started?

$\frac{16-10}{16}=\frac{6}{16}=\frac{3}{8}$, or as 8 to 3 against the hare, 1 hour=3600 seconds.

Sec. Feet. Sec. Feet. 10 miles = 52800 feet.
As 3600 : 52800 :: 45 : 660 distance the hare had run before the [dog discovered her.
Add 12 rods = 198

858 = the distance of the hare when the [dog started.
8

3)6864

Feet 2288 = the ground run over by the dog.

Miles. Feet. Sec. Feet. Sec.
Now, as 16 = 84480 : 3600 :: 2288 : $97\frac{1}{2}$

29. In a series of proportional numbers, the first is 4, the third 12, and the product of the second and third is 112·8: What is the difference of the second and fourth?

112·8÷12=9·4 the second. As 4 : 9·4 :: 12 : 28·2, and 28·2—9·4 = 18·8 Ans.

30. A fellow said that when he counted his nuts, two by two, three by three, four by four, five by five, and six by six, there was still an odd one; but when he told them seven by seven, they came out even: How many had he?

2×3×4×5×6=720, and $\overline{720+1}\div7$ = 103 even, Ans. 721.

$\frac{721}{2, 3, 4, 5 \text{ and } 6}$ respectively will leave an odd one.

31. There

31. There is an island 50 miles in circumference, and three men start together to travel the same way about it: A goes 7 miles per day, B 8, and C 9: When will they all come together again, and how far will each travel?

$\overline{50\times7}+\overline{50\times8}+\overline{50\times9}\div\overline{7+8+9}=50$ days.—A 350 miles, B 400, and C 450, Ans.

32. Suppose A leaves Newburyport at 6 o'clock on Monday morning, and travels towards Providence, at the rate of 4 miles per hour without intermission; and that, at 3 in the afternoon, B sets out from Providence for Newburyport, and travels constantly at the rate of 7 miles an hour: Now suppose the distance between the two towns to be 90 miles; whereabout on the road will they meet?

$6+3=9$ hours, and $9\times4=36$ miles, the time and distance A had travelled before B started. Then $90-36=54$ miles remain to be travelled by both; now, as both together lessen the distance $7+4=11$ miles an hour, therefore $\frac{4}{11}$ of $54+36=55\frac{7}{11}$ miles from Newburyport; which is near Ames's, at Dedham.

33. If, during ebb tide, a wherry should set out from Haverhill to come down the river, and at the same time, another should set out from Newburyport, to go up the river, allowing the distance to be 18 miles; suppose the current forwards one and retards the other $1\frac{1}{2}$ mile per hour; the boats are equally laden, the rowers equally good, and, in the common way of working in still water, would proceed at the rate of 4 miles per hour: Where, in the river will the two boats meet?

M. M. M.   M. M. M.   M. M. M.
$4+1\frac{1}{2}=5\frac{1}{2}$, and $4-1\frac{1}{2}=2\frac{1}{2}$, then, $5\frac{1}{2}+2\frac{1}{2}=8$ in one hour by both.
M. H. M. H.   M. H. M.   M. H. M.
As $8:1::18:2\frac{1}{4}$, then $5\frac{1}{2}\times2\frac{1}{4}=12\frac{3}{8}$ from Haverhill, and $2\frac{1}{2}\times2\frac{1}{4}=5\frac{5}{8}$ from Newburyport.

34. A gentleman making his addresses in a lady's family who had five daughters; she told him that their father had made a will, which imported that the first four of the girls' fortunes were, together, to make D.50000; the last four, D.66000; the three last with the first, D.60000; the three first with the last, 56000; and the two first with the two last, D.64000, which, if he would unravel, and make it appear, what each was to have, as he appeared to have a partiality for Harriet, her third daughter, he should be welcome to her: Pray, what was Miss Harriet's fortune?

$$\left.\begin{array}{r}
A+B+C+D = 50000 \\
B+C+D+E = 66000 \\
A +C+D+E = 60000 \\
A+B+C +E = 56000 \\
A+B +D+E = 64000 \\
\hline
296000
\end{array}\right\}$$

Then, $296000\div4$ the number of combinations $=74000$ the sum of their fortunes.

Then, $A+B+C+D+E=74000$
And $ A+B +D+E=64000$

Ans. Harriet's fortune = D.10000

35. Three persons purchase a vessel in company, towards the payment

ment whereof A advanced $\frac{3}{7}$, B $\frac{\ }{7}$, and C, D.900: What did A and B pay, each, and what part of the vessel had C?

$$\frac{2}{5}+\frac{3}{7}=\frac{14+15}{35}=\frac{29}{35}, \text{ and } \frac{35}{35}-\frac{29}{35}=\frac{6}{35} \text{ C's part of the vessel.}$$

$$\text{As } \frac{6}{35} : \frac{900}{1} :: \begin{cases} \frac{14}{35} : \text{D.2100 A advanced.} \\ \frac{15}{35} : \text{D.2250 B advanced.} \end{cases}$$

36. A and B cleared, by an adventure at sea, 45 guineas, which was 35l. per cent. upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B: What money did each adventure?

As £.35 : 100 :: 45 guineas, : £.180 = the whole adventure.

$$\text{As } 11+8 : 180 :: \begin{cases} 11 : £.104\ 4s.\ 2\frac{10}{19}d.\ \text{A's.} \\ 8 : £.75\ 15s.\ 9\frac{9}{19}d.\ \text{B's.} \end{cases}$$

37. A, B and C are to share 100l. in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively; but C dying, it is required to divide the whole sum properly, between the other two?

$$\text{As } \frac{1}{3}+\frac{1}{4}+\frac{1}{5} : \overset{£.}{100} :: \begin{cases} \frac{1}{3} : \overset{£.}{42}\ \overset{s.}{11}\ \overset{d.}{0\frac{36}{47}},\ \text{A's share.} \\ \frac{1}{4} : 31\ 18\ 3\frac{27}{47},\ \text{B's share.} \\ \frac{1}{5} : 25\ 10\ 7\frac{31}{47},\ \text{C's share.} \end{cases}$$

$$\text{Again, as } \frac{1}{3}+\frac{1}{4} : \overset{£.}{25}\ \overset{s.}{10}\ \overset{d.}{7\frac{31}{47}} :: \begin{cases} \frac{1}{3} : \overset{£.}{14}\ \overset{s.}{11}\ \overset{d.}{9\frac{171}{329}},\ \text{A's share of C's do.} \\ \frac{1}{4} : 10\ 18\ 10\frac{146}{329},\ \text{B's share of C's do.} \end{cases}$$

$$\left.\begin{array}{l} \text{Then, } \overset{£.}{42}\ \overset{s.}{11}\ \overset{d.}{0\frac{36}{47}}+\overset{£.}{14}\ \overset{s.}{11}\ \overset{d.}{9\frac{171}{329}}=\overset{£.}{57}\ \overset{s.}{2}\ \overset{d.}{10\frac{2}{7}}\ \text{A's share in all,} \\ \text{And, } 31\ 18\ 3\frac{27}{47}+10\ 18\ 10\frac{46}{329}=42\ 17\ 1\frac{5}{7}\ \text{B's share in all,} \end{array}\right\} \text{Ans.}$$

Proof 100

38. A, B and C have among them 135 guineas; A's+B's are to B's+C's, as 5 to 7, and C's—B's to C's+B's as 1 to 7: How many had each?

A+B. B+C.

Suppose A's+B's = 50; then, as 5 : 7 :: 50 : 70; as 7 : 1 :: 70 : 10 = C's—B's; then, 70—10 = 60, and 60÷2 = 30 = B's; 50—30 = 20 = A's, and 30+10 = 40 = C's, by the supposition: Now 20+30+40 = 90, which should have been 135, therefore,

$$\text{As } 90 : 135 :: \begin{cases} 20 : 30 = \text{A's.} \\ 30 : 45 = \text{B's.} \\ 40 : 60 = \text{C's.} \end{cases}$$

Sum = 135 proof.

39. There are three horses, belonging to different men, employed team to draw a load of salt from Newburyport to Boston for

21.

21. 10s. : A and B are supposed to do $\frac{3}{11}$ of the work; A and C $\frac{5}{13}$, and B and C $\frac{4}{14}$ of it; they are to be paid proportionally: Can you divide it as it should be?

$$\left.\begin{array}{l} A+B = \frac{3}{11} = \cdot 2727 \\ A+C = \frac{5}{13} = \cdot 3846 \\ B+C = \frac{4}{14} = \cdot 2857 \\ \hline \text{Sum} = \cdot 943 \end{array}\right\}$$

And $\cdot 943 \div 2$, the number combined $= \cdot 4715 = A+B+C$
$- \cdot 2727 = A+B$

s.

Then, as $\cdot 4715 : 50 :: \cdot 1988 : £.1\ 1s.\ 0\frac{1}{4}d. = \cdot 1988 = C.$

And in the same manner proceed for the rest.

40. I would put 20 hogsheads of London beer into 10 wine pipes, and desire to know what the cask must contain, which will receive the difference, 231 solid inches being the wine gallon, and 282 that of beer.

Beer hhd. = 54 gall. and $54 \times 282 \times 20 = 304560$ solid inches.
Wine pipe. = 126 gall. and $126 \times 231 \times 10 = 291060$ solid inches, and

$$\frac{304560 - 291060}{282} = 47\tfrac{41}{47} \text{ beer gallons, Ans.}$$

41. Being about to plant 5292 trees equally distant in rows, the length of the grove is to be three times the breadth: How many of the shorter rows will there be?

$$\sqrt{\frac{5292}{3}} \times 3 = 126 \text{ rows, Ans. viz. } \tfrac{1}{3} \text{ of the trees are to form an exact}$$

square, the side whereof being 42, shews how many come into a short row.

42. A general, disposing his army into a square battalion, found he had 231 over and above, but increasing each side with one soldier, he wanted 44 to fill up the square: How many men did his army consist of?

$231+44=275$, and $\overline{275-1} \div 2 = 137$, then $\overline{137 \times 137} + 231 = 19000$ Ans.
Proof, $138 \times 138 = 19044$.

43. I want the length of a shoar, the bottom of which, being set 9 feet from the perpendicular side of a house, will support a weak place in the wall, $22\frac{1}{2}$ feet from the ground?

$$\sqrt{\overline{22 \cdot 5 \times 22 \cdot 5} + \overline{9 \times 9}} = 24 \text{ feet, } 2\tfrac{1}{4} \text{ inches, Ans.}$$

44. A line 35 yards long will exactly reach from the top of a fort, standing on the brink of a river, known to be 27 yards broad, to the opposite bank: What is the height of the wall?

$$\sqrt{\overline{35 \times 35} - \overline{27 \times 27}} = 22 \text{ yards, } 9\tfrac{1}{4} \text{ inches, nearly.}$$

45. Suppose a light-house built on the top of a rock; the distance between the place of observation and that part of the rock level with the eye 620 yards; the distance from the top of the rock to the place of observation, 846 yards, and from the top of the light-house 900 yards: the height of the light-house is required?

$\sqrt{900}$

$\sqrt{\overline{900\times900}-\overline{620\times620}}-\sqrt{\overline{846\times846}-\overline{620\times620}}=76{\cdot}77$ yards, Ans.

46. *The sum and difference of the squares of two numbers given, to find those numbers.*

RULE.—From the sum take the difference, and half the remainder is the square of the less, which, taken from the sum of the squares, wlll give the square of the greater.

A and B have between them a number of guineas, which are to be so divided, that the sum of their squares may be 208, and the difference of their squares 80; supposing A's the greater number, how many has he more than B?

$\overline{208-80}\div2=64$ the square of B's, and $208-64=144$ the square of A's; therefore $\sqrt{144}-\sqrt{64}=4$ Ans.

47. *Having the sum of two numbers, and the sum of their squares given, to find those numbers.*

RULE.—From the square of their sum take the sum of their squares: then from the sum of their squares take this remainder, and the square root of the difference will be the difference of the two numbers. To half their sum add their difference, and the sum will be the greater. From half the sum take half their difference, and the remainder will be the less.

A and B have 50 guineas between them, which are to be so divided, as that the sum of the squares of the two numbers shall be 1300: How many had each, supposing A to have the greater number?

$\overline{50\times50}-1300=1200$; then, $\sqrt{\overline{1300-1200}}=10$ difference. Now $\overline{50\div2}+\overline{10\div2}=30=$A's. And $\overline{50\div2}-\overline{10\div2}=20=$ B's, Ans.

48. *Having the difference of two numbers, and the sum of their squares given, to find those numbers.*

RULE.—From the sum of their squares take the square of their difference: to the sum of the squares add the remainder, and the square root of this sum will be the sum of the required numbers; then, with the half sum and half difference proceed as in the last question.

A number of guineas are to be divided between A and B, in such a manner that A may have 50 more than B, and that the sum of the squares of the respective shares may be 12500: What number had each?

$12500-\overline{50\times50}=10000$, and $\sqrt{\overline{12500+10000}}=150=$sum of their shares. Then, $\overline{150\div2}+\overline{50\div2}=100$ A's; and $\overline{150\div2}-\overline{50\div2}=50$ B's, Ans.

49. *Having the sum of the squares of two numbers, and the square of their half sum given, to find those numbers.*

RULE.—From the sum of the squares take *twice* the square of the half sum, and the square root of half the remainder will be their half difference, with which and the half sum proceed as before directed.

Let

Let the sum of the squares of two numbers be 3161, and the square of their half sum 1560·25: Required those numbers?

$3161 - \overline{1560 \cdot 25 \times 2} = 40 \cdot 5$ $40 \cdot 5 \div 2 = 20 \cdot 25$, and $\sqrt{20 \cdot 25} = 4 \cdot 5 = \frac{1}{2}$ difference, and $\sqrt{1560 \cdot 25} = 39 \cdot 5 = \frac{1}{2}$ sum; then, $39 \cdot 5 + 4 \cdot 5 = 44$ the greater, and $39 \cdot 5 - 4 \cdot 5 = 35$ the less, Ans.

50.—1. *If the quantity of matter, (or weights) of any two bodies, put in motion, be equal, the force by which they are moved will be in proportion to their velocities, or swiftness of motion.*

2. *If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them, that is, as their weights.*

3. *If both the quantities of matter and the velocities be unequal, the forces, with which the bodies are moved, will be in a proportion compounded of their quantities of matter and velocities.*

Suppose the battering ram of Vespasian weighed 60000lb.; that it was moved at the rate of 24 feet in one second, and that this was sufficient to demolish the walls of Jerusalem: With what velocity must a cannon ball, which weighs 42lb. be moved, to do the same execution;

The velocity of the ram being 24, and the weight of the ball 42, compounded, will make a fraction $= \frac{24}{42} = \frac{4}{7}$, and $\frac{4}{7} \times 60000 = 34285\frac{5}{7}$ feet in a second, Ans.

51. A body weighing 30lb. is impelled by such a force as to send it 20 rods in a second: With what velocity would a body weighing 12lb. move, if it were impelled by the same force?

$$\frac{30 \times 20}{12} = 50 \text{ rods in a second, Ans.}$$

Of Gravity.

52. *The gravity of bodies above the surface of the earth decreases in a duplicate ratio (or as the squares of their distances) in semidiameters of the earth, from the earth's centre.*

Supposing a body to weigh 400lb. at 2000 miles above the earth's surface: What would it weigh at the surface, estimating the earth's semidiameter at 4000 miles?

From the centre to the given height being $1\frac{1}{2}$ semidiameter; multiply the square of $1\frac{1}{2}$ by the weight, and the product will be the answer. $1 \cdot 5 \times 1 \cdot 5 \times 400 = 900$lb. Ans.

53. If a body weigh 900lb. at the surface of the earth, what will it weigh at 2000 miles above the surface?

This being the reverse of the last, therefore, $1 + \cdot 5 = 1 \cdot 5$ and $900 \div 1 \cdot 5 \times 1 \cdot 5 = 400$lb. Ans.

54. A certain body on the surface of the earth, weighs 180lb.: How high must it be carried to weigh but 20lb.?

$\sqrt{180 \div 20} = 3$, Ans. 3 semidiameters from the earth's centre, that is 8000 above its surface.

55. To what height must a ball be raised to lose half its weight?

As 1 : $\overline{3982\cdot06\times3982\cdot06}$ :: 2 : 31713603·6872, and $\sqrt{31713603\cdot6872}$ =5631·48 : and 5631·48—3982·06=1649·42 miles, Ans.

56. *At what distance from the earth would a balloon be suspended between the earth and moon ?*

Rule.—As the sum of the square roots of their quantities of matter is to the distance of their centres, so is the square root of the quantity of matter in the earth, to the distance from the earth's centre.

The proportional quantity of matter in the earth being to that in the moon as 41·24 to 1: and the distance of their centres 240000+3982·06+1090 : therefore, as $\sqrt{41\cdot24}+\sqrt{1}$: $\overline{240000+3982\cdot06+1090}$ :: $\sqrt{41\cdot24}$: 212051·49. And 212051·49—3982·06=208069·43 miles from the earth's surface, Ans.

57.—1. *If the diameters of two globes be equal, and their densities different, the weight of a body on their surfaces will be as their densities.*

2. *If their densities be equal, and diameters different, the weight will be as their diameters.*

3. *If their diameters and densities be both different, the weight will be as the product of their diameters and densities.*

If a stone weigh 100lb. at the surface of the earth, required its weight at the surfaces of the sun and the several planets, whose densities are known respectively ?

	Sun.	Jupiter.	Saturn.	Earth.	Moon.
Their densities	100	78·5	36	392·5	464
Diameters in Eng. miles	883217·58.	89170·81.	79042·35.	7964·12.	2180

As $\overline{7964512\times392\cdot5}$: 100 :: $\left\{\begin{array}{l}\overline{883217\cdot58\times100} : 2825\cdot46\text{lb. at the Sun.}\\ \overline{89170\cdot81\times78\cdot5} : 220\cdot41\text{lb. at Jupiter.}\\ \overline{79042\cdot35\times36} : 91\cdot06\text{lb. at Saturn.}\\ \overline{2180\times464} : 32\cdot35\text{lb. at the Moon.}\end{array}\right.$

58. If the attraction of the moon raise a tide on the earth 5 feet : What will be the height of a tide raised by the earth on the surface of the moon under similar circumstances ?

The attraction of one of those bodies on the other's surface is directly as its quantity of matter, and inversely as its diameter ; therefore, as $\overline{2180\times2180\times2180\times464}$: 5 :: $\overline{7964\times7964\times7964\times392\cdot5}$: 206·22 directly. And as 2180 : 206·22 :: 7964 : 56·448 inversely, Ans.

Of the Fall of Bodies.

59. Heavy bodies near the surface of the earth, fall one foot the first quarter of a second ; three feet the second quarter ; five feet in the third, and seven feet in the fourth quarter ; that is, 16 feet in the first second.*

The

* The exact velocity in *vacuo* is 16·1 in the second ; but in the air it will be scarcely 16 feet.

The velocities, acquired by bodies in falling, are in proportion to the squares of the times in which they fall; for instance, Let go three bullets together; stop the first at one second, and it will have fallen 16 feet. Stop the next at the end of the second second, and it will have fallen ($2\times2=4$) four times 16, or 64 feet; and stop the last at the end of the third second, and the distance fallen will be ($3\times3=9$) nine times 16, or 144 feet, and so on.

Or, which is the same, the space fallen through (in feet) is always equal to the square of the time in 4ths of a second.

Or, by multiplying 16 feet by so many of the odd numbers, beginning at unity, as there are seconds in any given time; viz. by 1 for the first second, by 3 for the second, by 5 for the third, and so on, these several products will give the spaces fallen through, in each of the several seconds, and their sum will be the whole distance fallen.

The velocity given, to find the space fallen through.

RULE.

1. The square root of the feet, in the space fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall; therefore,

2. Divide the velocity by 8, and the square of the quotient will be the distance fallen through, to acquire that velocity.

Suppose the velocity of a cannon ball to be about $\frac{1}{8}$ of a mile, or 660 feet per second: From what height must a body fall, to acquire the same velocity per second?

$660\div8=82{\cdot}5$ and $82{\cdot}5\times82{\cdot}5=6806\frac{1}{4}$ feet, $=1\frac{37}{123}$ mile, Ans.

60. *The time given, to find the space fallen through.*

RULE.

1. The square root of the feet, in the space fallen through, will ever be equal to four times the number of seconds the body has been falling; therefore,

2. Multiply the time by 4, and the square of the products will be the space fallen through in the given time.

How many feet will a body fall in 5 seconds?

$5\times4=20$, and $20\times20=400$ feet, Ans.

61. A bullet is dropped from the top of a building, and found to reach the ground in $1\frac{3}{4}$ second: Required its height?

$1{\cdot}75\times4=7$, and $7\times7=49$ feet, Ans. Or, $1\frac{3}{4}=7$ qrs. and $7\times7=49$. Or, $1{\cdot}75\times1{\cdot}75\times16=49$ feet, Ans.

62 What is the difference between the depths of two wells, into each of which should a stone be dropped in the same instant, one would reach the bottom in 5 seconds, and the other in 3?

$5\times4=20$, and $20\times20=400$ feet.
$3\times4=12$, and $12\times12=144$ feet.

Ans. 256 feet.

63. Ascending bodies are retarded in the same ratio that descending bodies are accelerated; therefore, if a ball, discharged from a gun, returned to the earth in 12 seconds: How high did it ascend?

The

The ball being half of the time, or 6 seconds, in its ascent, therefore, $6 \times 4 = 24$, and $24 \times 24 = 576$ feet, Ans.

64. *The velocity per second given, to find the time.*

RULE.

1. Four times the number of seconds, in which a body has been falling, is equal to one eighth of the velocity, in feet, per second, acquired at the end of the fall; therefore,

2. Divide the given velocity by 8, and one fourth part of the quotient will be the answer.

How long must a bullet be falling, to acquire a velocity of 160 feet per second? $160 \div 8 = 20$, and $20 \div 4 = 5$ seconds, Ans.

65. *The space, through which a body has fallen, given, to find the time it has been falling.*

RULE.

1. Four times the number of seconds, in which the body has been falling, will ever be equal to the square root of the space, in feet, through which it has fallen; therefore,

2. Divide the square root of the space fallen through by 4, and the quotient will be the time, in which it was falling.

In how many seconds will a bullet fall through a space of 10125 feet? $\sqrt{10125} = 100{\cdot}6$, and $100{\cdot}6 \div 4 = 25{\cdot}15$ seconds $= 25'' \; 9'''$ Ans.

66. In what time will a musket ball, dropped from the top of a steeple, 484 feet high, come to the ground?

$\sqrt{484} = 22$, and $22 \div 4 = 5\frac{1}{2}$ seconds, Ans.

67. *To find the velocity, per second, with which a heavy body will begin to descend, at any distance from the earth's surface.*

RULE.

As the square of the earth's semidiameter is to 16 feet, so is the square of any other distance from the earth's centre, inversely, to the velocity with which it begins to descend per second.

With what velocity, per second, will an iron ball begin to descend if raised 3000 miles above the earth's surface?

As $4000 \times 4000 : 16 :: \overline{4000+3000} \times \overline{4000+3000} : 5{\cdot}22449$ feet, Ans.

68. How high must a ball be raised above the earth's surface, to begin to descend with a velocity of 5·22449 feet per second?

As $16 : 4000 \times 4000 :: 5{\cdot}22449 : 49000000$, and $\sqrt{49000000} = 7000$.
Wherefore, $7000 - 4000 = 3000$ miles, Ans.

69. *To find the mean velocity of a falling body.*

RULE.

Divide the space fallen through by the number of seconds it was falling, and the quotient will be the mean velocity.

A musket ball dropped from the top of a steeple 484 feet high in $5\frac{1}{2}$ seconds: Required its mean velocity?

$484 \div 5{\cdot}5 = 88$ feet per second, Ans.

70. *To*

70. *To find the velocity acquired by a falling body, per second, (or by a stream of water, having the perpendicular descent given) at the end of any given period of time.*

RULE.

1. The velocity acquired at the end of any period is equal to twice the mean velocity, with which it passed during that period.

Or, 2. Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity required.

If a ball fall through a space of 484 feet in $5\frac{1}{2}$ seconds, with what velocity will it strike?

By the former part of the rule.
$484 \div 5{\cdot}5 = 88$, and
$88 \times 2 = 176$, Ans.

By the latter part, without regarding the time.
$\sqrt{484 \times 64} = 176$, Ans.

71. There is a sluice, or flume, one end of which is $2\frac{1}{2}$ feet lower than the other: What is the velocity of the stream per second?

$2{\cdot}5 \times 64 = 160$, and $\sqrt{160} = 12{\cdot}649$ feet, Ans.

72. *The velocity, with which a falling body strikes, given, to find the space fallen through.*

RULE.

Divide the square of the velocity by 64, and the quotient will be the height required.

If a ball strike the ground with a velocity of 56 feet per second, from what height did it fall?

$\overline{56 \times 56} \div 64 = 49$ feet, Ans.

73. The mean velocity of a fluid, or stream, is 12·649 feet per second: What is the perpendicular fall of the stream?

$\overline{12{\cdot}649 \times 12{\cdot}649} \div 64 = 2\frac{1}{2}$ feet, Ans.

74. *The weight of a body, and the space fallen through, given, to find the force with which it will strike.*

RULE.

The momentum, or force, with which a falling body strikes, is equal to its weight multiplied by its velocity; therefore, find the velocity, by Problem 70, and multiply it by the weight, which will produce the force required.

If the rammer, used for driving the piles of Charlestown bridge, weighed $2\frac{1}{4}$ tons, or 4500lb. and fell through a space of 10 feet, with what force did it strike the pile?

$\sqrt{10 \times 64} = 25{\cdot}3 =$ velocity, and $25{\cdot}3 \times 4500 = 113850$lb. momentum, Answer.

75. *The weight and momentum, or striking force, given, to find the space fallen through.*

RULE.

Divide the momentum by the weight, and the quotient will be the velocity; then divide the square of the velocity by 64, and the quotient will be the space fallen through.

If

If the aforementioned rammer weighed 4500lb. and struck with a force of 113850lb. : From what height did it fall ?

$113850 \div 4500 = 25 \cdot 3$, and $\overline{25 \cdot 3 \times 25 \cdot 3} \div 64 = 10$ feet, Ans.

76. If it were required to know with what quantity of motion, momentum or force, a fluid, moving with a given velocity, strikes upon a fixed obstacle.

RULE.

By Problem 72 find the fall, which will produce the given velocity ; multiply that height by 62·5lb Avoird. for clean river water, by 63 lb. for dirty water, and by 64 for sea water.

Suppose a stream of clear water to move at the rate of 5 feet per second, and to meet with a fixed obstacle (or bulk head) 15 feet wide and 4 feet high : What is the momentary, instantaneous pressure of the stream ?

$\overline{5 \times 5} \div 64 = \frac{25}{64}$ and $25 \div 64 = \cdot 39$ of a foot, for the perpendicular fall of the water. Now $62 \cdot 5 \times \cdot 39 = 24 \cdot 375$lb. the pressure upon each square foot, which, multiplied by 60, (the number of square feet in the obstacle) gives 1462·5lb. going with the given velocity of 5 feet per second ; therefore, $1462 \cdot 5 \times 5 = 7312 \cdot 5$lb. Ans.*

77. The velocity of water, spouting through a sluice, or aperture in a reservoir, or bulk head, is the same that a body would acquire by falling through a perpendicular space equal to that between the top of the water in the reservoir and the aperture.

What is the velocity of water, issuing from a head of 5 feet deep ?

By Problem 70th $64 \times 5 = 320$, and $\sqrt{320} = 18$ feet, nearly, Ans.

78. If the velocity of a stream issuing through the bulk head of a mill, be 16 feet per second, what head of water is there.

$\overline{16 \times 16} \div 64 = 4$ feet, Ans.

79. The quantity of water discharged from a hole in a vessel, is as the square root of the height of water above the aperture.

A miller has a head of water 4 feet above the sluice : How high must the water be raised above the opening, so that half as much again water may be discharged from the sluice in the same time ?

$\sqrt{4} = 2$, and half as much again as 2, is $2+1=3$, for the square root of the required depth ; therefore, $3 \times 3 = 9$ feet high, Ans.

OF PENDULUMS.

80. The time of a vibration, in a cycloid, is to the time of a heavy body's descent through half its length, as the circumference of a circle to its diameter, that is, as 3·1416 to 1 : therefore, (as a body descends freely, by gravity, through about 193·5 inches in the first second) to find the length of a pendulum vibrating seconds.

RULE.—As $3 \cdot 1416 \times 3 \cdot 1416 : 1 \times 1 :: 193 \cdot 5 : 19 \cdot 6$ inches, the half length, and $19 \cdot 6 \times 2 = 39 \cdot 2$ inches, the length.

18. *To*

* Water, being a yielding fubftance, lofes two thirds of its power in producing effects.

81. *To find the length of a pendulum, that will swing any given time.*

Rule.—Multiply the square of the seconds in any given time by 39·2 and the product will be the length required, in inches.

Required the lengths of several pendulums, which will respectively swing $\frac{1}{4}$ seconds, $\frac{1}{2}$ seconds, seconds, minutes and hours?

$\cdot25\times\cdot25\times39\cdot2=2\cdot45$ inches for $\frac{1}{4}$ seconds. $\cdot5\times\cdot5\times39\cdot2=9\cdot8$ inches for $\frac{1}{2}$ seconds. $1\times1\times39\cdot2=39\cdot2$ inches for seconds, as above; $60\times60\times39\cdot2=$the inches in 2 miles and 1200 feet, for minutes; and 1 hour $=3600$ seconds, therefore $3300\times3600\times39\cdot2=$ the inches in 8018 miles and 96 feet, for hours, Ans.

82. What is the difference between the length of a pendulum, which vibrates half seconds and one which swings three seconds?

$\overline{3\times3\times39\cdot2}-\overline{\cdot5\times\cdot5\times39\cdot2}=343$ feet, Ans.

83. *To find the time which a pendulum of any given length will swing.*

Rule.—Divide the given length by 39·2, and the quotient will be the square of the time in seconds.

Or, as 6·261 (the square root of 39·2) is to the square root of the given length, so is 1 second to the time of 1 oscillation: that is, divide the square root of the given length by 6·261, and the quotient will be the time of one vibration of that pendulum.

How often will a pendulum of 9·8 inches vibrate in a second?

By the former part of the rule, $9\cdot8\div39\cdot2=\cdot25$ of a second, and $\sqrt{\cdot25}=\cdot5$ of a second, the time of one vibration, that is, it vibrates half seconds, or $60\div\cdot5=120$ times in a minute.

By the latter part. $\sqrt{9\cdot8}=3\cdot13$, and $\sqrt{39\cdot2}=6\cdot261$, therefore, $3\cdot13\div6\cdot261=\cdot5$ of a second.

84. I observed, that while a stone was falling from a precipice, a string, (with a bullet at the end) which measured 25 inches, (to the middle of the ball,) had made 5 vibrations: What was the height of the precipice?

$25\div39\cdot2=\cdot6377+$, and $\sqrt{\cdot6377}=\cdot7986$—of a second, the time of one vibration, and $\cdot7986\times5=4$ seconds, nearly, the time of the stone's descent; then $4\times4=16$, and $16\times16=256$ feet, Ans.

85. *To find the true depth of a well, by dropping a stone into it, also the time of the stone's descent and of the sound's ascent.*

Rule.—1. Take a line of any length, and by the last Problem find the time from the dropping of the stone till you hear it strike the bottom.

2. Multiply 73088 ($=16\times4\times1142$; 1142 feet being the distance, which sound moves in a second) by the number of seconds till you hear the stone strike the bottom.

3. To this product add 1304164 (= the square of 1142) and from the square root of the sum take 1142.

4. Divide the square of the remainder by 64 ($=16\times4$) and the quotient will be the depth of the well in feet.

5. Divide

5. Divide the depth by 1142, and the quotient will be the time of the sound's ascent, which, being taken from the whole time, will leave the time of the stone's descent in seconds.

Suppose I drop a stone into a well, and a string with a plummet, which measured to the middle of the ball, 25 inches, made 5 vibrations before I heard the stone strike the bottom: Required the depth, time of the stone's descent, and of the sound's ascent:

$25 \div 39{\cdot}2 = {\cdot}6377$, and $\sqrt{{\cdot}6377} = {\cdot}7986$, and ${\cdot}7986 \times 5 = 4$ seconds to the hearing of it strike; then, $\sqrt{\overline{73088 \times 4} + 1304164} - 1142 = 121{\cdot}53$; and $121{\cdot}53 \times 121{\cdot}53 \div 64 = 230{\cdot}77$ feet, the depth, and $23077 \div 1142 = {\cdot}2$ of a second, the time of the sound's ascent, and $4 - {\cdot}2 = 3{\cdot}8$ seconds, the time of the stone's descent.

Of the Lever or Steelyard.

86. It is a principle in mechanicks, that the power is to the weight, as the velocity of the weight, to the velocity of the power. Therefore, to find what weight may be raised or balanced by any given power, say;

As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied; so is the power to the weight which it will balance.

If a man, weighing 160lb. rest on the end of a lever 10 feet long, what weight will he balance on the other end, supposing the prop one foot from the weight?

The distance between the weight and prop being 1 foot, the distance from the prop to the power is $10 - 1 = 9$ feet; therefore, as 1 foot : 9 feet :: 160lb. : 1440lb. Ans.

87. If a weight of 1440lb. were to be raised with a lever 10 feet long, and the prop fixed one foot from the weight, what power or weight, applied to the other end of the lever would balance it?

As 9 : 1 :: 1440 : 160 lb. Ans.

88. If a weight of 1440lb. be placed 1 foot from the prop, at what distance from the prop must a power of 160lb. be applied, to balance it? As 160 : 1440 :: 1 : 9 feet, Ans.

89. At what distance from a weight of 1440lb. must a prop be placed, so as that a power of 160 lb. applied 9 feet from the prop may balance it? As 1440 : 160 :: 9 : 1 foot, Ans.

90. In giving directions for making a chaise, the length of the shafts between the axletree and backband, being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed. The chaise maker advised to place it 30 inches before the axletree; others supposed 20 inches would be a sufficient incumbrance for the horse: Now, supposing two passengers to weigh 3 cwt. and the body of the chaise $\frac{3}{4}$ cwt. more: What will the beast in both these cases bear, more than his harness?

Weight of the chaise and passengers $3\frac{3}{4}$ cwt. = 420lb. and 9 feet = 180 inches.

$$\text{Then, as } \overset{\text{In.}}{108} : \overset{\text{lb.}}{420} :: \left\{ \begin{array}{l} \overset{\text{In.}}{30} : \overset{\text{lb.}}{116\frac{2}{3}} \\ 20 : 77\frac{7}{9} \end{array} \right\} \text{Ans.}$$

Of

Of the Wheel and Axle.

91. The proportion for the wheel and axle (in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, which coils about the axle as the wheel turns round) is, as the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel, to the weight suspended by the axle.

A mechanick would make a windlass in such a manner, as that 1lb. applied to the wheel, should be equal to 10lb. suspended from the axle; now, supposing the axle to be 6 inches diameter, required the diameter of the wheel?

lb. in. lb. in.
As 10 : 6 :: 1 : 60 inversely, the diameter required.

92. Suppose the diameter of the wheel to be 60 inches: Required the diameter of the axle, so as that 1lb. on the wheel may balance 10lb. on the axle?

lb. in. lb in.
Inversely, as 1 : 60 :: 10 : 6 diameter required.

93. Suppose the diameter of the axle 6 inches, and that of the wheel 60 inches, what power at the wheel will balance 10lb. at the axle?

in. lb. in. lb.
Inversely, 6 : 10 :: 60 : 1 Ans.

94. Suppose the diameter of the wheel 60 inches, and that of the axle 6 inches: what weight at the axle will balance 1lb. at the wheel?

in. lb. in. lb.
Inversely, as 60 : 1 :: 6 : 10 Ans.

Of the Screw.

95. The power is to the weight, which is to be raised, as the distance between two threads of the screw, is to the circumference of a circle described by the power applied at the end of the lever.

Rule.—Find the circumference of the circle described by the end of the lever; then, as that circumference is to the distance between the spiral threads of the screw: so is the weight to be raised, to the power which will raise it, abating the friction, which is not proportional to the quantity of surface; but to the weight of the incumbent part; and, at a medium, $\frac{1}{3}$ part of the effect of the machine is destroyed by it, sometimes more and sometimes less.

There is a screw, whose threads are an inch asunder; the lever by which it is turned 30 inches long, and the weight to be raised a ton, or 2240lb.: What power or force must be applied to the end of the lever, sufficient to turn the screw—that is, to raise the weight. The lever being the semidiameter of the circle, the diameter is 60 inches; then, $3{\cdot}1416\times60=188{\cdot}496$ inches, the circumference:

in. in. lb. lb.
Therefore, as 188·496 : 1 :: 2240 : 11·88, Ans.

96. Let the lever be 30 inches, (the circumference of which is found to be 188·496) the threads 1 inch asunder, and the power 11·88 lb.: Required the weight to be raised?

in. in. lb. lb.
As 1 : 188·496 :: 11·88 : 2240 nearly, Ans.

97. Let the weight be 2240lb. the power 11·88lb. and the lever 30 inches: Required the distance between the threads?

lb. lb. in. in.
As 2240 : 11·88 :: 288·496 : 1 nearly, Ans.

98. Let the power be 11·88lb. the weight 2240lb. and the threads an inch asunder, to find the length of the lever.

lb. lb. in. in.
As 11·88 : 2240 :: 1 : 188·5 ; then, as 355 : 113 :: 188·5 : 60 inches nearly, the diameter, and 60÷2=30 inches, Ans.

99. Suppose one of those meteors, called fire balls, to move parallel to the earth's surface, and 50 miles from it, at the rate of 20 miles per second: In what time would it move round the earth?

The Earth's diameter is 7964 English miles; then, $7964+\overline{50\times2}=$ 8064=the diameter of the circle, described by the ball. Then, $\overline{8064\times3\cdot1416}=25333\cdot8624$ miles, its circumference, and 25333·8624 ÷20=1266·69312 seconds=21′ 6″ 41‴ 35⁗ 13′′′′′ 55′′′′′′ 12′′′′′′′, Ans.

100. Sound, uninterrupted, moves about 1142 feet in a second: How long, then, after firing a cannon at Newburyport, before it will be heard at Ipswich, estimating the distance at 10 miles in a right line?

10 miles = 52800 feet, and 52800÷1142 = $46\frac{134}{571}$ seconds, Ans.

101. In a thunder storm I observed by my clock that it was 6 seconds between the lightning and thunder: at what distance was the explosion? 1142×6=6852 feet = $1\frac{131}{440}$ mile, Ans.

102. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1623}$ of a grain per foot: What would be the weight of such a tube, which would extend across the Atlantick, from Boston to London, estimating the distance at 1000 leagues?

1000×3=3000 miles, and 3000×5280=15840000 feet, and 15840000 $\times\frac{1}{1623}=9747\frac{9}{13}$gr. or rather, 1lb. 8oz. 6pwt. $3\frac{0}{13}$gr. Ans.

103. The mean distances of the Planets from the Sun, in English miles, are as follow: viz. Mercury 36686617·5; Venus 68552135·83; Earth 94772980; Mars 144404783·33; Jupiter 492912533·33; Saturn 903957657·5: Now, as a cannon ball, at its first discharge, flies about a mile in 8 seconds, and sound 1142 feet in a second: In what time, at the above rate, would a bullet pass from the Earth to the Sun? and sound move from the Sun to Saturn?

94772980×8″=758183840=24 years, 15 days, 6 hours, 27 minutes, 20 seconds, for the passage of the ball. And 903957657·5×5280= 4772896431600 feet, and 4772896431600÷1142=132 years, 192 days, 21h. 42m. $21\frac{489}{571}$s. sound passing from the Sun to Saturn, Ans.

104. Light passes from the Sun to the Earth in 8·2 minutes: In what time would it pass from the sun to the *Georgium Sidus*, it being 1803930416·66 English miles?

As 94772980 : 8·2 :: 1803930416·66 : 2h. 36m. 4″ 50‴, Ans.

105. The Sun's diameter is 883217·58 English miles; Jupiter's is 89170·81; Saturn's 79042·35; Georgium 35109; Mercury's 3222·48; Venus' 7687·85; Earth's 7964·12; Mars' 4189·69; and

and the Moon's 2180: Required the comparative magnitude between each of those bodies and the Earth?

$$\left.\begin{array}{l}\overline{883217{\cdot}58\times883217{\cdot}58\times883217{\cdot}58}\\ \overline{89170{\cdot}81\times89170{\cdot}81\times89170{\cdot}81}\\ \overline{97042{\cdot}35\times97042{\cdot}35\times97042{\cdot}35}\\ \overline{35109\times35109\times35109}\end{array}\right\}\div\overline{7964{\cdot}12\times7964{\cdot}12\times7964{\cdot}12}=\left\{\begin{array}{r}1368724\\1402{\cdot}65\\982\\99{\cdot}57\end{array}\right\}\text{greater.}$$

$$\overline{7964{\cdot}12\times7964{\cdot}12\times7964.12}\div\left\{\begin{array}{l}3222{\cdot}48\times3222{\cdot}48\times3222{\cdot}48=93{\cdot}12\\7687{\cdot}85\times7687{\cdot}85\times7687{\cdot}85=\ 1{\cdot}11\\4189{\cdot}69\times4189{\cdot}69\times4189{\cdot}69=\ 6{\cdot}86\\ \quad 2180\times2180\quad\times2180\quad=48{\cdot}74\end{array}\right\}\text{lefs.}$$

N. B. The above diameters and mean distances in English miles answer to the same in geographical miles, as they were deduced from observations on the transits of Venus over the Sun in 1761 and 1769.

106. Suppose the density of the Moon 464, and that of the Earth 392·5: Required the proportion between the quantity of matter in the Earth and in that of the Moon, allowing the Earth's diameter to be 7964·12, and the Moon's 2180 miles, and supposing the Earth a complete sphere, which, however, it is not?

There is $\dfrac{7964{\cdot}12\times7964{\cdot}12\times7964{\cdot}12\times392{\cdot}5}{2180\quad\times2180\quad\times2180\quad\times464}=41{\cdot}24$ times the quantity of matter in the Earth that there is in the Moon; or, the Earth's weight is so many times that of the Moon.

107. The mean diameter of the Earth's orbit, (or annual path round the Sun) supposing it truly spherical, is, in English miles, 190437141·7: Required its mean motion, (or the space through which it moves in its orbit,) per minute?

190437141·7×3·1416=598277324·36 miles in circumference; then,
Days.
As 365·25 : 598277324·36 :: 1′ : 1137·49 miles, Ans.

N. B. The Earth's diurnal motion round its axis is 17¼ miles per minute, at the equator.

Of the Specifick Gravities of Bodies.

The specifick gravities of bodies are as their densities, or weights, bulk for bulk; thus, a body is said to have two or three times the specifick gravity of another, when it contains two or three times as much matter in the same space.

A body, immersed in a fluid, will sink, if it be heavier than its bulk of the fluid. If it be suspended therein, it will lose so much of what it weighed in the air, as its bulk of the fluid weighs. Hence, all bodies of equal bulk, which will sink in fluids, lose equal weights when suspended therein, and unequal bodies lose in proportion to their bulks.

The *hydrostatick balance* differs very little from a common balance that is nicely made; only it has a hook at the bottom of each scale, on which small weights may be hung by horse hairs, so that a body suspended by the hair, may be immersed in water without wetting the scales.

How

How to find the Specifick Gravities of Bodies.

If the body, thus suspended under the scale, at one end of the balance, be first counterpoised in air by weights in the opposite scale, and then immersed in water, the equilibrium will be immediately destroyed; then, if as much weight be put into the scale, to which the body is suspended, as will restore the equilibrium, (without altering the weights in the opposite scale) that weight, which restores the equilibrium, will be equal to a quantity of water as big as the immersed body; and if the weight of the body in air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus, if a guinea, suspended in air, be counterbalanced by 129 grains in the opposite scale, and then, upon being immersed in water, it becomes so much lighter as to require $7\frac{1}{4}$ grains to be put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs 7·25 grains; by which divide 129 (the weight of the guinea in air) and the quotient will be 17·793; which shews that the guinea is 17·793 times as heavy as its bulk of water.

Thus may any piece of gold be tried, by weighing it first in air, and then in water; and if, upon dividing the weight in air by the loss in water, the quotient comes out 17·793, the gold is good: If the quotient be 18, or between 18 and 19, the gold is very fine: but, if it be less than 17, the gold is too much alloyed by being mixed with some other metal.

If silver be tried in this manner and found to be 11 times as heavy as water, it is very fine: If it be $10\frac{1}{2}$ times as heavy, it is standard; but if it be of any less weight compared with water, it is mixed with some lighter metal, such as tin, &c.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, the different losses of weight therein will shew how much heavier it is than its bulk of the fluid; that fluid being lightest, in which the immersed body loses least of its aerial weight.

Common clear water, for common uses, is generally made a standard for comparing bodies by, whose gravity may be represented by unity, or 1, or, in case great accuracy be required, by 1·000, where 3 cyphers are annexed to give room to express the ratios of other gravities in larger numbers in the table. In doing this there is a two-fold advantage; the first is, that, by this mean, the specifick gravities of bodies may be expressed to a much greater degree of accuracy.— The second is, that the numbers of the Table, considered as whole numbers, do also express the ounces Avoirdupois contained in a cubick foot of every sort of matter therein specified; because a cubick foot of common water, is found by experiment to weigh very nearly 1000 ounces Avoirdupois, or $62\frac{1}{2}$ pounds.

A TABLE *of the Specifick Gravities of several solid and fluid Bodies ; where the second column contains their Absolute weight, and the third, their Relative Weight, in Avoirdupois Ounces.*

A Cubick Foot of	Abso. wt.	Rela. wt.	A Cubick Foot of	Abso. wt.	Rela. wt.
Platina rendered malleable and hammered	20170	20·170	Brick - - - - - - -	2000	2·000
			Liver Sulphur - - -	2000	2.000
Very fine Gold - - -	19637	19·637	Nitre - - - - - - -	1900	1·900
Standard Gold - - -	18888	18·888	Alabaster - - - -	1875	1·875
Guinea Gold - - - -	17793	17·793	Dry Ivory - - - -	1825	1·825
Moidore Gold - - -	17140	17·140	Brimstone - - - -	1800	1·800
Quickfilver - - - -	13600	13·600	Solid fubs. of Gun Pow.	1745	1·745
Lead - - - - - - -	11325	11·325	Alum - - - - - -	1714	1·714
Fine Silver - - - -	11087	11·087	Ebony - - - - - -	1117	1·117
Standard Silver - - -	10535	10·535	Human Blood - - -	1054	1·054
Rofe Copper - - - -	9000	9·000	Amber - - - - - -	1030	1·030
Copper - - - - - -	8843	8·843	Cow's Milk - - -	1030	1·030
Plate Brafs - - - -	8000	8·000	Sea Water - - - -	1030	1·030
Steel - - - - - - -	7852	7·852	Pure Water - - -	1000	1·000
Caft Brafs - - - - -	7850	7·850	Red Wine - - - -	993	0·993
Iron - - - - - - -	7645	7·645	Oil of Amber - - -	978	0·978
Block Tin - - - - -	7321	7·321	Proof Spirits - - -	925	0·925
Caft Iron - - - - -	7135	7·135	Dry Oak - - - -	925	0·925
Lead Ore - - - - -	6800	6·800	Olive Oil - - - - -	913	0·913
Copper Ore - - - -	3775	3·775	Loofe Gun Powder -	872	0·872
Diamond - - - - -	3400	3·400	Spirits of Turpentine -	864	0·864
Chryftal Glafs - - -	3150	3·150	Alcohol or pure Spirit	850	0·850
White Marble - - -	3707	2·707	Elm and Afh - - -	800	0·800
Black Marble - - -	2704	2·704	Oil of Turpentine -	772	0·772
Rock Cryftal - - - -	2658	2·658	Dry Crab Tree - - -	765	0·765
Green Glafs - - - -	2620	2·620	Æther - - - - - -	732	0·732
Clear Glafs - - - -	2600	2·600	White Pine - - - -	569	0·569
Stone: Flint - - -	2582	2·582	Saffafras Wood - -	482	0·482
Stone: Paving - - -	2570	2·570	Cork - - - - - - -	240	0·240
Stone: Cornelian -	2568	2·568	Common Air - - -	$1\frac{25}{100}$	0·00125
Stone: Free - - - -	2352	2·352	Inflammable Air - -	$0\frac{12}{100}$	0·00012

The use of the Table of Specifick Gravities will best appear by several Examples.

How to discover the quantity of adulteration in metals.

Suppose a body be compounded of gold and silver, and it be required to find the quantity of each metal in the compound.

First, find the specifick gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loses thereof in water, and the quotient will shew its specifick gravity, or how many times heavier it is than its bulk of water. Then, subtract the specifick gravity of silver (found in the Table) from that of the compound, and the specifick gravity of the compound from that of the gold : the first remainder will shew the bulk of gold, and the latter, the bulk of silver in the whole compound ; and if these remainders be multiplied by the respective specifick gravities, the products will shew the proportional weights of each metal in the body.

Suppose

Suppose the specifick gravity of the compounded body be 14; that of standard silver (by the Table) is 10·535, and that of standard gold 18·888; therefore, 10·535 from 14, remains 3·465, the proportional *bulk* of the gold in the compound; and 14 from 18·888, remains 4·888, the proportional *bulk* of silver in the compound: then, 18·888, the specifick gravity of gold, multiplied by the first remainder 3·465, produces 65·447 for the proportional *weight* of gold; and 10·535, the specifick gravity of silver, multiplied by the last remainder, produces 51·495 for the proportional weight of silver in the whole body: So that for every 65·447 ounces or pounds of gold, there are 51·495, ounces or pounds of silver in the body.

Hence it is easy to know whether any suspected metal be genuine, or alloyed or counterfeit, by finding how much heavier it is than its bulk of water, and comparing the same with the Table; if they agree, the metal is good; if they differ, it is alloyed or counterfeited.

How to try Spirituous Liquors.

A cubick inch of good brandy, rum, or other proof spirits, weighs 234 grains; therefore, if a true inch cube of any metal weighs 234 grains less in spirits than in air, it shews the spirits are proof: If it lose less of its aerial weight in spirits, they are above proof; if it lose more, they are under proof; for, the better the spirits are, the lighter they are, and the worse, the heavier.

Or, let any solid, of sufficient specifick gravity, be weighed first in air, then in water, and then in another liquid; from its weight in the air take its weight in water, and the remainder is the weight of its bulk of water. From its weight in air take its weight in the other liquid, and the remainder is the weight of the same quantity of that liquid. Divide the weight of this quantity of liquid by the weight of the same quantity of water, and the quotient will be the specifick gravity of the liquid.

All bodies expand with heat and contract with cold; but some more, and some less than others: therefore, the specifick gravities of bodies are not precisely the same in summer as in winter.

The four following Problems, relating to spirituous liquors, are wrought by Alligation.

108. What proportion of rectified spirits of wine must be mixed with water, to make proof spirit, the specifick gravity of the rectified spirits being 850, that of proof spirit 925, and of water 1000?

$$925 \left\{ \begin{matrix} 1000 \\ 850 \end{matrix} \right\rangle \begin{matrix} 75 \\ 75 \end{matrix} \Big\} \text{ Or equal measures.}$$

109. What proportional *weight* of rectified spirits of wine and water must be mixed, to make proof spirit, the specifick gravities as before?

$$\text{Ans. } \frac{1000}{850} = \frac{20}{17}, \text{ or as 20 to 17.}$$

110. What is the specifick gravity of best French brandy, consisting of 5 parts, measure, of rectified spirits of wine, and 3 parts water? 850×

$$\begin{array}{rcr} 850 \times 5 & = & 4250 \\ 1000 \times 3 & = & 3000 \\ \hline 5+3= \; 8 &) & 7250 \\ \hline & & 906\cdot25 = \text{specifick gravity.} \end{array}$$

111. A retailer has 30 gallons of rum, whose specifick gravity is 900 : How much water must he add to reduce it to standard proof ?

$$925\left\{\begin{array}{ll} 1000 & 25 \\ 900 & 75 \end{array}\right\} \quad \begin{array}{l} \text{g. rum. g. wat. g. rum. g. wat.} \\ \text{As } 75 : 25 :: 30 : 10 \text{ to be added.} \end{array}$$

112. The cubick inch of common glass weighs about 1·36oz. Troy : ditto of salt water ·5427oz. ditto of brandy ·48927oz. Suppose then, a seaman has a gallon of brandy in a bottle, which weighs 4½lb. Troy, out of water, and, to conceal it, throws it overboard into salt water : Pray, will it sink or swim, and by how much is it heavier or lighter than the same bulk of salt water ?

$$4\tfrac{1}{2}\text{lb.}=54\text{oz.}=\text{weight of bottle } \frac{54}{1\cdot36} = 39\cdot7059 \text{ cub. in. in the bottle.}$$

$$\begin{array}{lr} \text{Add } 231\cdot & = \text{do. in the brandy.} \\ \hline 270\cdot7059 & = \text{ditto in both.} \end{array}$$

Then, 270·7059×·5427=146·912oz.=weight of salt water occupied by the bottle and brandy. And ·48927 (=weight of a cubick inch of brandy)×231=113·02+oz. and 113·02×54=167·02oz.= weight of the bottle and brandy. From this take the weight of the salt water, viz. 146·192oz. Ans. Supposing the bottle full, it is 20·11oz. heavier than the same bulk of salt water, and therefore will sink.

Given the weight to be raised by a balloon, to find its diameter.

RULE.

1. As the specifick difference between common and inflammable air, is to one cubick foot ; so is any weight to be raised, to the cubick feet contained in the balloon.

2. Divide the cubick feet by ·5236, and the cube root of the quotient will be the diameter required, to balance it with common air ; but, to raise it, the diameter must be somewhat greater, or the weight somewhat less.

113. I would construct a spherical balloon, of sufficient capacity to ascend with 4 persons, weighing, one with another, 160lb. and the balloon and a bag of sand weighing 60lb. : Required the diameter of the balloon ?

By the Table of Specifick Gravities, page 349th. I find a cubick foot of common air weighs 1·25 ounces Avoirdupois, and a cubick foot of inflammable air ·12 of an ounce Avoirdupois ; therefore,

$$1\cdot25 - \cdot12 = 1\cdot13\text{oz. difference. And } \overset{\text{lb.}}{160} \times 4 + \overset{\text{lb.}}{60} = \overset{\text{lb.}}{700} = \overset{\text{oz.}}{11200}.$$

$$\text{As } \overset{\text{oz.}}{1\cdot13} : \overset{\text{cub. foot.}}{1} :: \overset{\text{oz.}}{11200} : \overset{\text{cub. foot.}}{9911\cdot5044}. \text{ And } \sqrt[3]{\frac{9911\cdot5044}{\cdot5236}} = 26\cdot65 \text{ feet. Ans.}$$

Given

Given the diameter of a balloon, to find what weight it is capable of raising.

RULE.

1. Multiply the cube of the diameter by ·5236, and the product will be the content in cubick feet.

2. As one cubick foot is to the specifick difference between common and inflammable air; so is the content of the balloon to the weight it will raise.

114. The diameter of a balloon is 26·65 feet: What weight is it capable of raising?

26·65×26·65×26·65×·5236=9911·4+ cubick feet. And

cub. foot.		oz.		cub. feet.		oz.
As 1	:	1·13	::	9911·4+	:	11199·882 = 700lb. nearly.

If the magnitude of any body be multiplied by its specifick gravity, the product will be its absolute weight.

115. What weight of lead will cover a house, the area of whose roof is 6000 feet, and the thickness of the lead $\frac{1}{120}$ of a foot?

6000×$\frac{1}{120}$=50 cub. feet, and its specifick gravity 11325×50=566250

	tons.	cwt.	qrs.	lb.	oz.	
ounces =	15	15	3	26	10	Ans.

To find the magnitude of any thing, when the weight is known.

Divide the weight by the specifick gravity in the Table, and the quotient will be the magnitude sought.

116. What is the magnitude of several fragments of clear glass, whose weight is 13 ounces?

13÷2600 = ·005 of a cubick feet, and ·005×1728 = 8·640 cubick inches, Ans.

Having the magnitude and weight of any body given, to find its specifick gravity.

Divide the weight by the magnitude, and the quotient will be the specifick gravity.

117. Suppose a piece of marble contains 8 cubick feet, and weighs 1353½lb. or 21656 ounces: What is the specifick gravity?

21656÷8=2707 the specifick gravity required, as by the Table.

To find the quantity of pressure against the sluice or bank, which pens water.

Multiply the area of the sluice, under water, by the depth of the centre of gravity, (which is equal to half the depth of the water) in feet, and that product again by 62½ (the number of pounds Avoirdupois in a cubick foot of fresh water) or by 64·4lb. (the Avoirdupois weight of a cubick foot of salt water) and the product will be the number of pounds required.

118. Suppose the length of a sluice or flume be 30 feet, the width at bottom 4 feet, and the depth of the water 4 feet; what is the pressure against the side of the sluice?

30×4=120 feet the area of the bottom, and 120×2 (the depth of the centre of gravity) gives 240 cubick feet, and 240×62·5=15000lb. =6T. 13cwt. 3qrs. 20lb. Ans.

The perpendicular pressure of fluids on the bsttoms of vessels is estimated by the area of the bottom multiplied by the altitude of the fluid.

119. Suppose a vessel 3 feet wide, 5 feet long, and 4 feet high, what is the pressure on the bottom, it being filled with water to the brim?

3×5=15

3×5=15 square feet, the area of the bottom, and 15×4=60 cubick feet, and 60×62·5=3750lb.=33cwt. 1qr. 26lb.

The Use of the Barometer.

The Barometer is so formed, that a column of quicksilver is supported within it to such a height as to counterbalance the weight of a column of air, of an equal diameter, extending from the barometer to the top of the atmosphere.

120. At the surface of the earth, the height of this column of quicksilver is, at an average, almost 30 inches; when the barometer is at that height; what is the pressure of atmosphere on a square foot, and on the surface of a man's body, estimated at 14 square feet?

As the cubick foot of quicksilver is 13600 ounces, Avoirdupois, and as the height in the barometer, is 2·5 feet, therefore 13600×2·5 =34000 ounces, =2125 pounds on a square foot; and 2125×14= 29750 pounds on a man's body.

121. If the mercury in a barometer, at the bottom of a tower, be observed to stand at 30 inches, and, on being carried to the top of it, be observed at 29·9 inches: What is the height of the tower?

Divide 13600, the specifick gravity of quicksilver, by 1·25, the specifick gravity of air, and the quotient will be the height of the tower, in tenths of an inch.

$$\frac{13600}{1\cdot25} = 10880 \text{ tenths, and } \frac{10880}{10} = 1088 \text{ inch.} = 90\tfrac{2}{3} \text{ feet, Ans.}$$

The number of feet, in height, of the atmosphere, corresponding with $\frac{1}{10}$ of an inch on the barometer is variable, depending on the temperature and density of the atmosphere.

The variation, depending on the temperature, is shewn in the following Table, calculated for every 5 degrees, from 32 to 80, Farenheit's Thermometer, from whence it may be easily calculated for the intermediate degrees, by allowing $\frac{21}{100}$ of a foot for each degree.

TABLE.

Thermo	Feet
32°	86·86
35	87·49
40	88·54
45	89·60
50	90·66
55	91·72
60	92·77
65	93·82
70	94·88
75	95·93
80	96·99

The altitude, thus found, will be to the altitude corrected for the density of the air, inversely, as the mean height of the barometer, at the two stations, is to 30 inches; therefore,

Rule.—Multiply the mean height corresponding to the mean temperature of the two barometers (found in the Table) by the *tenths* of an inch in the difference of the two barometers, and this product by 30; divide this last product by the mean height of the two barometers, and the quotient will be the answer, or height required, with the errour of a few feet only, if the height be less than a mile.* 122. At

* Let b = mean height of the barometer at its two stations, (or of two barometers, one at each station) in inches; d = difference of the two barometers in *tenths* of an inch; and n = number from the Table answering to the mean temperature of the two thermometers accompanying the barometer, then $\frac{30dn}{b}$ = the altitude required nearly.

122. At the first station, suppose the barometer to stand at 29, and the thermometer at 60; at the 2d station, the barometer at 28, and the thermometer at 40: What is the height of the 2d station or the distance between the two places of observation?

Barometer.

Add { First station = 29
Second station = 28

$\frac{1}{2}$)57

$\frac{1}{2}$ sum = 28·5 = mean height of the two barometers.
29
28

Difference = 1 = 10 tenths of an inch.

Thermometer.

First station = 60
Second station = 40

$\frac{1}{2}$)100

50 = mean height of the two thermometers, against which, in the Table you will find 90·66, the mean temperature of the two barometers. Now, according to the rule $90{\cdot}66 \times 10 \times 30 \div 28{\cdot}5 = 954{\cdot}3$ feet, the Answer, nearly.

TABLES.

Value of English and Portuguese Gold, in dollars, cents and mills, throughout the United States.

Gr.	Cts.	m.	Pwts.	Dls.	cts.
1	3	7	1	0	89
2	7	4	2	1	$77\frac{3}{4}$
3	11	1	3	2	$66\frac{2}{3}$
4	14	8	4	3	55
5	$18\frac{1}{2}$	0	5	4	44
6	22	2	6	5	$33\frac{1}{3}$
7	25	9	7	6	22
8	29	6	8	7	11
9	$33\frac{1}{3}$	0	9	8	0
10	37	0	10	8	89
11	40	7	11	9	$77\frac{3}{4}$
12	44	4	12	10	$66\frac{2}{3}$
13	48	1	13	11	$55\frac{1}{2}$
14	51	8	14	12	44
15	55	5	15	13	$33\frac{1}{3}$
16	$59\frac{1}{4}$	0	16	14	22
17	63	0	17	15	11
18	$66\frac{2}{3}$	0	18	16	0
19	70	4	19	16	89
20	74	0	1oz.	17	$77\frac{3}{4}$
21	$77\frac{3}{4}$	0	*Note.* 89 cents is the value of 1 penny-weight of English and Portugs. Gold.		
22	$81\frac{1}{2}$	0			
23	85	2			

Value of French and Spanish Gold, in dollars, cents and mills, throughout the United States.

Gr.	Cts.	m.	Pwts.	Dls.	cts.	m.
1	3	6	1	0	87	6
2	7	3	2	1	75	2
3	11	0	3	2	$62\frac{3}{4}$	0
4	14	6	4	3	50	3
5	18	2	5	4	38	0
6	21	9	6	5	25	5
7	25	5	7	6	13	1
8	29	2	8	7	0	7
9	32	8	9	7	88	3
10	$36\frac{1}{9}$	0	10	8	76	0
11	40	1	11	9	$63\frac{1}{2}$	0
12	43	8	12	10	51	1
13	47	4	13	11	38	7
14	51	1	14	12	26	3
15	$54\frac{3}{4}$	0	15	13	13	9
16	58	4	16	14	$1\frac{1}{2}$	0
17	62	0	17	14	89	0
18	65	7	18	15	76	6
19	69	3	19	16	64	2
20	73	0	1oz.	17	51	8
21	76	6	*Note.* 87 cents 6 mills, the value of 1 penny-weight French and Spanish Gold.			
22	80	3				
23	83	9				

Federal Money.	*N. Hampſhire, Maſſachuſetts, R. Iſland, Connecticut, & Virginia.*	*New-York and N. Carolina.*	*New-Jerſey, Pennſylvania, Delaware & Maryland.*	*S. Carolina & Georgia.*	*Canada and Novaſcotia.*	*French.*
Dol. c.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	Liv. Tor. Sou
0·0 1	$\frac{18}{25}$	$\frac{24}{25}$	$\frac{9}{10}$	$\frac{14}{25}$	$\frac{3}{5}$	$1\frac{1}{20}$
0·0 2	$1\frac{11}{25}$	$1\frac{23}{25}$	$1\frac{8}{10}$	$1\frac{3}{25}$	$1\frac{1}{5}$	$2\frac{1}{10}$
0·0 3	$2\frac{4}{25}$	$2\frac{22}{25}$	$2\frac{7}{10}$	$0\frac{17}{25}$	$1\frac{4}{5}$	$3\frac{3}{20}$
0·0 4	$2\frac{22}{25}$	$3\frac{21}{25}$	$3\frac{6}{10}$	$2\frac{6}{25}$	$2\frac{2}{5}$	$4\ \frac{1}{5}$
0·0 5	$3\ \frac{3}{5}$	$4\ \frac{4}{5}$	$4\frac{5}{10}$	$2\ \frac{4}{5}$	3	$5\ \frac{1}{4}$
0·0 6	$4\frac{3}{25}$	$5\frac{19}{25}$	$5\frac{4}{10}$	$3\frac{9}{25}$	$3\frac{3}{5}$	$6\frac{3}{10}$
0·0 7	$5\frac{1}{25}$	$6\frac{18}{25}$	$6\frac{3}{10}$	$3\frac{23}{25}$	$4\frac{1}{5}$	$7\frac{7}{20}$
0·0 8	$5\frac{19}{25}$	$7\frac{17}{25}$	$7\frac{2}{10}$	$4\frac{12}{25}$	$4\frac{4}{5}$	$8\ \frac{2}{5}$
0·0 9	$6\frac{12}{25}$	$8\frac{16}{25}$	$8\frac{1}{10}$	$5\frac{1}{25}$	$5\frac{2}{5}$	$9\frac{9}{20}$
0·1 0	$7\ \frac{1}{5}$	$9\frac{3}{5}$	9	$5\ \frac{3}{5}$	6	$10\ \frac{1}{2}$
0·2 0	$1\ 2\ \frac{2}{5}$	$1\ 7\frac{1}{5}$	1 6	$11\ \frac{1}{5}$	1 0	1 1
0·3 0	$1\ 9\ \frac{3}{5}$	$2\ 4\frac{4}{5}$	2 3	$1\ 4\ \frac{4}{5}$	1 6	$1\ 11\ \frac{1}{2}$
0·4 0	$2\ 4\ \frac{4}{5}$	$3\ 2\frac{2}{5}$	3 0	$1\ 10\ \frac{2}{5}$	2 0	2 2
0·5 0	3 0	4 0	3 9	2 4	2 6	$2\ 12\ \frac{1}{2}$
0·6 0	$3\ 7\ \frac{1}{5}$	$4\ 9\frac{3}{5}$	4 6	$2\ 9\ \frac{3}{5}$	3 0	3 3
0·7 0	$4\ 2\ \frac{2}{5}$	$5\ 7\frac{1}{5}$	5 3	$3\ 3\ \frac{1}{5}$	3 6	$3\ 13\ \frac{1}{2}$
0·8 0	$4\ 9\ \frac{3}{5}$	$6\ 4\frac{4}{5}$	6 0	$3\ 8\ \frac{4}{5}$	4 0	4 4
0·9 0	$5\ 4\ \frac{4}{5}$	$7\ 2\frac{2}{5}$	6 9	$4\ 2\ \frac{2}{5}$	4 6	$4\ 14\ \frac{1}{2}$
1·0 0	6 0	8 0	7 6	4 8	5 0	5 5
2·0 0	12 0	16 0	15 0	9 4	10 0	10 10
3·0 0	18 0	1 4 0	1 2 6	14 0	15 0	15 15
4·0 0	1 4 0	1 12 0	1 10 0	18 8	1 0 0	21 0
5·0 0	1 10 0	2 0 0	1 17 6	1 3 4	1 5 0	26 5
6·0 0	1 16 0	2 8 0	2 5 0	1 8 0	1 10 0	31 10
7·0 0	2 2 0	2 16 0	2 12 6	1 12 8	1 15 0	36 15
8·0 0	2 8 0	3 4 0	3 0 0	1 17 4	2 0 0	42 0
9·0 0	2 14 0	3 12 0	3 7 6	2 2 0	2 5 0	47 5
10·0 0	3 0 0	4 0 0	3 15 0	2 6 8	2 10 0	52 10
20·0 0	6 0 0	8 0 0	7 10 0	4 13 4	5 0 0	105 0
30·0 0	9 0 0	12 0 0	11 5 0	7 0 0	7 10 0	157 10
40·0 0	12 0 0	16 0 0	15 0 0	9 6 8	10 0 0	210 0
50·0 0	15 0 0	20 0 0	18 15 0	11 13 4	12 10 0	262 10
60·0 0	18 0 0	24 0 0	22 10 0	14 0 0	15 0 0	315 0
70·0 0	21 0 0	28 0 0	26 5 0	16 6 8	17 10 0	367 10
80·0 0	24 0 0	32 0 0	30 0 0	18 13 4	20 0 0	420 0
90·0 0	27 0 0	36 0 0	33 15 0	21 0 0	22 10 0	472 10
100·0 0	30 0 0	40 0 0	37 10 0	23 6 8	25 0 0	525 0
200·0 0	60 0 0	80 0 0	75 0 0	46 13 4	50 0 0	1050 0
300·0 0	90 0 0	120 0 0	112 10 0	70 0 0	75 0 0	1575 0
400·0 0	120 0 0	160 0 0	150 0 0	93 6 8	100 0 0	2100 0
500·0 0	150 0 0	200 0 0	187 10 0	116 13 4	125 0 0	2625 0
600·0 0	180 0 0	240 0 0	225 0 0	140 0 0	150 0 0	3150 0
700·0 0	210 0 0	280 0 0	262 10 0	163 6 8	175 0 0	3675 0
800·0 0	240 0 0	320 0 0	300 0 0	186 13 4	200 0 0	4200 0
900·0 0	270 0 0	360 0 0	337 10 0	210 0 0	225 0 0	4725 0
1000·0 0	300 0 0	400 0 0	375 0 0	233 6 8	250 0 0	5250 0

N. Hamp. Massachu. R. Island, Connecticut, and Virgin.	*Federal Coin.*	*N. York and North-Carolina.*	*New-Jersey, Pennsylvania, Delaware and Maryland.*	*South-Carolina and Georgia.*	*English Money.*	*French Money.*
£. s. d.	Dls. c.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	Livr. Tour. } Sous.
1	0·01$\frac{7}{18}$	1$\frac{1}{3}$	1$\frac{1}{4}$	$\frac{7}{9}$	$\frac{3}{4}$	1$\frac{11}{24}$
2	0·02$\frac{7}{9}$	2$\frac{2}{3}$	2$\frac{1}{2}$	1$\frac{5}{9}$	1$\frac{1}{2}$	2$\frac{11}{12}$
3	0·04$\frac{1}{6}$	4	3$\frac{3}{4}$	2$\frac{3}{9}$	2$\frac{1}{4}$	4$\frac{3}{8}$
4	0·05$\frac{5}{9}$	5$\frac{1}{3}$	5	3$\frac{1}{9}$	3	5$\frac{5}{6}$
5	0·06$\frac{17}{18}$	6$\frac{2}{3}$	6$\frac{1}{4}$	3$\frac{8}{9}$	3$\frac{3}{4}$	7$\frac{7}{24}$
6	0·08$\frac{1}{3}$	8	7$\frac{1}{2}$	4$\frac{6}{9}$	4$\frac{1}{2}$	8$\frac{3}{4}$
7	0·09$\frac{13}{18}$	9$\frac{1}{3}$	8$\frac{3}{4}$	5$\frac{4}{9}$	5$\frac{1}{4}$	10$\frac{5}{24}$
8	0·11$\frac{1}{9}$	10$\frac{2}{3}$	10	6$\frac{2}{9}$	6	11$\frac{2}{3}$
9	0·12$\frac{1}{2}$	1 0	11$\frac{1}{4}$	7	6$\frac{3}{4}$	13$\frac{1}{8}$
10	0·13$\frac{8}{9}$	1 1$\frac{1}{3}$	1 0$\frac{1}{2}$	7$\frac{7}{9}$	7$\frac{1}{2}$	14$\frac{7}{12}$
11	0·15$\frac{5}{18}$	1 2$\frac{2}{3}$	1 1$\frac{3}{4}$	8$\frac{5}{9}$	8$\frac{1}{4}$	16$\frac{1}{24}$
1 0	0·16$\frac{2}{3}$	1 4	1 3	9$\frac{3}{9}$	9	17$\frac{1}{2}$
2 0	0·33$\frac{1}{3}$	2 8	2 6	1 6$\frac{6}{9}$	1 6	1 15
3 0	0·50	4 0	3 9	2 4	2 3	2 12$\frac{1}{2}$
4 0	0·66$\frac{2}{3}$	5 4	5 0	3 1$\frac{3}{9}$	3 0	3 10
5 0	0·83$\frac{1}{3}$	6 8	6 3	3 10$\frac{6}{9}$	3 9	4 7$\frac{1}{2}$
6 0	1·00	8 0	7 6	4 8	4 6	5 5
7 0	1·16$\frac{2}{3}$	9 4	8 9	5 5$\frac{3}{9}$	5 3	6 2$\frac{1}{2}$
8 0	1·33$\frac{1}{3}$	10 8	10 0	6 2$\frac{6}{9}$	6 0	7 0
9 0	1·50	12 0	11 3	7 0	6 9	7 17$\frac{1}{2}$
10 0	1·66$\frac{2}{3}$	13 4	12 6	7 9$\frac{3}{9}$	7 6	8 15
1 0 0	3·33$\frac{1}{3}$	1 6 8	1 5 0	15 6$\frac{6}{9}$	15 0	17 10
2 0 0	6·66$\frac{2}{3}$	2 13 4	2 10 0	1 11 1$\frac{3}{9}$	1 10 0	35 0
3 0 0	10·00	4 0 0	3 15 0	2 6 8	2 5 0	52 10
4 0 0	13·33$\frac{1}{3}$	5 6 8	5 0 0	3 2 2$\frac{6}{9}$	3 0 0	70 0
5 0 0	16·66$\frac{2}{3}$	6 13 4	6 5 0	3 17 9$\frac{3}{9}$	3 15 0	87 10
6 0 0	20·00	8 0 0	7 10 0	4 13 4	4 10 0	105 0
7 0 0	23·33$\frac{1}{3}$	9 6 8	8 15 0	5 8 10$\frac{6}{9}$	5 5 0	122 10
8 0 0	26·66$\frac{2}{3}$	10 13 4	10 0 0	6 4 5$\frac{3}{9}$	6 0 0	140 0
9 0 0	30·00	12 0 0	11 5 0	7 0 0	6 15 0	157 15
10 0 0	33·33$\frac{1}{3}$	13 6 8	12 10 0	7 15 6$\frac{6}{9}$	7 10 0	175 0
20 0 0	66·66$\frac{2}{3}$	26 13 4	25 0 0	15 11 1$\frac{3}{6}$	15 0 0	350 0
30 0 0	100·00	40 0 0	37 10 0	23 6 8	22 10 0	525 0
40 0 0	133·33$\frac{1}{3}$	53 6 8	50 0 0	31 2 2$\frac{6}{9}$	30 0 0	700 0
50 0 0	166·66$\frac{2}{3}$	66 13 4	62 10 0	38 17 9$\frac{3}{9}$	37 10 0	875 0
60 0 0	200·00	80 0 0	75 0 0	46 13 4	45 0 0	1050 0
70 0 0	233·33$\frac{1}{3}$	93 6 8	87 10 0	54 8 10$\frac{6}{9}$	52 10 0	1225 0
80 0 0	266·66$\frac{2}{3}$	106 13 4	100 0 0	62 4 5$\frac{3}{9}$	60 0 0	1400 0
90 0 0	300·00	120 0 0	112 10 0	70 0 0	67 10 0	1575 0
100 0 0	333·33$\frac{1}{3}$	133 6 8	125 0 0	77 15 6$\frac{6}{9}$	75 0 0	1750 0
200 0 0	666·66$\frac{2}{3}$	266 13 4	250 0 0	155 11 1$\frac{3}{9}$	150 0 0	3500 0
[illegible]0 0 0	1000·00	400 0 0	375 0 0	233 6 8	225 0 0	5250 0
[illegible] 0 0	1333·33	5[illegible]3 6 8	500 0 0	311 2 2$\frac{6}{9}$	300 0 0	7000 0
[illegible] 0 0	1666·66	666 13 4	625 0 0	388 17 9$\frac{3}{9}$	375 0 0	8750 0

TABLE *of the Value of several Pieces of Coin, in the Federal Coin, and the several Currencies of the United States.*

	pwt.	gr.	*Federal Coin.*	*N. Hampshire, Massachusetts, R. Island, Connecticut, Virginia.*			*New-York and North-Carolina.*			*New Jersey, Pennsylvania, Delaware and Maryland.*			*S. Carolina and Georgia.*		
			Cents.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
$\frac{1}{16}$ of a Dollar			0 06 $\frac{1}{4}$			$4\frac{1}{2}$			6			$5\frac{5}{8}$			$3\frac{1}{2}$
$\frac{1}{2}$ of a Pistareen			0·10			$7\frac{1}{5}$			9			9			$5\frac{3}{5}$
				Vir.		8									
$\frac{1}{9}$ of a Dollar			0·11 $\frac{1}{9}$			8			10			10			$6\frac{2}{9}$
$\frac{1}{8}$ of ditto			0·12 $\frac{1}{2}$			9		1	0			$11\frac{1}{4}$			7
A Pistareen			0·20		1	$2\frac{2}{5}$		1	4		1	6			$11\frac{1}{5}$
				Vir.	1	4									
An Eng. Shilling			0 22 $\frac{2}{9}$		1	4		1	7		1	8		1	$0\frac{4}{9}$
$\frac{1}{4}$ of a Dollar			0·25		1	6		2	0		1	$10\frac{1}{2}$		1	2
Half ditto			0·50		3	0		4	0		3	9		2	4
A Dollar			1·00		6	0		8	0		7	6		4	8
En. or Fr. Crown			1·11 $\frac{1}{9}$		6	8	N. York N. Caro.	9 8	0 9		8	4		5	$2\frac{2}{9}$
Fr. Guinea	5	5	4·62 $\frac{26}{27}$	1	7	6	1	16	0	1	14	6	1	1	5
In Massa.	5	6	4·55 $\frac{1}{9}$	1	7	4									
En. Guinea	5	6	4·66 $\frac{2}{3}$	1	8	0	1	17	0	1	15	0	1	1	9
In S. Caro.	5	7											1	1	10
$\frac{1}{2}$ Johann.	9	0	4·00	2	8	0	3	4	0	3	0	0	1	17	4
Pistole In Massa.	4 4	5 3	3·66 $\frac{2}{3}$	1	2	0	1	8	0	1	7	0		17	6
Moidore	6	18	6·00	1	16	0	2	8	0	2	5	0	1	8	0
Doubloon	17	0	14·66 $\frac{2}{3}$	4	8	0	5	16	0	5	12	0	3	10	0

The standard weight of an eagle 11 pwt. $4\frac{2}{3}$gr. Half ditto 5pwt. $14\frac{1}{3}$gr. A dollar 17pwt $1\frac{3}{4}$gr. Half ditto 8pwt. $12\frac{7}{8}$gr. A double dime 3pwt. $9\frac{4}{7}$gr. A dime 1pwt. $16\frac{9}{10}$gr.

TABLE OF REFINER'S WEIGHTS.

Blanks
24 = 1 Perrot.
480 = 20 = 1 Mite.
9600 = 400 = 20 = 1 Grain.

Note. What they denominate a carat, is the $\frac{1}{24}$ of a lb. an oz. or any other weight.

DUTCH WEIGHTS FOR GOLD AND SILVER.

Note, 32 aces = 1 engel, 20 engels = 1 ounce, 8 ounces = 1 mark, for gross gold. Also, 24 parts = 1 grain, 12 grains = 1 carat, 24 carats = 1 mark, for fine gold.

The mark weights are 1 per cent. lighter than our Troy weight.

A TABLE

A Table of Commission or Brokerage.

Goods or stock sold.	at ½ per cent.	at 1 per cent.	at 1½ per cent.	at 2 per cent.	at 2½ per cent.	at 3 per cent.
Shill. 1	£. s. d.	0 0 0	0 0 0	0 0 0	0 0 0¼	0 0 0¼
2	0 0 0	0 0 0¼	0 0 0¼	0 0 0¼	0 0 0½	0 0 0½
3	0 0 0¼	0 0 0¼	0 0 0½	0 0 0½	0 0 0¾	0 0 1
4	0 0 0¼	0 0 0½	0 0 0½	0 0 0¾	0 0 1	0 0 1¼
5	0 0 0¼	0 0 0½	0 0 0¾	0 0 1	0 0 1½	0 0 1¾
6	0 0 0¼	0 0 0¾	0 0 1	0 0 1¼	0 0 1¾	0 0 2
7	0 0 0½	0 0 0¾	0 0 1¼	0 0 1½	0 0 2	0 0 2¼
8	0 0 0½	0 0 1	0 0 1¼	0 0 1¾	0 0 2¼	0 0 2¾
9	0 0 0½	0 0 1	0 0 1½	0 0 2	0 0 2½	0 0 3
10	0 0 0½	0 0 1¼	0 0 1¾	0 0 2¼	0 0 3	0 0 3½
11	0 0 0½	0 0 1½	0 0 1¾	0 0 2½	0 0 3¼	0 0 3¾
12	0 0 0½	0 0 1½	0 0 2	0 0 2¾	0 0 3½	0 0 4¼
13	0 0 0¾	0 0 1½	0 0 2¼	0 0 3	0 0 3¾	0 0 4½
14	0 0 0¾	0 0 1¾	0 0 2½	0 0 3¼	0 0 4	0 0 5
15	0 0 0¾	0 0 1¾	0 0 2½	0 0 3½	0 0 4½	0 0 5¼
16	0 0 0¾	0 0 2	0 0 2¾	0 0 3¾	0 0 4¾	0 0 5¾
17	0 0 1	0 0 2	0 0 3	0 0 4	0 0 5	0 0 6
18	0 0 1	0 0 2¼	0 0 3	0 0 4¼	0 0 5¼	0 0 6¼
19	0 0 1	0 0 2¼	0 0 3¼	0 0 4½	0 0 5½	0 0 6¾
Pounds 1	0 0 1¼	0 0 2½	0 0 3½	0 0 4¾	0 0 6	0 0 7
2	0 0 2½	0 0 5	0 0 7½	0 0 9½	0 1 0	0 1 2½
3	0 0 3¾	0 0 7	0 0 10¾	0 1 2¼	0 1 6	0 1 9½
4	0 0 5	0 0 9½	0 1 2¼	0 1 7	0 2 0	0 2 4¾
5	0 0 6	0 1 0	0 1 6	0 2 0	0 2 6	0 3 0
6	0 0 7¼	0 1 2¼	0 1 9½	0 2 4¾	0 3 0	0 3 7
7	0 0 8½	0 1 4¾	0 2 1	0 2 9½	0 3 6	0 4 2¼
8	0 0 9¾	0 1 7	0 2 4¾	0 3 2¼	0 4 0	0 4 9½
9	0 0 10¾	0 1 9½	0 2 8¼	0 3 7	0 4 6	0 5 4¾
10	0 1 0	0 2 0	0 3 0	0 4 0	0 5 0	0 6 0
20	0 2 0	0 4 0	0 6 0	0 8 0	0 10 0	0 12 0
30	0 3 0	0 6 0	0 9 0	0 12 0	0 15 0	0 18 0
40	0 4 0	0 8 0	0 12 0	0 16 0	1 0 0	1 4 0
50	0 5 0	0 10 0	0 15 0	1 0 0	1 5 0	1 10 0
60	0 6 0	0 12 0	0 18 0	1 4 0	1 10 0	1 16 0
70	0 7 0	0 14 0	1 1 0	1 8 0	1 15 0	2 2 0
80	0 8 0	0 16 0	1 4 0	1 12 0	2 0 0	2 8 0
90	0 9 0	0 18 0	1 7 0	1 16 0	2 5 0	2 14 0
100	0 10 0	1 0 0	1 10 0	2 0 0	2 10 0	3 0 0
200	1 0 0	2 0 0	3 0 0	4 0 0	5 0 0	6 0 0
300	1 10 0	3 0 0	4 10 0	6 0 0	7 10 0	9 0 0
400	2 0 0	4 0 0	6 0 0	8 0 0	10 0 0	12 0 0
500	2 10 0	5 0 0	7 10 0	10 0 0	12 10 0	15 0 0
600	3 0 0	6 0 0	9 0 0	12 0 0	15 0 0	18 0 0
700	3 10 0	7 0 0	10 10 0	14 0 0	17 10 0	21 0 0
800	4 0 0	8 0 0	12 0 0	16 0 0	20 0 0	24 0 0
900	4 10 0	9 0 0	13 10 0	18 0 0	22 10 0	27 0 0
1000	5 0 0	10 0 0	15 0 0	20 6 0	25 0 0	30 0 0

A Table *of the Returns of the Neat Proceeds of an Account of Sales from a Factor to his Employer, reserving his Commissions for Remittance.*

Neat Proceeds.			*Sum to be remitted, reserving 2½ pr. cent. commission.*			*Sum to be remitted, reserving 5 pr cent. commission.*			*Neat Proceeds.*			*Sum to be remitted, reserving 2½ per cent. commission.*			*Sum to be remitted, reserving 5 per cent. commission.*		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
		3			3			2¾	6	0	0	5	17	0¾	5	14	5½
		4			4			3¾	7	0	0	6	16	7	6	13	4
		5			$5\frac{5}{4}$			4¾	8	0	0	7	16	1¼	7	12	4½
		6			5¾			5¾	9	0	0	8	15	7¼	8	11	5¼
		7			6¾			6¾	10	0	0	9	15	1½	9	10	5¾
		8			7¾			7½	20	0	0	19	10	3	19	0	11½
		9			8¾			8½	30	0	0	29	5	4¼	28	11	5¼
		10			9¾			9½	40	0	0	39	0	5¾	38	1	10¾
		11			10¾			10¼	50	0	0	48	15	7¼	47	12	4½
	1	0			11¾			11¼	60	0	0	58	10	8¾	57	2	10¼
	2	0		1	11½		1	10¾	70	0	0	68	5	10	66	13	4
	3	0		2	11¼		2	10¼	80	0	0	78	0	11½	76	3	9¾
	4	0		3	10¾		3	9¾	90	0	0	87	16	1	85	14	3½
	5	0		4	10½		4	9¼	100	0	0	97	11	2¾	95	4	9
	6	0		5	10¼		5	8½	200	0	0	195	2	5¼	190	9	6¼
	7	0		6	10		6	8	300	0	0	292	13	8	285	14	3¼
	8	0		7	9¾		7	7½	400	0	0	390	4	10½	380	19	0½
	9	0		8	9¼		8	6¾	500	0	0	487	16	1¼	476	3	9½
	10	0		9	9		9	6¼	600	0	0	585	7	3¾	571	8	6¾
1	0	0		19	6¼		19	0½	700	0	0	682	18	4¾	666	13	4
2	0	0	1	19	0¼	1	18	1¼	800	0	0	780	9	9	761	18	1
3	0	0	2	18	6½	2	17	1¾	900	0	0	878	0	11¾	857	2	10
4	0	0	3	18	0½	3	16	2¼	1000	0	0	975	12	2¼	952	7	7¼
5	0	0	4	17	6¾	4	15	2¾									

Suppose I have the neat proceeds, or balance of an account of sales 325l. 17s. 9d. in my hands and would make remittance to my employer, reserving my commission at 2½ per cent. What sum must be remitted, so that my employer's account may be closed?

	£.	s.	d.		£.	s.	d.
Against	300	0	0	stands	292	13	8
	20	0	0		19	10	3
	5	0	0		4	17	6¾
		10	0			9	9
		7	0			6	10
			9				8¾

To be remitted £.317 18 9½ Answer.

A Table, *shewing the number of Days from any Day in any Month to the same Day in any other Month through the Year.*

From	Jan.	Feb.	Mar.	Apr.	May.	Jun.	July.	Aug.	Sep.	Oct.	Nov.	Dec.
To Jan.	365	334	306	275	245	[illegible]	184	153	122	92	[illegible]	31
Feb.	31	365	337	306	[illegible]	[illegible]	215	184	153	123	92	[illegible]
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	214	[illegible]	183	153	122	91	61	30	365

The use of the preceding Table of number of days, will easily appear from the following examples.

Suppose the number of days between the 1st, or 10th, or 30th, &c. of January, and the 1st, or 10th, or 30th, &c. of October, were required? Look in the column under January for October, and against that month you will find 273, which is the number of days between the said times; and so for the days between any other two months.

If the *given days* be *different*, it is only adding or subtracting their inequality to or from the *tabular number*.

How many days from the 6th of April to the 12th of January? From the 6th of April to the 6th of Jannary is 275, and adding the 6 overplus days, it makes 281 days. And from the 5th of June to the 1st of February is 240 days.

Note. After February 31, (in leap years) increase each number with an unit or 1.

A TABLE *of the Measure of Length of the principal places in Europe, compared with the American yard.*

100	Aunes or Ells of England, - - - - -	= 125
100	—— of Holland or Amsterdam, Hærlem, Leyden, the Hague, Rotterdam, Nuremberg, and other cities of Holland,	= 75
100	—— of Brabant or Antwerp, - - - - -	= 76
100	—— of France and Oznaburg, - - - - -	= $128\frac{1}{2}$
100	—— of Hamburg, Franckfort, Leipsick, Bern, and Basil,	= $62\frac{1}{2}$
100	—— of Breslau, - - - - - - - -	= 60
100	—— of Dantzick, - - - - - - -	= $66\frac{3}{4}$
100	—— of Bergen and Drontheim, - - -	= $68\frac{1}{4}$
100	—— of Sweden and Stockholm, - - - - -	= $65\frac{3}{4}$
100	—— of St. Gall, for Linens, - - - - -	= $87\frac{1}{2}$
100	—— of ditto for Cloths, - - - - - -	= 67
100	—— of Geneva, - - - - - - -	= $124\frac{3}{4}$
100	Canes of Marseilles and Montpelier, - - -	= $214\frac{1}{2}$
100	—— of Thoulouse and High Languedoc, - -	= 200
100	—— of Genoa, of 9 palms, - - - - -	= $245\frac{1}{4}$
100	—— of Rome, - - - - - - -	= $227\frac{1}{4}$
100	Varas of Spain, - - - - - - - -	= $93\frac{3}{4}$
100	—— of Portugal, - - - - - - -	= 123
100	Cavidos of Portugal, - - - - - -	= 75
100	Brasses of Venice, - - - - - - -	= $73\frac{1}{2}$
100	—— of Bergamo, - - - - - - -	= $71\frac{1}{4}$
100	—— of Florence and Leghorn, - - - -	= 64
100	—— of Milan, - - - - - - - -	= $58\frac{1}{2}$

The use of the following TABLE, *directing how to buy and sell by the hundred.*

If you buy or sell any thing by the great hundred (112lb.) and desire to know, by the lb. what the hundred is valued at, observe the following examples.

1. If you buy sugar at $6\frac{3}{4}$d. per lb. look for $6\frac{3}{4}$d. in the left hand column of the Table, against it, in the second column, you will find £.3 3s. which is the value of 1cwt. at that rate.

2. If 1cwt. (112lb.) cost £.9 4s. 4d. to know how much it is per lb. look £.9 4s. 4d. in the fourth column, and against it, in the next left hand column, you will find 1s. $7\frac{3}{4}$d. which is the price per lb.

Again, If you buy one hundred weight of goods for 9l. 4s. 4d. and retail it at 1s. $9\frac{3}{4}$d. per lb. it comes at that rate to 10l. 3s.; then take 9l. 4s. 4d. from 10l. 3s. and, by the remainder, you will find that you have gained 18s. 8d.

And in this manner you may, with ease, calculate any quantity by the following Table.

A TABLE

A Table *directing how to buy and sell by the hundred.*

d.	£. s. d.	s. d.	£. s. d.	s. d.	£. s. d.
¼	0 2 4	1 0¼	5 14 4	2 0¼	11 6 4
½	0 4 8	1 0½	5 16 8	2 0½	11 8 8
¾	0 7 0	1 0¾	5 19 0	2 0¾	11 11 0
1	0 9 4	1 1	6 1 4	2 1	11 13 4
1¼	0 11 8	1 1¼	6 3 8	2 1¼	11 15 8
1½	0 14 0	1 1½	6 6 0	2 1½	11 18 0
1¾	0 16 4	1 1¾	6 8 4	2 1¾	12 0 4
2	0 18 8	1 2	6 10 8	2 2	12 2 8
2¼	1 1 0	1 2¼	6 13 0	2 2¼	12 5 0
2½	1 3 4	1 2½	6 15 4	2 2½	12 7 4
2¾	1 5 8	1 2¾	6 17 8	2 2¾	12 9 8
3	1 8 0	1 3	7 0 0	2 3	12 12 0
3¼	1 10 4	1 3¼	7 2 4	2 2¼	12 14 4
3½	1 12 8	1 3½	7 4 8	2 3½	12 16 8
3¾	1 15 0	1 3¾	7 7 0	2 3¾	12 19 0
4	1 17 4	1 4	7 9 4	2 4	13 1 4
4¼	1 19 8	1 4¼	7 11 8	2 4¼	13 3 8
4½	2 2 0	1 4½	7 14 0	2 4½	13 6 0
4¾	2 4 4	1 4¾	7 16 4	2 4¾	13 8 4
5	2 6 8	1 5	7 18 8	2 5	13 10 8
5¼	2 9 0	1 5¼	8 1 0	2 5¼	13 13 0
5½	2 11 4	1 5½	8 3 4	2 5½	13 15 4
5¾	2 13 8	1 5¾	8 5 8	2 5¾	13 17 8
6	2 16 0	1 6	8 8 0	2 6	14 0 0
6¼	2 18 4	1 6¼	8 10 4	2 6¼	14 2 4
6½	3 0 8	1 6½	8 12 8	2 6½	14 4 8
6¾	3 3 0	1 6¾	8 15 0	2 6¾	14 7 0
7	3 5 4	1 7	8 17 4	2 7	14 9 4
7¼	3 7 8	1 7¼	8 19 8	2 7¼	14 11 8
7½	3 10 0	1 7½	9 2 0	2 7½	14 14 0
7¾	3 12 4	1 7¾	9 4 4	2 7¾	14 16 4
8	3 14 8	1 8	9 6 8	2 8	14 18 8
8¼	3 17 0	1 8¼	9 9 0	2 8¼	15 1 0
8½	3 19 4	1 8½	9 11 4	2 8½	15 3 4
8¾	4 1 8	1 8¾	9 13 8	2 8¾	15 5 8
9	4 4 0	1 9	9 16 0	2 9	15 8 0
9¼	4 6 4	1 9¼	9 18 4	2 9¼	15 10 4
9½	4 8 8	1 9½	10 0 8	2 9½	15 12 8
9¾	4 11 0	1 9¾	10 3 0	2 9¾	15 15 0
10	4 13 4	1 10	10 5 4	2 10	15 17 4
10¼	4 15 8	1 10¼	10 7 8	2 10¼	15 19 8
10½	4 18 0	1 10½	10 10 0	2 10½	16 2 0
10¾	5 0 4	1 10¾	10 12 4	2 10¾	16 4 4
11	5 2 8	1 11	10 14 8	2 11	16 6 8
11¼	5 5 0	1 11¼	10 17 0	2 11¼	16 9 0
11½	5 7 4	1 11½	10 19 4	2 11½	16 11 4
11¾	5 9 8	1 11¾	11 1 8	2 11¾	16 13 8
12	5 12 0	2 0	11 4 0	3 0	16 16 0

A Comparison of the American Foot with the Feet of other Countries.

The American Foot being divided into 1000 parts, or into 12 inches, the feet of several other countries will be as follow.

	Parts.	Inch.	lin.	points.
America, - - - - -	1000	12	0	0 dec.
London, - - - - - -	1000	12	0	0
Antwerp, - - - - -	946	11	4	1·32
Bologna, - - - - - -	1204	14	5	2·25
Bremen, - - - - - -	964	11	6	4·89
Cologne, - - - - - -	954	11	5	2·25
Copenhagen, - - -	965	11	6	5·76
Amsterdam, - - - -	942	11	3	3·88
Dantzick, - - - - -	944	11	3	5·61
Dort, - - - - - - -	1184	14	2	2·97
Frankfort on the Main,	948	11	4	3·07
The Greek, - - - -	1007	12	1	0·04
Lorrain, - - - - - -	958	11	5	5·71
Mantua, - - - - - -	1569	18	9	5·61
Mecklin, - - - - -	919	11	0	2·01
Middleburg, - - - -	991	14	10	4·22
France, - - - - - -	1066	12	9	3·34
Prague, - - - - - -	1026	12	3	4·46
Rhyneland or Leyden,	1033	12	4	4·51
Riga, - - - - - - -	1831	21	11	3·98
Roman, - - - - - -	967	11	7	1·48
Old Roman, - - - -	970	11	8	0
Scotch, - - - - - -	1005	12	0	4·32
Strasburgh, - - - -	920	11	0	2·88
Toledo, - - - - - -	899	10	9	2·73
Turin, - - - - - - -	1062	12	8	5·66
Venice, - - - - - -	1162	13	11	1·96

A TABLE *representing the Conformity of the weights of the principal trading Cities of Europe with those of America.*

lb.	of America.		
100 of England, Scotland and Ireland,	Equal	100lb.	0oz.
100 of Amsterdam, Paris, Bourdeaux, &c.	——	109	8
100 of Antwerp, or Brabant, - - -	——	103	12
100 of Rouen, the Viscounty, - -	——	113	14
100 of Lyons, the city, - - - - -	——	94	8
100 of Rochelle, - - - - - -	——	110	9
100 of Toulouse, and Upper Languedoc, -	——	92	6
100 of Marseilles and Provence, - -	——	88	11
100 of Geneva, - - - - - -	——	123	
100 of Hamburg, - - - - - -	——	107	5
100 of Frankfort, - - - - - -	——	111	11
100 of Leipsick, - - - - - -	——	104	5

A TABLE

A TABLE *representing the Conformity of the Weights of the principal trading Cities of Europe with those of America.*

lb.			of America.	
100	of Genoa,	Equal	73	
100	of Leghorn,	——	75	8
100	of Milan,	——	65	3
100	of Venice,	——	65	11
100	of Naples,	——	64	10
100	of Seville, Cadiz, &c.	——	103	7
100	of Portugal,	——	95	4
100	of Liege,	——	104	
100	of Spain,	——	97	dr.

Note, The Spanish Arrobe is 25 Spanish pounds, —— 25 12 6

A TABLE to caſt up wages, or expenſes, for a year, at ſo much per day, week, or month.

Day.		*by week.*			*by month.*			*by year.*		
s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
0	1	0	0	7	0	2	4	1	10	5
0	2	0	1	2	0	4	8	3	0	10
0	3	0	1	9	0	7	0	4	11	3
0	4	0	2	4	0	9	4	6	1	8
0	5	0	2	11	0	11	8	7	12	1
0	6	0	3	6	0	14	0	9	2	6
0	7	0	4	1	0	16	4	10	12	11
0	8	0	4	8	0	18	8	12	3	4
0	9	0	5	3	1	1	0	13	13	9
0	10	0	5	10	1	3	4	15	4	2
0	11	0	6	5	1	5	8	16	14	7
1	0	0	7	0	1	8	0	18	5	0
2	0	0	14	0	2	16	0	36	10	0
3	0	1	1	0	4	4	0	54	15	0
4	0	1	8	0	5	12	0	73	0	0
5	0	1	15	0	7	0	0	91	5	0
6	0	2	2	0	8	8	0	109	10	0
7	0	2	9	0	9	16	0	127	15	0
8	0	2	16	0	11	4	0	146	0	0
9	0	3	3	0	12	12	0	164	5	0
10	0	3	10	0	14	0	0	182	10	0
11	0	3	17	0	15	8	0	200	15	0
12	0	4	4	0	16	16	0	219	0	0
13	0	4	11	0	18	4	0	237	5	0
14	0	4	18	0	19	12	0	255	10	0
15	0	5	5	0	21	0	0	273	15	0
16	0	5	12	0	22	8	0	292	0	0
17	0	5	19	0	23	16	0	310	5	0
18	0	6	6	0	25	4	0	328	10	0
19	0	6	13	0	26	12	0	346	15	0
20	0	7	0	0	28	0	0	365	0	0

Note. In these two Tables the month is only 28 days.

A TABLE to find wages or expenſes for a month, week or day, at ſo much by the year.

by yr.	*by month.*			*by week.*			*by day.*		
£.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1	0	1	6½	0	0	4½	0	0	0¾
2	0	3	0¾	0	0	9	0	0	1¼
3	0	4	7¼	0	1	1¾	0	0	2
4	0	6	1¾	0	1	6½	0	0	2¾
5	0	7	8	0	1	11	0	0	3¼
6	0	9	2½	0	2	3½	0	0	4
7	0	10	9	0	2	8¼	0	0	4½
8	0	12	3¼	0	3	0¾	0	0	5¼
9	0	13	9¾	0	3	5½	0	0	6
10	0	15	4	0	3	10	0	0	6½
11	0	16	10½	0	4	2¾	0	0	7¼
12	0	18	5	0	4	7¼	0	0	8
13	0	19	11¼	0	4	11¾	0	0	8½
14	1	1	5¾	0	5	4½	0	0	9¼
15	1	3	0¼	0	5	9	0	0	9¾
16	1	4	6½	0	6	1¾	0	0	10½
17	1	6	1	0	6	6¼	0	0	11¼
18	1	7	7½	0	6	10¾	0	0	11¾
19	1	9	1¾	0	7	3½	0	1	0½
20	1	10	8¼	0	7	8	0	1	1¼
30	2	6	0¼	0	11	6	0	1	7¾
40	3	1	4½	0	15	4	0	2	2¼
50	3	16	8	0	19	2¼	0	2	9
60	4	12	0¾	1	3	0¼	0	3	3½
70	5	7	4¾	1	6	10¼	0	3	10
80	6	2	9	1	10	8¼	0	4	4½
90	6	18	1	1	14	9¼	0	4	11¼
100	7	13	5	1	18	4¼	0	5	5¾
200	15	6	10¼	3	16	8½	0	10	11½
300	23	0	3¼	5	15	0¾	0	16	5¼
400	30	13	8½	7	13	5	1	1	11
500	38	7	1½	9	11	9¼	1	7	4¾
1000	76	14	3	19	3	6¾	2	14	9½

1787	Thurſday
1788	*Saturday*†
1789	Sunday
1790	Monday
1791	Tueſday
1792	*Thurſday*†
1793	Friday
1794	Saturday
1795	Sunday
1796	*Tueſday*†
1797	Wedneſday
1798	Thurſday
1799	Friday
1800	Saturday
1801	Sunday
1802	Monday
1803	Tueſday
1804	*Thurſday*†
1805	Friday
1806	Saturday
1807	Sunday
1808	*Tueſday*†
1809	Wedneſday
1810	Thurſday
1811	Friday
1812	*Sunday*†
1813	Monday
1814	Tueſday
1815	Wedneſday
1816	*Friday*†
1817	Saturday
1818	Sunday
1819	Monday

PERPETUAL ALMANACK.

February March November	*February* August.*	*May.*	*January October.*	*January* April July.*	*September. December.*	*June.*
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

To find on what day of the week any given day in any month will fall, and the contrary.

EXAMPLE.

On what day of the week will the 31st. day of January, 1810, fall?

Observe the day of the week, annexed to the year, in the outer column; then, in the Table, under the given month, in the upper row of figures, you will find the day of the month on which that day falls. According to this direction, I find that, in January, 1810, Thursday is the 4th. 11th, 18th. and 25th. then reckoning on, Friday 26, Saturday 27th. &c. I find the 31st. day falls on Wednesday; or, that the last Wednesday in January is the 31st. day.

NOTE, In leap years, January and February must be taken in the columns marked thus, *. Leap years are marked, in the outer columns, thus, †.

The years 1800, 1900, and all other 100th. years, not to be leap years, except the years 2000, 2400, 2800, and every 400th. year following, which must be leap years.

1820	*Wedneſday*†
1821	Thurſday
1822	Friday
1823	Saturday
1824	*Monday*†
1825	Tueſday
1826	Wedneſday
1827	Thurſday
1828	*Saturday*†
1829	Sunday
1830	Monday
1831	Tueſday
1832	*Thurſday*†
1833	Friday
1834	Saturday
1835	Sunday
1836	*Tueſday*†
1837	Wedneſday
1838	Thurſday
1839	Friday
1840	*Sunday*†
1841	Monday
1842	Tueſday
1843	Wedneſday
1844	*Friday*†
1845	Saturday
1846	Sunday
1847	Monday
1848	*Wedneſday*†
1849	Thurſday
1850	Friday
1851	Saturday
1852	*Monday*†
1853	Tueſday

A Table for reducing Troy wt. to Avoirdupois.

Tr.	Av.	Troy.	Avoirdupois		
gr.	drm.	oz	lb.	oz.	dr
1	04	1		1	1 55
2	·07	2		2	3·11
3	·11	3		3	4 66
4	·15	4		4	6·22
5	·18	5		5	7·77
6	·22	6		6	9·32
7	·26	7		7	10 89
8	·29	8		8	12 44
9	·33	9		9	14·
10	·36	10		10	15·56
11	·4	11		22	1·09
12	·44	lb.			
13	·47	1	0	13	2·65
14	·51	2	1	10	5·3
15	·55	3	2	7	8
16	·58	4	3	4	10·6
17	·62	5	4	1	13 25
18	·66	6	4	14	15·9
29	·69	7	5	12	2·56
20	·73	8	6	9	5·21
21	·77	9	7	6	7 86
22	·8	10	8	3	10 52
23	·84	20	16	7	5·03
pw.		30	24	10	15·54
1	0.88	40	32	14	10 05
2	1·75	50	41	2	4·57
3	2 63	60	49	5	15·08
4	3·51	70	57	9	9·6
5	4 39	80	65	13	4 11
6	5·27	90	74	0	13·62
7	6 14	100	82	4	9·15
8	7·02	200	164	9	2·28
9	7 9	300	246	13	11·42
10	8·78	400	329	2	4·57
11	9·65	500	411	6	13 71
12	10·53	600	493	11	6 85
13	11 41	700	576	0	0·
14	12·29	800	658	4	9·14
15	13·16	900	740	9	2·28
16	14·04	1000	822	13	11·42
17	14 92	2000	1645	11	6·84
18	15·79	3000	2528	9	2·26
19	16·67	4000	3291	6	13·68

A Table for reducing Avoirdupois weight into Troy.

Avoir.	Troy.				Avoir.	Troy			
dr.	lb.	oz.	pw	gr	lb.	lb.	oz.	pw.	gr.
½				13·67	1	1	2	11	16
¼			0	20·51	2	2	5	3	8
1			1	3·34	3	3	7	15	0
2			2	6·68	4	4	10	6	16
3			3	10·02	5	6	0	18	8
4			5	13 36	6	7	3	10	0
5			6	16·7	7	8	6	1	16
6			7	20 04	8	9	8	13	8
7			8	23 38	9	10	11	5	0
8			9	2 72	10	12	1	16	16
9			10	6·06	20	24	3	13	8
10			11	9·4	30	36	5	10	0
11			12	12·74	40	48	7	6	16
12			13	15 08	50	60	9	3	8
13			14	19 42	60	72	11	0	0
14			15	22·76	70	85	0	16	16
15			17	2·1	80	97	2	13	8
oz					90	109	4	10	0
1		0	18	5·5	100	121	6	6	16
2		1	16	11	200	243	0	13	8
3		2	14	16·5	300	364	7	0	0
4		3	12	22	400	486	1	6	16
5		4	11	3·5	500	607	7	13	8
6		5	9	9	600	729	2	0	0
7		6	7	14 5	700	850	8	6	16
8		7	5	20	800	972	2	13	8
9		8	4	1·5	900	1093	9	0	0
10		9	2	7	1000	1215	3	6	16
11		10	0	12·5	2000	2430	6	13	8
12		10	18	18	3000	3645	10	0	0
13		11	16	23·5	4000	4861	1	6	16
14	1	0	15	5	5000	6076	4	13	8
15	1	1	13	10·5	6000	7291	8	0	0

AN ACCOUNT of the Gregorian *or* New Style, *together with some* Chronological Problems, *for finding the* Epact, Golden Number, Moon's Age, *&c.*

POPE GREGORY the XIIIth. made a reformation of the calendar. The Julian calendar, or old style, had, before that time, been in general use all over Europe. The year, according to the Julian calendar, consists of three hundred and sixty five days and six hours; which six hours being one fourth part of a day, the common years consisted of three hundred and sixty five days, and every fourth year, one day was added to the month of February, which made each of those years three hundred and sixty six days, which are usually called leap years.

This computation, though near the truth, is more than the solar year by eleven minutes, which, in one hundred and thirty one years, amounts to a whole day. By which the Vernal Æquinox was anticipated ten days, from the time of the general council of Nice, held in the year 325 of the Christian Æra, to the time of Pope Gregory; who therefore caused ten days to be taken out of the month of October in 1582, to make the Æquinox fall on the 21st of March, as it did at the time of that council. And, to prevent the like variation for the future, he ordered that three days should be abated in every four hundred years, by reducing the leap year at the close of each century, for three successive centuries, to common years, and retaining the leap year at the close of each fourth century only.

This was at that time esteemed as exactly conformable to the true solar year; but Dr. Halley makes the solar year to be three hundred and sixty five days, five hours, forty eight minutes, fifty four seconds, forty one thirds, twenty seven fourths, and thirty one fifths: According to which, in four hundred years, the Julian year of three hundred and sixty five days and six hours will exceed the solar by three days, one hour and fifty five minutes, which is near two hours, so that in fifty centuries it will amount to a day.

Though the Gregorian calendar, or new style, had long been used throughout the greatest part of Europe, it did not take place in Great Britain and America till the first of January, 1752; and in September following, the eleven days were adjusted by calling the third day of that month the fourteenth, and continuing the rest in their order.

CRONOLOGICAL PROBLEMS.

Problem I.

As there are three leap years to be abated in every four centuries: to shew how to find in which century the last year is to be a leap year, and in which it is not.

Rule.—Cut off two cyphers, and divide the remaining figures by 4; if nothing remain, the year is a leap year.

Examp.

EXAMP. 1. The year 18|00.

```
4)18(4
  16
  --
   2
```

EXAMP. 2. The year 19|00.

```
4)19(4
  16
  --
   3
```

EXAMP. 3. The year 20|00.

```
4)20(5
  20
  --
   0
```

EXAMP. 4. The year 40|00.

```
4)40(10
  4
  -
  0
```

The first and second examples, having remainders, shew the years to be common years of three hundred and sixty-five days; but the third and fourth, having no remainders, are leap years of three hundred and sixty-six days.

PROBLEM II.

To find, with regard to any other years, whether any given year be leap year, and the contrary.

RULE.

Divide the proposed year by 4, and if there be no remainder, after the division, it is leap year; but if 1, 2 or 3 remain, it is the first, second or third after leap year.

EXAMP. 1. For the year 1784.

```
4)1784(446
  16
  --
   18
   16
   --
    24
    24
    --
     0
```

EXAMP. 2. For the year 1786.

```
4)1786(446
  16
  --
   18
   16
   --
    26
    24
    --
     2 { second after
       { leap year.
```

PROBLEM III.

To find the Dominical Letter for any year, according to the Julian method of calculation.

RULE.

Add to the year its fourth part and 4, and divide that sum by 7: if nothing remain, the Dominical Letter is G; but if there be any remainder, it shews the letter in a retrograde order from G, beginning the reckoning with F; or, if it be subtracted from 7, you will have the index of the letter from A, accounting as follows:

A	B	C	D	E	F	G
1	2	3	4	5	6	7

EXAMP.

EXAMP. For the year 1786.

Add { Given year = 1786
Its fourth = 446
And 4 }

7)2236(319
21
—
13
7
—
66
63
—

And 7—3=4=D, reckoning from A.

PROBLEM IV.

To find the Dominical Letter for any year according to the Gregorian computation.

RULE.—Divide the year and its fourth part, less 1 (for the present century) by 7; subtract the remainder after the division, from 7, and this remainder will be the index of the Dominical Letter, as before: if nothing remain it is G.

EXAMP. 1. For the year 1810.

Add { Given year = 1810
Its fourth = 452 }

2262
Subtract 1
—
7)2261(323
21
—
16
14
—
21
21
—

And 7—0=7=G.

EXAMP. 2. For the year 1812.*

1812
452
—
2265
1
—
7)2264(323
21
—
16
14
—
24
21
—

And 7—3=4=D.

PROBLEM

* Here it is to be observed, that every leap year has two Dominical Letters; that, found by this rule, is the Dominical Letter from the twenty-fifth day of February to the end of the year; and the next in the order of the alphabet serves from the first of January to the twenty-fourth of February.

In the 2d. Example, D is the Dominical Letter for the year; but E, the next in the order of the alphabet, is the Dominical Letter for January and February. From this interruption of the Dominical Letter every fourth year, it is twenty-eight years before the Dominical Letter returns to the same order, which, were it not for the leap years, would return to the same every seven years.

This Cycle of twenty-eight years is called the Cycle of the Sun.

PROBLEM V.

To find the Prime, or Golden Number.

RULE.

Add 1 to the given year ; divide the sum by 19, and the remainder, after the division, will be the Prime ; if nothing remain, then 19 will be the Golden Number.

EXAMP. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

1 Golden Number.

The Golden Number, or Lunar Cycle, is a period of 19 years, invented by *Meton*, an *Athenian*, and from him called the *Metonick* Cycle. The use of this cycle is to find the change of the moon ; because, after 19 years, the changes of the moon fall on the same days of the month as in the former 19 years ; though not at the same time of the day, there being an anticipation of one hour, twenty-seven minutes, forty-one seconds, and thirty-two thirds ; which, in 312 years, amount to a whole day. Hence, the Golden Number will not show the true change of the moon for more than three hundred and twelve years, without being varied. But the golden number is not so well adapted to the *Gregorian*, as the *Julian* calendar : The epact being more certain in the new style, to find which, the golden number is of use.

PROBLEM VI.

To find the Julian Epact.

RULE.—First find the Golden Number, which multiply by 11, and the product, if less than 30, will be the number required ; if the product exceed 30, then divide it by 30, and the remainder is the epact.

EXAMP. 1. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

Golden Number = 1 and $1 \times 11 = 11$ the Julian Epact.

EXAMP.

EXAMP. 2. For the year 1791.

```
   1791
      1
   ----
19)1792(94
   171
   ----
     82
     76
     --
      6=Golden Number, and 6×11=66, therefore 30)66(2
                                                 60
                                                 --
                                                  6 Epact.
```

PROBLEM VII.

To find the Gregorian Epact.

RULE. Subtract 11 from the Julian Epact: If the subtraction cannot be made, add 30 to the Julian Epact; then subtract, and the remainder will be the Gregorian Epact; if nothing remain, the Epact is 29.

Or, take 1 from the Golden Number, and divide the remainder by 3; if 1 remain, add 10 to the dividend, which sum will be the Epact; if 2 remain, add 20 to the dividend; but if nothing remain, the dividend is the Epact.

EXAMP. 1. For the year 1786.

```
The Julian Epact being 11
              Subtract 11
                       --
                        0
```

Because nothing remains, the Epact is 29.

Or,

EXAMP. 2. For the year 1786.

```
The Golden Number being 1
           Take from it 1
                        -
           Divide by 3)0(0
```

There being no remainder, the Epact is 29, as before.

EXAMP. 3. For the year 1791.

```
The Julian Epact being but 6
                 Add to it 30
                           --
                           36
                  Subtract 11
                           --
         Gregorian Epact=25
```

Or,

EXAMP. 4. For the year 1791.

```
The Golden Number being 6
           Take from it 1
                        -
                     3)5(1
                       3
                       -
                       2
```

Therefore, as 2 remains, add 20 to the dividend, and it gives the Epact 25, as before.

A general Rule for finding the Gregorian Epact forever.

Divide the *centuries* of any year of the Christian Æra by 4, (rejecting the subsequent numbers;) multiply the remainder by 17, and

to

to this product add the quotient multiplied by 43; divide this sum plus 86 by 25, multiplying the Golden Number by 11, from which subtract the last quotient, and rejecting the *thirties*, the remainder will be the Epact.

EXAMP. For the year 1786.

Rejecting the subsequent numbers 86, it will be 17.

```
        4)17(4
          16
          —
           1
Multiply by 17
          —
          17
Add 4×43=172
         ——
         189
     Add  86
         ——
     25)275(11
         25
         ——
         25
         25
```

```
             Golden Number = 1
                Multiply by 11
                            —
                            11
Subtract the last quotient = 11
                            —
                            00
```

Therefore, as nothing remains, the Epact is 29, as before.

A TABLE *of the nineteen Epacts for the* Julian *and* Gregorian *Accounts, by the Golden Number.*

G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.
1	11	29	7	17	6	13	23	12
2	22	11	8	28	17	14	4	23
3	3	22	9	9	28	15	15	4
4	14	3	10	20	9	16	26	15
5	25	14	11	1	20	17	7	26
6	6	25	12	12	1	18	18	7
						19	29	18

PROBLEM VIII.

To calculate the Moon's Age on any given day.

RULE.—To the given day of the month, add the Epact and number of the month: If the sum be less than 30, it is the Moon's age, but if it exceed 30, then take 30 from it, and the remainder will be the Moon's age.

Note. The numbers to be added to the following months, are as follow:

To	January	0	July	5
	February	2	August	6
	March	1	September	8
	April	2	October	8
	May	3	November	10
	June	4	December	10

EXAMPLE.

EXAMPLE. For January 25th, 1786.

Add	Given day	=25
	Epact	=29
	No. of the month	=00
		54
	Subtract	30
		24 = Moon's age.

PROBLEM IX.

To find the times of the New and Full Moon, and the first and last Quarters.

RULE.—Find the Moon's age on the given day, then, if it be 15, the Moon will be full on that day, and by counting $7\frac{1}{2}$ days backward and forward you will have the first and last quarters, and by counting backward and forward 15 days, you will have the times of the last and next change ; but if the age of the Moon be greater than 15, take 15 from it, and the remainder will shew how many days have passed since the last full moon, and counting these backward, you will have the day the last full moon happened on, and by knowing that, we can find the change, or either of the quarters, as before.

Again, if the age of the moon, on the assumed day, be less than 15, then take that from 15, and the remainder will shew how many days are to run till the next full moon, which you will have by adding the remainder to the assumed day ; and, proceeding as before, you will have the days of the change, and either quarter as above.

EXAMP. For Jan. 25th. 1786.

Add	Assumed day	=25
	Epact	=29
	Number of the month=	[illegible]
		54
	Subtract	3[illegible]
	Moon's age=	24
	Subtract	15
	Take the days since the last full moon=	9
	From the assumed day=	25
	To the day of the full moon=	16th.
	Add	15
	NewMoon	31st.
	From the full moon	16
	Take	$7\frac{1}{2}$
	First quarter	9th.
	To the full Moon=	16
	Add	$7\frac{1}{2}$
	Last quarter=	23

PROBLEM

Problem X.

The time of the Moon's coming to the South, after the Sun, being given, to find the age of the Moon.

Rule.—As 24 hours, the whole difference of time, are to 30, the number of days from change to change, so is the difference of time, to the Moon's age.

Example. I observed the Moon to be on the meridian, or due south, at 6 o'clock in the afternoon: What is the Moon's age?

24 : 30 :: 6 : $7\frac{1}{2}$ days, Ans.

Problem XI.

To find the time of the Moon's Southing.

Rule. Multiply the Moon's age, on the given day, by 48 minutes, and divide the product by 60, the minutes in an hour, (or multiply by 4, and divide by 5) and the quotient will show how many hours and minutes the moon is later, in coming on the meridian, than the sun, and counting so many hours and minutes forward from 12 o'clock, we have the time of the Moon's southing; if the hours and minutes, found as above, be less than 12, then, that will be the time of the Moon's southing after noon; but, if greater than 12, then, take 12 from them, and the remainder will be the time of the Moon's southing in the morning.

Examp. 1. Required the time of the Moon's southing on the 25th day of January 1786?

Moon's age = 24
48
——
192
96
—— h. m.
60)1152(19 12
60
——
552
540
——
12

h. m.
From 19 12
Take 12 00
——
7 12

Hence, the Moon souths at 12 minutes past 7 in the morning.

Or,
24
4
——
5)96
—— h. m.
$19\frac{1}{5}$=19 12 as before.

Examp. 2. For the 9th. of February 1786?

Moon's age=10
48
—— h. m.
60)480(8 0 afternoon, is the time of the Moon's southing.
48

Note. From the change to the full, the Moon comes to the south afternoon; but from the full to the change, before noon.

Problem

Problem XII.

To find on what day of the week, any given day in any month will fall.

As one of the first seven letters of the alphabet is prefixed to every day in the year beginning with A, which is always prefixed to the first day of January: And as, in common years, the letter, annexed to the first Sunday in January, shews the Dominical Letter for that year; but every leap year having two Dominical Letters, the first of which serving to the twenty fourth of February, and the other for the rest of the year, consequently, in any common year, the Dominical Letter being known, the first of January may be easily found, reckoning from A according to the natural order of the letters: and in any leap year, the first of its two Dominical Letters will shew as above, counting from A 1, B 2, C 3, &c. and by counting backward, you may have the day of the week, on which the first of January will happen.

Rule.—Find the day of the week answering to the first of January that year, then add together the days contained in each month from the beginning of the year to the proposed day of the month inclusively; divide this sum by 7, and if any thing remain, after the division, then, count so many forward, beginning with that day on which the first of January falls, and yon will have the day of the week, on which the proposed day will fall: but if nothing remain, then the day of the week, preceding that day on which the first of January falls, answers to the proposed day.

Example.

On what day of the week will the 5th day of May 1786 fall?

By the preceding observations, and by Prob. 4th, the first of January is found to fall on Sunday.

Now, counting forward six days from Sunday, the first of January (inclusively) the 5th of May falls on Friday.

Jan.	31
Feb.	28
March	31
April	30
May	5th.

7)125(17
7
—
55
49
—
6 from Jan. 1.

Problem XIII.

To Find the Cycle of the Sun.

Rule.—Add 25* to the given year; divide the sum by 28, and the remainder, after division, is the Cycle required; but if nothing remain, the Cycle is 28.

Example.

* From the commencement of this century, 9+16=25 muft be added to the given year. The leap year having been omitted in the year 1800, makes it neceffary to add 25 to the date of the year, and then dividing by 28, it will give the Cycle right during the prefent century. And this is a general rule to be obferved, that when a leap year has been abated, add 16 to the number which was before added to the year, rejecting 28, when it exceeds it, and this number being added to the year, and the fum divided by 28, the remainder after divifion, will be the Cycle for finding the Dominical Letter. Thus in the nineteenth century, it will be 9+16=25, and in the twentieth century $\overline{25+16}-28=13$, which number will ferve two centuries, for the year 2000 is a leap year.

EXAMPLE.

For the year 1807?

```
To 1807
Add   25

28)1832(65
   168

    152
    140

     12 = Cycle required.
```

The use of this Cycle is to find the Dominical Letter by the following Table.

A TABLE *of the Dominical Letters for the New Style, according to the Cycle of the Sun.*

Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.
1	DC	8	B	15	G	22	E
2	B	9	A G	16	F	23	D
3	A	10	F	17	ED	24	C
4	G	11	E	18	C	25	B A
5	F E	12	D	19	B	26	G
6	D	13	C B	20	A	27	F
7	C	14	A	21	GF	28	E

PROBLEM XIV.

To find the year of the Dionysian Period.

RULE.—Add to the given year 457; divide the sum by 532, and the remainder will be the number required.

EXAMPLE.

Required the year of the Dionysian Period for the year 1786?

```
To 1786
Add 457

532)2243(4
    2128

     115=Dionysian Period.
```

PROBLEM XV.

To find the year of Indiction.

RULE.—Add 3 to the given year; divide the sum by 15, and the remainder, after division, will be the Indiction; if nothing remain, it will be 15.

EXAMPLE.

EXAMPLE.

Required the year of Indiction for 1786?

To 1786
Add 3

15)1789(119
15

28
15

139
135

4=Indiction.

PROBLEM XVI.

To find the Julian Period.

RULE.—Add 4713 to the given year and the sum will be the Julian Period.

EXAMPLE.

What year of the Julian period will answer to the year 1786?

To 1786
Add 4713

6499 Ans.

PROBLEM XVII.

To find the Cycle of the Sun, Golden Number, and Indiction, for any current year.

RULE.—To the current year add 4729;* divide the sum by 28, 19 and 15, respectively, and the several remainders will be the numbers required; when nothing remains, the divisor is the number required.

EXAMPLE.

What are the Cycle of the Sun, Golden Number, and Indiction, for the year 1807?

1807	19)6536(344	15)6536(435
4729	57	60
28)6536(232	83	53
56	76	45
93	76	86
84	76	75
96	0	Indiction = 11
84	Golden Number = 19	
12 Cycle of the Sun.		

PROBLEM

* For any year in the nineteenth century add 4713+16=4729.

PROBLEM XVIII.

To find the time of High Water.

RULE.—Find the Moon's southing, to which add the point of the compass making full sea, on the full and change days, for the place proposed, and the sum will be the time required.

EXAMPLE.

I demand the time of high water at Boston, January 25th, 1786, admitting the tide to flow and ebb N. W. and S. E. on the days of change and full?

We have before found the Moon's southing to be 7h. 12m. in the morning.

	h.	m.	
Therefore to	7	12	
Add	4	0	= the point of the compass, and it
Gives	11	12	in the morning, for the time of high water.

PROBLEM XIX.

To find on what day Easter will happen.

It was ordered by the Nicene Council, that Easter Sunday should be kept on the first *Sunday* after the first full moon, which happened upon or after the twenty first day of March, the day on which they thought the Vernal Æquinox happened. Though this was a mistake, for the Vernal Æquinox, that year, fell on the twentieth of March—But yet, the full moon, which fell on, or next after the twenty first of March, they called the Paschal full moon. And by the introduction of the Gregorian, or New Style, the Æquinox will now always happen on the twentieth or twenty first of March. And the feast of Easter is now to be kept on the next Sunday after the Paschal full moon, or the full moon which happens after the twenty first of March; but, if the full moon happens on a Sunday, Easter day is to be the next Sunday after.

RULE.—Find the age of the moon on the 21st of March, in the given year, and if it be 14, then find the day of the week answering to it, and the Sunday following is Easter Sunday; but if the moon's age on the 21st day of March be not 14, then reckon forward to the day on which the moon's age is 14, and find the day of the week answering to that day; the Sunday following will be the day required.

N. B. On leap year take the 20th of March.

EXAMP. When does Easter happen in the year 1786?

	21 of March	Jan.	31
	29 Epact.	Feb.	28
	1 No. of the month.	March	31
	—	April	13th
	51		—
Subt.	30		7)103(14
	—		7
	21 Moon's age.		—
Add	23 { No. of days to the Moon's being 14 days old.		33
			28

Carried over.

Add 23

—

44

Take 31 = days in March.

—

13th of April, the day of the full moon, or Easter limit.

Brought over. 28

—

5 Therefore, the first of January being Sunday, reckon forward 5 days, including Sunday, and you will find the 13th of April falls on Thursday, consequently the next Sunday is the 16th, which is Easter Sunday.

Easter may be found, for any future time, by the following Table which is calculated from 1753, the time of the commencement of the New Style in America, and which shews, by the Golden Number, the days of the Paschal full moons ; by which, and the Dominical Letter, the day, on which Easter will fall, may be found.

The Use of the Table.

First, find the Golden Number as before taught, which seek in the column of Golden Numbers under the time in which the given year is included ; right against the Golden Number of the year, in the last column but one, you have the day of the month on which the Paschal full moon happens, which is the limit of Easter ; from thence run your eye down among the Dominical Letters, till you come to the Letter of the given year, and against it you have the day of the month, on which Easter falls that year.

Example. To know when Easter falls in 1786.

The Golden Number for the year being one, and the Dominical Letter A ; therefore seek in the first column (the given year being included between the years 1753 and 1899) for the Golden Number : then cast your eye along to the last column but one, under the title, Paschal full ●, and you will find the thirteenth of April to be the day of the full moon ; against which, in the last column, stands E, which shews it to be Thursday, therefore the next Sunday following is Easter Sunday, which, by going down the column of Letters to the next A, you will find to be the sixteenth of April.

GOLDEN NUMBERS from 1753 to 1899, and so on to 4199, inclusively

1753 to 1899	1900 to 2199	2200 to 2299	2300 to 2399	2400 to 2499	2500 to 2599	2600 to 2899	2900 to 3099	3100 to 3399	3400 to 3499	3500 to 3599	3600 to 3699	3700 to 3799	3800 to 4099	4100 to 4199	Paschal Full ● Days of the Moon.		Dom. Letter.
14	—	6	17	6	17	—	9	—	1	12	1	12	—	4	March	21	C
3	14	—	6	—	6	17	—	9	—	9	—	1	12	—		22	D
—	3	14	—	14	—	6	17	—	9	—	9	—	1	12		23	E
11	—	3	14	3	14	—	6	17	—	9	—	9	—	1		24	F
—	11	—	3	—	3	14	—	6	17	—	17	—	9	—		25	G
19	—	11	—	11	—	13	14	—	6	17	6	17	—	9		26	A
8	19	—	11	—	11	—	3	14	—	6	—	6	17	—		27	B
—	8	19	—	19	—	11	—	3	14	—	14	—	6	17		28	C
16	—	8	19	8	19	—	11	—	8	14	3	14	—	6		29	D
5	16	—	8	—	8	19	—	11	—	3	—	3	14	—		30	E
—	5	16	—	16	—	8	19	—	11	—	11	—	3	14		31	F
13	—	5	16	5	16	—	8	19	—	11	—	11	—	3	April	1	G
2	13	—	5	—	5	16	—	8	19	—	19	—	11	—		2	A
—	2	13	—	13	—	5	16	—	8	19	8	19	—	11		3	B
10	—	2	13	2	13	—	5	16	—	8	—	8	19	—		4	C
—	10	—	2	—	2	13	—	5	16	—	16	—	8	19		5	D
18	—	10	—	10	—	2	13	—	5	10	5	16	—	8		6	E
7	18	—	10	—	10	—	2	13	—	5	—	5	16	—		7	F
—	7	18	—	18	—	10	—	2	13	—	13	—	5	16		8	G
15	—	7	18	7	18	—	10	—	2	13	2	13	—	5		9	A
4	15	—	7	—	7	18	—	10	—	2	—	2	13	—		10	B
—	4	15	—	15	—	7	18	—	10	—	10	—	2	13		11	C
12	—	4	15	4	15	—	7	81	—	10	—	10	—	2		12	D
1	12	—	4	—	4	15	—	7	18	—	18	—	10	—		13	E
—	1	12	—	12	—	4	15	—	7	18	7	18	—	10		14	F
9	—	1	12	1	12	—	4	15	—	7	—	7	18	—		15	G
—	9	—	1	—	1	12	—	4	15	—	15	—	7	18		16	A
17	17	9	—	9	—	1	12	12	4	15	4	15	15	7		17	B
6	6	17	9	17	9	9	1	1	12	4	12	4	4	15		18	C
																19	D
																20	E
																21	F
																22	G
																23	A
																24	B
																25	C

THE USE OE LOGARITHMS.

I. IN MULTIPLICATION.

Given two numbers, viz. 275 *and* 12·6, *to find their product.*

RULE.—To the logarithm of 275, viz. - - - - - - - 2·43933
Add the logarithm of 12·6, viz. - - - - - - - - - 1·10037

And their sum is the logarithm of their prod. viz. 3465 = 3·53970

2. IN

2. In Division.

Let it be required to find the quotient, which arises by dividing one number by another; suppose 1425 by 57.

From the logarithm of the dividend, viz. 1425	$=3{\cdot}15381$
Take the logarithm of the divisor, viz. 57	$=1{\cdot}75587$
And the remainder is the logar. of the quotient, viz. 25	$=1{\cdot}39794$

3. In the Rule of Three.

Three numbers given, to find a fourth, in direct proportion.

Rule.—From the Tables take the logarithms of each of the proposed numbers, then, add the logarithms of the second and third together, and from the sum take the logarithm of the first, and the remainder will be the logarithm of the fourth number.

Let the three proposed numbers be 18, 24, and 33, and the operation will stand thus:

$1{\cdot}38021$ = the logarithm of 24, the 2d term.
$1{\cdot}51851$ = the logarithm of 33, the 3d term.

$2{\cdot}89872$ = the logarithm of their product.
$-1{\cdot}25527$ = the logarithm of the first term 18.

$1{\cdot}64345$ = the logarithm of the fourth term required, which, by the Table, answers to the natural number 44, the 4th proportional to the three proposed numbers.

4. In Involution, or Raising Powers.

To find any power of any proposed number, or to involve any number to any proposed power, by logarithms.

Rule.—Multiply the logarithm of the given root by the power, viz. by 2 for the square, by 3 for the cube, &c. and the product is the logarithm of the power sought.

Required to find the cube of 12?

$1{\cdot}07918$ = the logarithm of 12.
$\times 3$ = the third power, or cube.

$3{\cdot}23754 = 1728$ the cube of 12.

5. In Evolution, or Extracting Roots.

To extract any root of any proposed number.

Rule.—Divide the logarithm of the proposed number, by the index of the required root, viz. by 2 for the square, by 3 for the cube, &c. and the quotient will be the logarithm of the root required.

Required to find the cube root of 1728?

$3{\cdot}23754$ = the logarithm of 1728, and $3{\cdot}23754 \div 3 = 1{\cdot}07918$ is the logarithm of the cube root of 1728, viz. 12.

6. In Compound Interest.

To find the amount of any sum for any time, and at any rate, at Compound Interest.

Rule.—Multiply the logarithm of the ratio (i. e. the amount of £.1 or D.1 for one year) by the number of years, and to the pro-

duct

duct add the logarithm of the principal; the sum will be the logarithm of the amount.

What will £.45 amount to, forborne 12 years, at 6 per cent. per annum, compound interest?

Log. of 1·06 the ratio, is	·02533
Multiply by the time	12
	·30396
Add log. of 45, the principal	1·65321

The sum is 1·95717 which is the logarithm of 90·7= £.90 14s. Ans.

7. In Discount at Compound Interest.

To find the present worth of any sum of money due any time hence, at any rate, at Compound Interest.

Rule.—From the logarithm of the sum to be discounted, subtract the logarithm of the rate multiplied by the time; and the remainder is the logarithm of the present worth.

What present money will pay a debt of £.90 14s. due 12 years hence, discounting at the rate of 6 per cent. per annum?

From the logarithm of £.90 14 =	1·95717
Subt. prod. of the log. of the ratio × by the time =	·30396

The remainder 1·65321 is the logarithm of £.45 Ans.

PLAIN GEOMETRY.

Definitions.

1. A *POINT* in the Mathematicks is considered only as a mark, without any regard to dimensions.

2. A *Line* is considered as length, without regard to breadth or thickness.

3. A *Plain* or *Surface* has two dimensions, length and breadth, but is not considered as having thickness.

4. A *Solid* has three dimensions, length, breadth and thickness, and is usually called a *Body*.

5. A line is either *straight*, which is the nearest distance between two Points; or *crooked*, called a Curve Line, whose ends may be drawn further asunder.

6. If two Lines are at equal distance from one another in every part, they are called *parallel* Lines, which, if continued infinitely, will never meet.

7. If two lines incline one towards another, they will, if continued, meet in a point: by which meeting is formed an *Angle*.

8. If

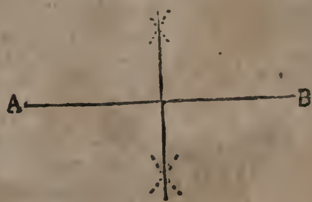
8. If one Line fall directly upon another, so that the Angles on both sides are equal, the Line, so falling, is called a *perpendicular*, and the Angles, so made, are called *right* Angles, and are equal to 90 degrees, each.

9. All Angles, except right Angles, are called oblique Angles, whether they are *acute*, that is, less than a right Angle; or *obtuse*, that is, greater than a right Angle.

GEOMETRICAL PROBLEMS.

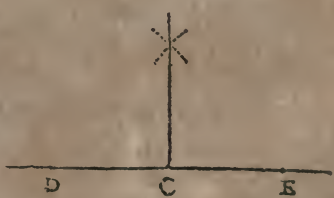
PROBLEM I. *To divide a Line* A B *into two equal parts.*

Set one foot of the compasses in the point A, and, opening them beyond the middle of the line, describe arches above and below the line; with the same extent of the compasses, set one foot in the point B, and describe two arches crossing the former: draw a line from the intersection of the arches above the line, to the intersection below the line, and it will divide the line A B into two equal parts.



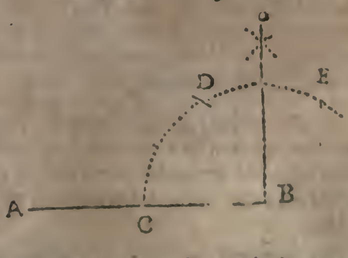
PROBLEM II. *To erect a perpendicular on the point* C *in a given line.*

Set one foot of the compasses in the given point C, extend the other foot to any distance at pleasure, as to D, and with that extent make the marks D, and E. With the compasses, one foot in D, at any extent above half the distance of D and E, describe an arch above the line, and with the same extent, and one foot in E, describe an arch crossing the former; draw a line from the intersection of the arches to the given point C, which will be perpendicular to the given line in the point C.



PROBLEM III. *To erect a perpendicular upon the end of a line.*

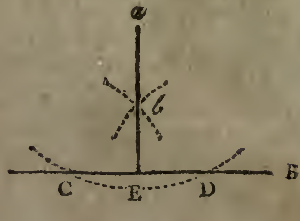
Set one foot of the compasses in the given point B, open them to any convenient distance, and describe the arch C D E; set one foot in C, and with the same extent, cross the arch at D: with the same extent cross the arch again from D to E; then with one foot of the compasses in D, and with any extent above the half of ED, describe an arch *a*; take the compasses from D, and, keeping them at the same extent with one foot in E, intersect the former arch *a* in *a*; from thence draw a line to the point B, which will be a perpendicular to AB.



PROBLEM

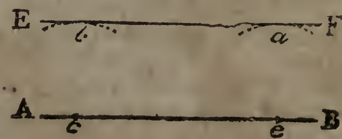
PROBLEM IV. *From a given point, a, to let fall a perpendicular to a given line* A B.

Set one foot of the compasses in the point *a*, extend the other so as to reach beyond the line A B, and describe an arch to cut the line A B in C and D; put one foot of the compasses in C, and, with any extent above half C D, describe an arch *b*; keeping the compasses at the same extent, put one foot in D, and intersect the arch *b* in *b*; through which intersection, and the point *a*, draw *a* E, the perpendicular required.



PROBLEM V. *To draw a Line parallel to a given Line* A B.

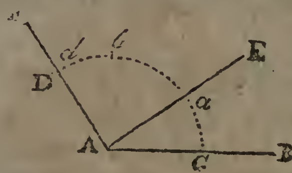
Set one foot of the compasses in any part of the line, as at *c*; extend the compasses at pleasure, unless a distance be assigned, and describe an arch *b*; with the same extent, in some other part of the line AB, as at *e*, describe the arch *a*; lay a ruler to the extremities of the arches, and draw the line E F, which will be parallel to the line A B.



PROBLEM VI. *To make an Angle equal to any number of Degrees.*

It is required to lay off an acute Angle of 35° on a given line AB.

Take 60 degrees from the line of chords in the compasses, set one foot of the compasses in the point A, describe an arch C D, at pleasure; then set one foot of the compasses in the brass centre, in the beginning of the line of chords, and bring the other to 35 on the line; with this extent set one foot in C, with the other intersect the arch CD, in *a*, and through *a* draw the line AE, so will EAB be an angle of 35 degrees.



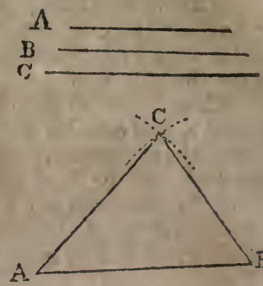
If the angle had been obtuse, suppose 125°, then take 90° from the line of chords; set one foot in C, and intersect the arch in *b*; then take 35° from the same line of chords, and set them from *b* to *d*; a line drawn from A through *d* to F will make an angle, FAB, of 125°.

To measure an angle by the line of chords, is only to take the distance on the arch between the lines AB and AE, or AB and AF, and laying it on the line of chords.

PROBLEM

PROBLEM VII. *To make a Triangle, whose sides shall be equal to three given lines, provided any two of them be longer than the third.*

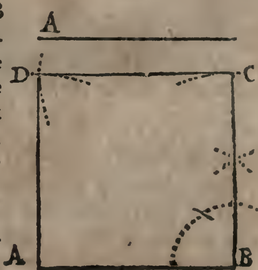
Let A, B, C, be the three given lines; draw a line AB, at pleasure; take the line C in the compasses, set one foot in A, and with the other make a mark at B; then take the given line B in the compasses, and setting one foot in A, draw the arch C; then take the line A in the compasses, and intersect the arch C in C; lastly, draw the lines AC and BC, and the triangle will be completed.



PROBLEM VIII. *To make a Square, having equal sides, equal to any given line.*

Let A be the given line; draw a line A B equal to the given line; from B raise a perpendicular to C equal to AB, with the same extent, set one foot in C and describe the arch D; also with the same extent, set one foot in A and intersect the arch D; lastly, draw the lines AD and CD, and the square will be completed.

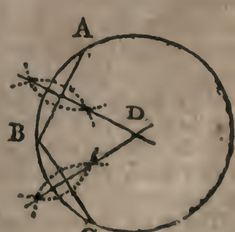
In like manner may a Parallelogram be constructed, only attending to the difference between the length and breadth.



PROBLEM IX. *To describe a Circle, which shall pass through any three given Points, which are not in a straight line.*

Let the three given points be A, B, C, through which the circle is to pass. Join the points AB and BC with right lines, and bisect these lines; the point D, where the bisecting lines cross each other, will be the centre of the circle required. Therefore, place one point of the compasses in D, extending the other to either of the given points, and the circle, described by that radius, will pass through all the points.

Hence, it will be easy to find the centre of any given circle; for, if any three points are taken in the circumference of the given circle, the centre will be readily found as above. The same may also be observed, when only a part of the circumference is given.



PROBLEM X. *To describe an Ellipsis or Oval mechanically.*

Draw two parallel lines, as L and M, at a moderate distance, by

Prob. 5; then draw two others at the same distance, across the former, as N and O; by the crossing of these lines will be made a figure ABCD, of four sides; extend the compasses at pleasure, and setting one foot in D, describe the arch *cde*; with the seme extent, set one foot in B, and describe the arch *fgh*; then set one foot in C, and contract them so as to reach the point *e*, and describe the arch *lm*; with the same extent, and one foot in A, describe the arch *ik*, and the oval will be completed. In the same manner, with a greater or less extent of the compasses, may a greater or less oval be made by the same four sided figure ABCD.

OF PLAIN TRIGONOMETRY.

RIGHT AND OBLIQUE ANGLED.

PLAIN Trigonometry is that science, by which we measure the sides and angles of plain triangles.

SECTION I. *Of Rectangular Trigonometry.*

In a right triangle, the longest side is usually called the hypothenuse, the next longest, the base, and the shortest the perpendicular.

Logarithmick sines, tangents, and secants, are called the *tabular* sides of a triangle, and are the sines, &c. of the opposite angles. The length of the sides are called the *natural* sides.

All the three angles of a triangle are equal to two right angles, or 180°.

The proportion ought to be made between sides and sides; and between angles and angles.

When a side is required, any side (whether known or not) may be made radius; but when an angle is required, then a known side only, must be made radius.

Note. A side is said to be made radius, when one foot of the dividers is set in one end of the side, and such a circle described, of which the side is the semidiameter: Also, that when the hypothenuse is radius, it is the sign of the right angle, or 90°, and the base and perpendicular, usually called the legs, become sines of their opposite angles: but when one of the legs is made radius, the other becomes the tangent of the opposite angle, and the hypothenuse, the secant of the same angle.* Tangent's radius is 45°.

When a side is to be found, the two first terms of the proportion must be tabular sides, and the last a real one: but when an angle is to be found, the two first terms must be real sides, and the third, a tabular one.

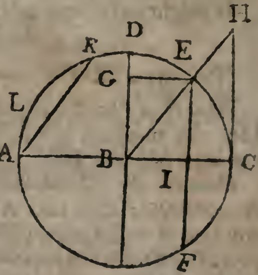
The given parts, whether sides or angles, are marked with —, and the part required, with O.

Angles are measured by the arch of a circle. The periphery of every

* To work on the ſcale with a ſecant, you muſt take the ſines backward, that is 80 ſines for 10 ſecants, &c.

every circle, whether great or small, is divided into 360 degrees, each degree into 60 minutes, every minute into 60 seconds, and so on, to thirds, fourths, &c.

Any portion of the periphery of a circle, as ECF, is called an *arch*, and a line drawn from the ends of an arch, as, EIF, is called the *chord* of the arch. Half the chord of any arch, as EI, is called the *sine* of the arch EC, and IC it called the *versed sine* of the same arch EC: So, also, EG is the *sine* of the arch ED. A line drawn perpendicular to the diameter of a circle, so as to touch the circle and not cut it, is called a tangent, as CH, which is the tangent of the arch EC, because the line BH, drawn from the centre B, through E, called the *secant*, meets it in the point H.

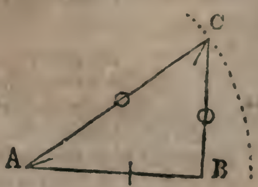


The complement of an arch is the remainder, after the arch is tak-from 90°; thus KD is the complement of the arch ALK, taken from the arch AD. The cosine or sine complement of an arch is the sine of the complement of that arch, as ED is the complement of EC.

Problem I. *The Angles and one of the Legs given, to find the Hypothenuse and other Leg.*

Example. In the triangle ABC, right angled at B, suppose the leg AB, 86 equal parts (as feet, yards, miles, &c.) the angle A=33°, 40′, and the angle C=56°, 20′: Required the length of the hypothenuse AC, and the leg BC?

Geometrically. Draw AB equal to 86, from any line of equal parts, then upon the point B, erect the perpendicular BC; lastly, from the point A, draw the line AC, making with AB an angle=33°, 40′, and that line produced will meet BC in C, and so constitute the triangle The length of AB and BC may be found by taking them in your compasses, and applying them to the same line of equal parts that AB was taken from.



By Calculation. By making the hypothenuse radius, the legs will become the sines of the opposite angles; and as natural sides are required, the proportions must begin with tabular sides: therefore, for the hypothenuse,

As the sine of C	56°,20′	9·92027
Is to radius	90 ,00	10·
So is the side AB	86	1·93450
		11·93450
		9·92027
To the side AC	103·3	2·01423

Here, I add the logarithms of the 2d and 3d terms, and from their sum subtract the first, and the remainder is the logarithm of the side sought, which gives 103·3. The same must be done in all the following cases.

For

For the Leg BC.

As the sine of the angle	C	56°,20′	9·92027
Is to the sine of the angle	A	33 ,40	9·74380
So is the side AB		86	1·93450
To the side BC		57·28	1·75803

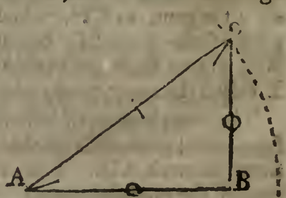
It might have been as easily found by the following proportion :

As R : S,A :: AC : BC.

Problem II. *The Angles and Hypothenuse given, to find the Legs.*

Example. In the triangle ABC, suppose the hypothenuse AC = 146, the angle A = 36°,25′, and the angle C = 53°,35′ : Required the legs AB and BC ?

Geometrically. Draw the line AB at pleasure, and make the angle A = 36°,25′ ; then take AC = 146 from any line of equal parts ; lastly, from the point C let fall the perpendicular CB on the line AB : so the triangle is constructed, and AB and BC may be measured from the line of equal parts.



By Calculation. Making AC radius, the legs become sines, as before, and as the angles are given to find the sides, we must begin the proportion with angles, or tabular sides.

For the Leg AB.

As radius	90°,00′	10·	
Is to the sine of C	53 ,35		9·90565
So is side AC	146		2·16435
To side AB	117·5		2·07000

For the Leg BC.

As radius	90°,00	10·	
Is to the sine of A	36 ,25		9·77353
So is side AC	146		2·16435
To side BC	86·67		1·93788

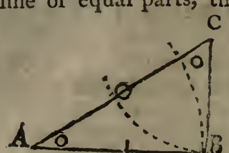
As we had before found AB, the proportion might have been,

As S.C : S,A :: AB : BC.

Problem III. *and* **IV.** *The two Legs given, to find the Angles and Hypothenuse.*

Example. In the triangle ABC, suppose the leg AB = 94, and BC = 56 : Required the angles and hypothenuse.

Geometrically. Draw AB = 94 from any line of equal parts, then, from the point B raise BC perpendicular to AB, and take BC from the former line of equal parts = 56 ; lastly, join the points A and C with the straight line AC, so the triangle is constructed. AC may be found by taking it in your dividers and applying it to the line of equal parts ; and the angles may be measured by the 6th Geometrical Problem.



By

By Calculation. 1st. For the angle A; supposing the base AB the radius, then the hypothenuse becomes secant of the angle A, and the perpendicular BC, the tangent of the angle A: and as an angle is required, we must begin the analogy with a natural side.

As AB	94	1·97313
Is to BC	56	1·74819
So is tangent's radius	45°,00	10·
To the tangent of A	30°,47′	9·77506

The perpendicular might have been made radius, and then the proportion would have been, as BC : AB :: tan. rad. : tang. of C.

Now, as we have found the angle A, and as the angles A and C, taken together, are equal to 90°, therefore from 90°,00′

Take the angle A = 30°,47

And we have the angle C = 59°,13′

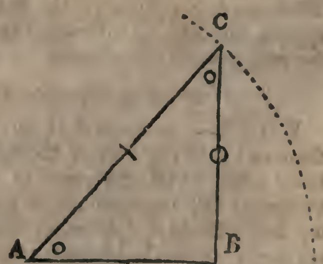
2d. *For the Hypothenuse.* The base still being radius, we have this analogy for finding the hypothenuse: as T. R : Sec. A :: AB : AC. But this may be done without the help of secants; for, having found the angles, we may now make the hypothenuse radius; and as a natural side is required, we must begin the proportion with a tabular side; therefore,

As the sine of C	59°,13′	9·93405
Is to radius	90 ,00	10·
So is AB	94	1·97318
To AC	109·4	2·03908

Or the analogy might have been, as S. C : R :: BC : AC.

PROBLEM V. *and* VI. *The Hypothenuse and one of the Legs given, to find the Angles and other Leg.*

EXAMPLE. In the triangle ABC, suppose the leg AB = 83, and the hypothenuse AC = 126: Required the angles A and C, and the leg BC?



Geometrically. Draw AB = 83 from any line of equal parts; and from the point B, raise the perpendicular BC of any length, then take the length of AC 126 from the same line of equal parts, and setting one foot of the dividers in A, with the other cross the perpendicular BC in C; lastly, join AC, so the triangle will be constructed, and the angles may be measured as directed in Problem 3d and 4th.

By Calculation. First, for the angle C; and as an angle is required, we must begin with a side, making the hypothenuse radius.

As AC	126	2·10037
Is to AB	83	1·91908
So is radius	90°,00	10·
To sine of C	41 12	9·81871

From

From	90°,00′
Take the angle at C =	41 ,12
And we have the angle A =	48°,48′

For the side BC. As a side is now required, we must begin with an angle; therefore,

As radius	90°,00′	10·
Is to the sine of A	48°,48′	9·87646
So is AC	126	2·10034
To BC	94·8	1·97683

SECTION II. *Of oblique angular Trigonometry.*

In any triangle, the sides are proportional to the sines of the opposite angles.

When two angles of any triangle are given, their sum, being subtracted from 180°, leaves the third angle; and when one angle is given, that being subtracted from 180°, leaves the sum of the two unknown angles.

When any angle exceeds 90°, subtract it from 180°, and work with the remainder.

When the given and required parts, viz. sides and angles are opposite.

RULE 1.—As in right angled triangles.

As the sine of any angle is to the sine of any other angle; so is the side opposite to the first angle, to the side opposite to the other angle.

Or, as one side is to any other side; so is the sine of the angle opposite to the first side, to the angle opposite to the other side.

When any two sides, with the angle included between them are given.

RULE 2.—As the sum of any two sides is to their difference; so is the tangent of the half sum of the two opposite angles, to the tangent of half the difference of those two opposite angles; which half difference being added to the half sum, gives the greater of the two angles, and, being subtracted from the half sum, leaves the less of the two unknown angles.

When the three sides are given, to find the Angles.

RULE 3.—As the base of any triangle (or sum of the segments of the base) is to the sum of the other two sides: so is the difference of those sides, to the difference of the two segments of the base, made by letting fall a perpendicular to the base from the angle opposite to it; half of which difference, being added to half the sum of the two segments, gives the longest, and being subtracted, leaves the shortest.

The learner being now somewhat acquainted with the common method of working by logarithms, it will be proper to shew how to perform those proportions without subtracting the first number from the sum of the second and third, which is done by setting down the arithmetical complement of the first term instead of the logarithm. This may be readily done thus; subtract the first figure of the logarithm from 10, and set down the remainder: then subtract each of the

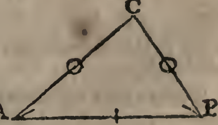
the other figures, index and all, from 9, setting down the remainders, and place a dot before the index, as in the case of the logarithm.— Thus the arithmetical complement (usualy marked Co. Ar.) of the logarithm 9·66004 is 0·33996, and so of any other.

When the arithmetical complement of the first term is used instead of the logarithm, add all the three numbers together, and reject 10 out of the index of their sum, as in those cases where the radius is the first term.

PROBLEM I. *In the olique angled triangle ABC, given two angles and a side opposite to one of them, to find the two other sides.*

Suppose the angle at A 36°,40′, the angle at B, 60°,51′, and the base AB 85·6: Required AC and BC?

Geometrically. Draw the base AB, and from any scale of equal parts, lay thereon 85·6 from A to B; then, from the line of chords, lay off an angle of 36°,40′ at A, and an angle of 60°,51′ at B, and the meeting of these two lines in C completes the triangle, and AB and BC may be measured by the same line of equal parts.



From the sum of all the angles	180°,00′
Take the sum of the angles A and B, viz.	97°,31′
And we have the angle C equal to	82°,29′

Here we have the angle at C opposite to the given base, and the angles at A and B opposite the two required sides, which may be found by the first rule, as follows:

By Calculation. For the side BC. Having to find a side, we begin with an angle,

As the sine of the angle at C,	82°,29′	Co. Ar.	0·00375
Is to the sine of the angle A,	36°,40′		9·77609
So is the base AB	85·6		1·93247
To side BC	51·55		1·71231

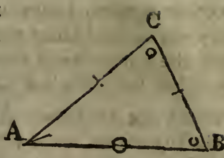
For the side AB.

As the sine of C	82°,29′	Co. Ar.	0·00375
Is to the sine of B	60°,51′		9·94118
So is AB	85·6		1·93247
To AC	75·4		1·87740

PROBLEM II. *and* III. *Two sides and an angle opposite to one of them, given, to find the two other angles and remaining side.*

In the oblique angled triangle ABC, given the side AC 75·4, the side BC 51·56, and the angle at A 36°,40′, to find the base AB, and the angles at B and C.

Geometrically.



Geometrically. Draw the base AB, at pleasure, and on any point assumed, as A, make an angle of 36°,40′; take 75·4 from the scale of equal parts and set it from A to C, then take 51·56 from the same scale; set one foot of the dividers in C, and with the other intersect the base in B; lastly, draw BC, and the triangle is completed, and the base may be measured by the same scale of equal parts.

By Calculation. Here we have the side BC opposite the known angle at A, and the side AC opposite the unknown angle at B, which may be found by Rule 1st.

To find the angle at B. Having to find an angle we begin with a side.

As BC	51·55	Co. Ar.	8·28778
Is to AC	75·4		1·87737
So is the sine of the angle A	36°,40′		9·77609
To the sine of the angle B	60°,51′		9·94124

From the sum of all the angles	180°,00′
Take the sum of the angles A and B	97°,31′
And we have the angle C equal to	82°,29

For the base AB. Having to find a side we begin with an angle.

As the sine of A	36°,40′	Co. Ar.	0·22392
Is to the sine of C	82°,29′		9·99625
So is BC	51·55		1·71223
To AB	85·6		1·93240

To find the tangent of half the difference of two unknown angles by the scale.

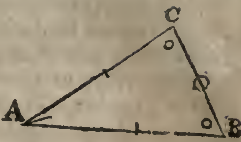
Rule 1.—When the tangent of half the sum of the unknown angles is less than 45°, the extent from the half sum of the sides to their half difference on the line of numbers will reach from the half sum of the unknown angles to their half difference on the tangents.

2. When the tangent of half the sum of the unknown angles exceeds 45°, take the extent between the sum and difference of the sides; set one foot in tangent's radius, 45°, and fix the other foot wherever it falls on the tangent line, and contract the foot that stands on 45°, to the tangent of the half sum of the unknown angles; then with that extent set one foot in 45°. of tangents, and the other will point out the tangent of the half difference.

Problem IV. *and* V. *Two sides, and the angle included between them at* A, *given, to find the two other angles and the other side.*

In the oblique angled triangle ABC, given the side AC 75·4, the base AB 85·6, and the included angle at A 36°,40′, to find the angle B and C, and the side BC.

Geometrically.



Geometrically. Draw the base AB, and from any scale of equal parts, set off 85·6 from A to B; make an angle at A of 36°, 40′ and draw AC, and from the same scale of equal parts, set 75·4 from A to C; lastly, draw the line BC, and the triangle is completed: BC may be measured by the same scale of equal parts, and the angles B and C, on the line of chords.

By Calculation. Here we have given the two sides AB and AC, with the angle included between them; and therefore these cases must be solved by Rules 2d and 1st. Now, as the three angles of every triangle are equal to 180°, the angle at A 36°,40′ being subtracted from 180° leaves 143°,20′, the sum of the two unknown angles B and C, half of which is 71°,40′; and half their difference may be found by the following proportion, according to Rule 2.

As the sum of the two sides AB and AC	161	Co. Ar. 7·79318
Is to their difference	10·2	1·00860
So is the tangent of half the sum of the unknown angles B and C	71°,40′	10·47969
To the tangent of half their difference	10°,49′	9·28147

To the half sum	71°,40′	From the half sum	71°,40′
Add the half difference	10 ,49	Take the half diff.	10 ,49
The sum is the greater ang. C	82 ,29	The remainder is the less angle B	60 ,51

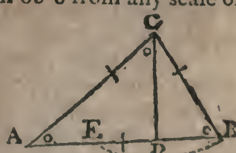
Having found the angles B and C, the side BC may be found by Rule 1.

As the sine of C	82°,29′	Co. Ar. 0·00375
Is to the sine of A	36°,40′	9·77609
So is AB	85·6	1·03247
To BC	51·56	1·71231

PROBLEM VI. *The three sides given, to find the angles.*

In the oblique angled triangle ABC, given the base AB 85·6, the side AC 75·4, and the side BC 51·56; Required the angles?

Geometrically. Draw the base AB, and set off 85·6 from any scale of equal parts from A to B; take 75·4 from the same scale, and setting one foot in A, describe an arch; then from the scale take 51·56, and setting one foot in B, intersect the former arch in C; from C draw lines to A and B, and the triangle is completed. The angles may all be measured upon the line of chords.



By Calculation. Here being no angle given, these cases must be solved by Rule 3d, in the following manner: Place one foot of the dividers in C, and extend the other so as to take in the shortest side BC, and describe the arch BE; then, from C let fall a perpendicular on the base AB, which will divide it into two segments, AD the greater, and DB the less, whose difference is AE: Then,

As the Base AB	85·6	Co.Ar.8·06753
Is to the sum of the two sides AC and BC	126·96	2·10366
So is the difference of the sides AC and BC	23·84	1·37730
To the difference of the segments of the base, or AE — —	35·36	1·54849

Half the difference of the segments is 17·68

To half the base	42·8	From half the base	42·8
Add half the difference	17·68	Take the half difference	17·68
And the sum is the greater segment AD	60·48	And the remainder is the less segment DB	25·12

Thus is the oblique angled triangle ABC divided into two right angled triangles ADC and BDC, both right angled at D, in each of which are given the base and hypothenuse, to find the other parts.

First, For the angle at C in the right angled triangle ADC, making the hypothenuse radius.

As AC	75·4	Co. Ar. 8·12263
Is to AD	60·48	1·78161
So is radius	90°,00′	10·
To the sine of C	53°,20′	9·90424

The angle A, being the complement of the angle C, is 36°,40′. Then for the angle C in the right angled triangle BDC.

As BC	51·56	Co. Ar. 8·28778
Is to BD	25·12	1·40002
So is radius	90°,00′	10·
To the sine of C	29°,09′	9·68780

Whence the angle B is 60°,51′, being the complement of 29°,09′; and the angle at C, in one triangle, being added to the angle C in the other, is 82°,29′; thus the solution of the problem is finished.

Trigonometry is easily applied to Navigation, and the Mensuration of Heights and Distances. With respect to the former; suppose in the first Problem of right angled Trigonometry, the angle at A is the ship's course, the base to be the true, (or meridional,) difference of latitude, the perpendicular to be the departure, or difference of longitude, and the hypothenuse to be the distance the ship is to run; then we have the course and true (or meridional) difference of latitude given, to find the distance, and departure from the meridian, (or difference of longitude.)

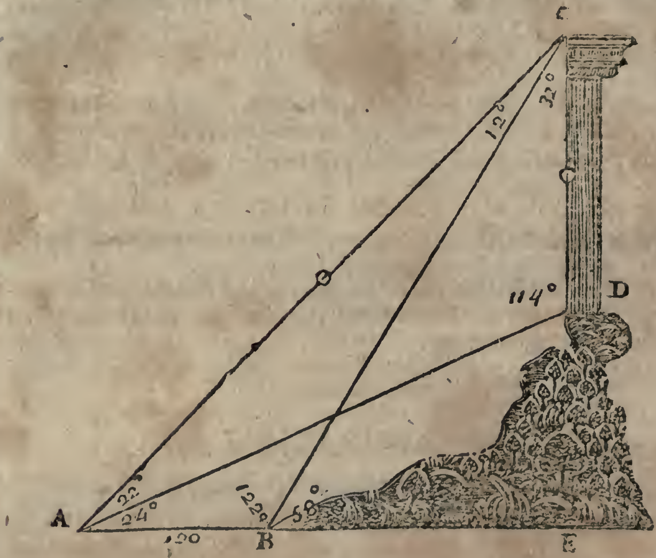
In Problem 2d, we have the course and distance given, to find the true (or meridional) difference of latitude, and the departure, (or difference of longitude.)

With respect to heights and distances: If we suppose in the first Problem before mentioned, the angle at A to be the angle which the top of any distant object makes with the surface of the earth, where we stand; the base to be the distance of the object, (on level ground,) and the perpendicular, the object's height; then we have the angle A, and the distance AB, to find the height BC; but this will serve only on level ground, and where the object is accessible.

The

The distance of any *inaccessible* object may be found by Problem 1st, of oblique Trigonometry: for, if we suppose the object at C, then, at *two* stations, as at A and B, take the bearing of the place; also, measure the stationary distance AB, and you will then have *two* angles and a side opposite to one of them, to find either of the other sides.

To take the height of an Object standing on a hill, which is inaccessible.



At two stations, as at A and B, take the angles, viz. CAE and CBE, which the top of the object makes with an horizontal line, and that which the bottom of the object makes with the first station, at A, viz. DAE, then take DAE from CAE, and the remainder is CAD.

Note. When an angle is expressed by three letters, the middle one shews the angle. Now, suppose the stationary distance AB 120, the angle ACB 12°, and angle CBA 122°, then by Problem 1st of oblique Trigonometry, we have two angles and a side opposite to one of them given, to find the side AC. Therefore,

		Co. Ar.
As S. of ACB	12°,00′ —	0·68213
Is to S. of CBA	122°,00′ —	9·92842
So is stationary distance	120 —	2·07918
To side AC	489·5 —	2·68973

Note. I. subtracted 122° from 180°, and worked with the remainder, and in the following, 114° from 180°. Now, having found AC 489·5, suppose the angle CDA 114°, and the angle CAD 22°, and we have two angles and a side opposite to one of them, as before to find the perpendicular height of the object CD. Therefore,

As

		Co. Ar.			Co. Ar.
As S. of CDA	114°,00′	0·03927	As S. of CDA	114°	– 0·03927
Is to S. of CAD	22°,00′	9·57358	Is to S. of ACD	44°	– 9·84177
So is side AC	489·5	2·68973	So is side AC	489·5	– 2·68973
To perpend. hht. CD	200·7	2·30258	To side AD	372·2	– 2·57077

To find the height of the mountain and object together; we have the right angled triangle ACE, in which are given the hypothenuse AC 489·5, angle CAE 46°, and the angle ACE 44°, whence, by Problem 2d. of right angled Trigonometry, we have these proportions.

As radius	90°	10·00000	As radius	90°	–10·00000
Is to S. of CAE	46°	9·85693	Is to S. of ACE	44°	– 9·84177
So is hypoth. AC	489·5	2·68973	So is AC	489·5	– 2·68973
To per. hht. CE	352·1	2·54666	To AE	340	– 2·53150

If you subtract CD from CE, you will have the height of the hill 151·4.

Any figure in Navigation, or Mensuration of Heights and Distances, may be measured Geometrically, as directed in the foregoing Problems of Trigonometry.

MENSURATION

OF SUPERFICIES AND SOLIDS.

Section I. Of Superficies.

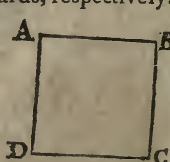
SUPERFICIES, or surfaces, are measured by the superficial inch, foot, yard, &c. according to the measures peculiar to different artists.

The superficial inch, foot, &c. is one inch, foot, &c. in length and breadth; and, because 12 inches make one foot of Long Measure, therefore, 12×12=144 inches make 1 superficial foot, 3×3=9 feet, a yard, &c.

The superficial content of every surface is found by the proper rule of its figure, whether square, triangle, polygon, or circle.

Article I. *To measure a Square, having equal sides.*

Rule.—Multiply the side of the square into itself, and the product will be the area or superficial content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.



Let ABCD represent a square, whose side is 12 inches or 12 feet. Multiply the side 12 by itself, thus,

	12 inches.	12 feet.
	12 inches.	12 feet.
Area =	144 inches.	144 feet.

By the Sliding Rule.

Set 1 to the length on B, then, find the breadth on A, and opposite to this on B, you will have the content.

By

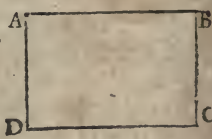
By Gunter's Scale.

Extend the dividers from 1, on the line of numbers, to the length; that distance, laid the same way from the breadth, will point out the answer.

ART. 2. *To measure a Parallelogram, or long Square.*

RULE.—Multiply the length by the breadth, and the product will be the area, or superficial content.

Let ABCD represent a parallelogram, whose length is 16 feet, and breadth, 12 feet. Multiply 16 by 12.



Length 16
Breadth 12
———
192 area.

The content of this figure is found on the sliding rule and scale, as the former.

ART. 3. *When the breadth of a Superficies is given, to find how much in length will make a square foot, yard, &c.*

RULE.—As the breadth is to a foot, yard, &c. so is a foot, yard, &c. to the length required to make a foot, yard, &c. Or divide 144 by the breadth, and the quotient will be the length required.

How much, in length, of a board 2½ feet wide, will make a square foot?

In. br.		In. leng.		In. br.		In. leng.
As 30	:	12	::	12	:	4·8

12
———
30)144(4·8 inches, length required.
120
———
240
240

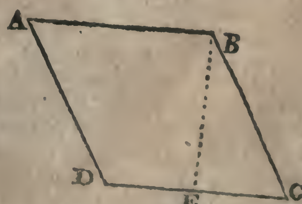
In.
Breadth = 30)144(4·8 inches, Ans.

ART. 4. *To measure a Rhombus.*

Definition. A rhombus is a figure with four equal sides, in the form of a diamond on cards, having two angles greater and two less, than the angles of a square: the former are called *obtuse* angles, and the latter, *acute*, or sharp, angles.

RULE.—Multiply the side by the length of a perpendicular, let fall from one of the obtuse angles to the side opposite such angle.

Let ABCD represent a rhombus, each of whose sides is 16 feet: A perpendicular let fall from the obtuse angle, at B, on the side DC, will intersect it in the point E, so will BE be 12 feet; and this being multiplied into the given side, the product will be the area of the rhombus.



Side

Side = 16
Per. = 12
——
192 area.

By the Sliding Rule.

Set 1 on A to the length on B; find the perpendicular height on A, against which on B is the content.

By Gunter.

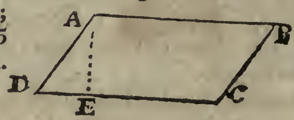
The extent from 1 to the perpendicular height will reach from the length to the content.

Art. 5. *To find the Area of a Rhomboides.*

Definition. A rhomboides is a figure, whose opposite sides and opposite angles are equal.

Rule.—Multiply one of the longest sides by the perpendicular let fall from one of the obtuse angles on one of the longest sides.

Let ABCD represent a rhomboides; the longest sides AB and CD being 16·5 feet, and the perpendicular AE, 9·7 feet.



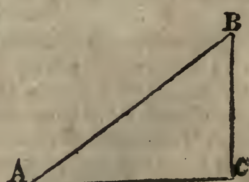
Side = 16·5
Perp. 9·7
——
1155
1485
——
Ans. 160·05 feet.

The content is found on the sliding rule, and scale, as in the last figure.

Art. 6. *To measure a Triangle.*

Rule.—If it be a right angled triangle, multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area: but if it be an oblique angled triangle, (whether obtuse, or acute) multiply half the base by the length of the perpendicular let fall on the base from the angle opposite to it, and the product will be the area. The longest side of a triangle is usually called the base, except in a right angled triangle, where the longest of the two legs, which include the right angle, is called the base.

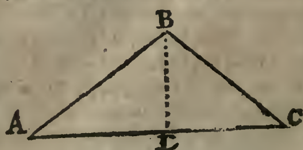
In the right angled triangle ABC right angled at C; the base AC is 18·8 feet, and the perpendicular BC = 12·6.



Base = 18·8
½ Perp. = 6·3
——
564
1128
——
118·44 area.

Or, Perp. = 12·6
½ Base = 9·4
——
504
1134
——
118·44 area.

The oblique angled triangle ABC being given, let fall a perpendicular from the angle at B on the base AC, and that perpendicular is the height of the triangle. The base AC being 15·6, and the perpendicular BD = 9, to find the area.



7·8 = half

7·8 = half the base.
9 = height of the angle.

70·2 = area.

By the Sliding Rule.

Set 1 on A to the length of the base on B, and opposite to half the length of the perpendicular, on A, you will have the content on B.

By Gunter,

The extent from 1 to half the length of the perpendicular will reach from the length of the base to the content.

In this place it may be proper to instruct the learner in one of the properties of a right angled triangle: viz. That the square of the longest side of a right angled triangle, usually called the hypothenuse, is equal to the sum of the squares of the two other sides, usually called the legs; which is of great use, for by this mean, any two sides of a right angled triangle being given, the other may be found by common Arithmetick. Thus, in the right angled triangle ABC, the base AC and perpendicular BC being given, the hypothenue AB may be found by extracting the square root of the sum of the squares of the base and perpendicular.

```
Base 18·8      Perp. 12·6        353·44 = square of the base.
     18·8            12·6        158·76 = square of the perp.
     ----            ----        ------
     1504             756         . . .
    1504             252         512·20(22·63 hypothenuse.
    188             126          4
    ------          ------       --
    353·44          158·76     42)112
                                  84
                                 ----
                              446)2820
                                  2676
                                  ----
                             4523)14400
                                  13569
                                  -----
                                    831
```

And, if the hypothenuse and one of the legs be given, the other may be found by subtracting the square of the given leg from the square of the hypothenuse.

There are some numbers, the sum of whose squares make a perfect square, of which sort are 3 and 4, whose squares, being added together, make 25, which is the square of 5: therefore, if the base of a triangle be 4, and the perpendicular 3, the hypothenuse will be 5; and if any of these numbers be multiplied by any other number, those products will be the sides of right angled triangles, as 6, 8, 10, and 15, 20, 25, &c. Thus artificers, when they set off the corner of a building, usually measure 6 feet on one side, and 8 feet on the other, then laying a 10 feet pole across, it makes the corner a true right angle.

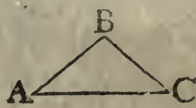
ART.

Art. 7. *There is another method of finding the area of triangles, the three sides being given.*

Rule.—Add the three sides together, then take the half of that sum, and out of it subtract each side severally; and multiply the half of the sum and these remainders continually, and the square root of this product will be the area of the triangle.

In the oblique triangle ABC, the base AC is given 15·6, the side AB is 10·4, and the side BC is 9·2, to find the area.

15·6	17·6	17·6	17·6
10·4	—15·6	—10·4	— 9·2
9·2	2·	7·2	8·4
35·2 sum.			
17·6 = half the sum.			



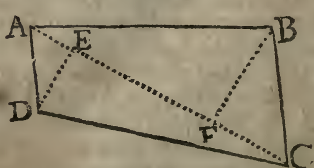
17·6	2128·8960(46·139 = area.
2	16
35·2	86)528
7·2	516
704	921)1289
2464	921
253·44	9223)36860
8·4	27669
101376	92269)919100
202752	830421
2128·896	88679

Art. 8. *To measure a Trapezium.*

Definition. A trapezium is an irregular figure of four unequal sides, and unequal angles.

Rule.—Draw a diagonal line from one of the angles to the opposite angle, as AC, and then will the trapezium be divided into two triangles, of which the diagonal is the common base: then, letting fall perpendiculars from the other opposite angles on the diagonal, add those perpendiculars together, and multiply half that sum into the diagonal, or half of the diagonal into the sum of the perpendiculars, and that product will be the area of the trapezium.

In the trapezium ABCD, the diagonal AC is 24, the perpendicular DE 6, and the perpendicular BF 10. The sum of the perpendiculars is 16, whose half is 8, which being multiplied into 24, will give the area.



$$\begin{array}{r} 24 \\ 8 \\ \hline 192 = \text{area.} \end{array}$$

By the Sliding Rule.

Set 1 on A to ½ the sum of the perpendiculars on B, and opposite the length of the diagonal on A, you will have the area on B.

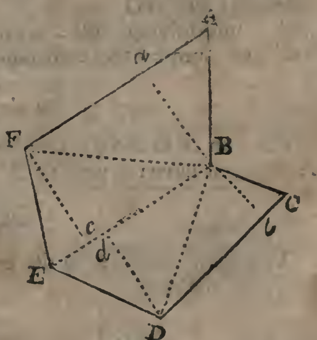
By Gunter.

The extent from 1 to ½ the sum of the perpendiculars will reach from the length of the diagonal to the area.

Art. 9. *To measure any irregular Figure.*

Rule.—Divide the figure into triangles, by drawing diagonals from one angle to another; then measure all the triangles by either of the rules, already taught, at Article 6 or 7, and the sum of the several areas of all the triangles will be the area of the given figure.

The irregular figure ABCDEF being given, divide it into triangles by the diagonals FB, EB, and DB: then may the triangles be measured by letting fall perpendiculars on their respective bases, as B*a*, B*b*, D*c*, F*d*, and multiplying those perpendiculars by half their respective bases.



In the triangle AFB the base FA is 100, and the perpendicular B*a* 49; in the triangle FBE the base BE is 92, and the perpendicular F*d* 52; in the triangle EBD, the base BE is the same as before, and the perpendicular D*c* 44; and in the triangle DCB, the base DC is 80, and the perpendicular B*b* 38; by which the area of each may be found by Art. 6, as follows.

$$\begin{array}{r} 50 = \text{half AF.} \\ 49 = \text{perp. } a\text{B.} \\ \hline 2450 = \text{area of AFB.} \end{array}$$

$$\begin{array}{r} 46 = \text{half BE.} \\ 44 = \text{perp. D}c\text{.} \\ \hline 184 \\ 184 \\ \hline 2024 = \text{area of EBD.} \end{array}$$

$$\begin{array}{r} 46 = \text{half BE.} \\ 52 = \text{perp. F}d\text{.} \\ \hline 92 \\ 230 \\ \hline 2392 = \text{area of FBE.} \end{array}$$

$$\begin{array}{r} 38 = \text{perp. B}b\text{.} \\ 40 = \text{half DC.} \\ \hline 1520 = \text{area of DCB.} \end{array}$$

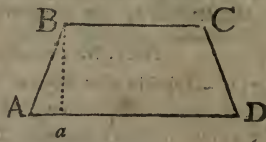
$$\begin{array}{r} 2450 \\ 2024 \\ 2392 \\ 1520 \\ \hline 8386 = \text{area of the figure ABCDEF.} \end{array}$$

In dividing any irregular figure into triangles, the triangles will be less, by two, and the diagonals less by three, than the number of the sides of the figure.

Art. 10. *To measure a Trapezoid.*

Definition. A trapezoid is the segment of a triangle, cut by a line parallel to the base.

Rule.—Add the parallel sides together, and multiply half that sum by the perpendicular breadth.



In the trapezoid ABCD, the side AD is 24, the side BC is 16, and the perpendic. breadth Ba is 10, to find the area by adding the sides BC and AD and multiplying half their sum by the perpendicular breadth Ba.

$$\begin{array}{r l} 24 & = AD \\ 16 & = BC \\ \hline 40 & = \text{sum.} \\ \hline 20 & = \tfrac{1}{2}\text{ sum.} \\ 10 & = Ba. \\ \hline 200 & = \text{area.} \end{array}$$

By the Sliding Rule.

Set 1 on A to the equated length on B, and against the breadth on A you will have the area on B.

By Gunter.

The extent from 1 to the breadth will reach from the equated length to the area.

Art. 11. *To measure any regular Polygon.*

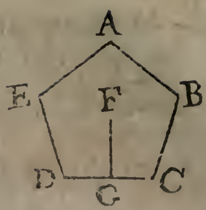
Definition. A regular polygon is a figure whose sides and angles are all equal; they are usually denominated from the number of their sides.

Thus, A figure having — equal sides and angles is a —

Sides	Name
3	Trigon.
4	Tetragon.
5	Pentagon.
6	Hexagon.
7	Heptagon.
8	Octagon.
9	Enneagon.
10	Decagon.
11	Endecagon.
12	Dodecagon.

Rule.—Multiply the length of one of the sides by the number of of sides; then, this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the sides, and the product will be the area of the polygon.

In



In the pentagon ABCDE, each side is 95, and the perpendicular FG 65·36, to find the area.

95 = length of a side.
5 = number of sides.

475 = sum of the sides.
32·68 = ½ of the perpendicular.

3800
2850
950
1425

15523·00 = area of the pentagon.

By the Sliding Rule.

Set 1 on A to ½ the perpendicnlar on B, and against the sum of the sides on A you will have the area on B.

By Gunter.

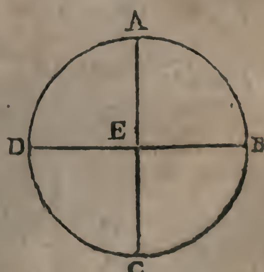
The extent from 1 to half the length of the perpendicular, will reach from the sum of the sides to the content.

But for the more ready measuring regular polygons, the following Table, containing multipliers for all regular figures from the triangle to the dodecagon, will be of use to the learner.

Number of ſides.	Names.	Multipliers.	Number of ſides.	Names.	Multipliers.
3	Trigon.	·433013	8	Octagon.	4·828427
4	Tetragon.	1·	9	Enneagon.	6·181827
5	Pentagon.	1·720477	10	Decagon.	7·694209
6	Hexagon.	2·589076	11	Endecagon.	9·361
7	Heptagon.	3·633959	12	Dodecagon.	11·196

If the square of the side of a polygon be multiplied by the multiplier of the like figure, the product will be the area of the figure sought.

To measure a Circle and its Parts.



In the annexed circle ABCD, the arch line ABCD is called the *periphery*, the length of which is called the *circumference:* Any line, as DB or AC, passing through the centre E, cuts the circle into two equal parts, called *semicircles*, or half circles; and such lines are called diameters of the circle: If two diameters be drawn through a circle, at right angles to each other, then, the four equal divisions of the circle are called *qurdrants:* half the diameter as EB, is called the *radius*, or semidiameter.

ART.

ART. 12. *The Diameter of a Circle being given, to find the Circumference.**

RULE.—This may be done by either of the following proportions in whole numbers, as 7 is to 22, or more exactly, as 113 is to 355; or in decimals, as 1 is to 3·14159; so is the diameter of a circle to the circumference.

EXAMP. A circle whose diameter is 12, to find the circumference.

As

* *Note.* 1. If the diameter of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}3\cdot14159,\ \text{the product}\\ \cdot31831,\ \text{the quotient}\end{matrix}\right\}$ is the circumference.

2. If the diameter of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot886227,\ \text{the product}\\ 1\cdot128379,\ \text{the quotient}\end{matrix}\right\}$ is the ſide of an equal ſquare.

3. If the diameter of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot866024,\ \text{the product}\\ \cdot1547,\ \text{the quotient}\end{matrix}\right\}$ is the ſide of the equilateral triangle inſcribed.

4. If the diameter of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot707016,\ \text{the product}\\ 1\cdot414213,\ \text{the quotient}\end{matrix}\right\}$ is the ſide of the ſquare inſcribed.

5. If the ſquare of the diameter of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot785398,\ \text{the product}\\ 1\cdot273241,\ \text{the quotient}\end{matrix}\right\}$ is the area.

6. If the circumference of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot31831,\ \text{the product}\\ 3\cdot14159,\ \text{the quotient}\end{matrix}\right\}$ is the diameter.

7. If the circumference of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot282094,\ \text{the product}\\ 3\cdot544907,\ \text{the quotient}\end{matrix}\right\}$ is the ſide of the ſquare equal.

8. If the circumference of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot2756646,\ \text{the product}\\ 3\cdot6275939,\ \text{the quotient}\end{matrix}\right\}$ is the ſide of the equilateral triangle inſcribed.

9. If the circumference of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot225079,\ \text{the product}\\ 4\cdot442877,\ \text{the quotient}\end{matrix}\right\}$ is the ſide of the ſquare inſcribed.

10. If the ſquare of the circumference of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}\cdot079577525,\ \text{the product}\\ 12\cdot56636217,\ \text{the quotient}\end{matrix}\right\}$ is the area.

11. If the area of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}1\cdot273241,\ \text{the product}\\ \cdot785398,\ \text{the quotient}\end{matrix}\right\}$ is the ſquare of the diameter.

12. If the area of any circle be $\left\{\begin{matrix}\text{multiplied}\\ \text{divided}\end{matrix}\right\}$ by $\left\{\begin{matrix}12\cdot56636217,\ \text{the product}\\ \cdot079577525,\ \text{the quotient}\end{matrix}\right\}$ is the ſquare of the circumference.

13. When the diameter of one circle is 1, and the diameter of another is 2, the circumference of the firſt is equal to the area of the ſecond, = 3·141592.

14. If the circumference be 4, the diameter and area are equal, = 1·273241.

15. If the diameter be 4, the circumference and area are equal, = 12·566368.

Hence, becauſe circles are the moſt capacious of all figures, if the *fourth* part of a circle be *ſquared*, it will not be equal to the *area* of that circle (as is commonly ſuppoſed) although the *four* ſides added together are equal to the *circumference* of that circle.

In a circle whoſe diameter is 24, circumference 75·4, and area 452·4, the *fourth* part of the circumference is 18·85, the *ſquare* of which is only 355·3225, that is, 97·0775 leſs than the truth: and the larger the circle is, the greater will the errour be.

For

```
As 7 : 22 :: 12
        12
      ----
  7)264(37·71 = circumference.
    21
    --
     54
     49
     --
      50
      49
      --
       10
        7
       --
        3
```

```
As 113 : 355 :: 12
          12
        ----
  113)4260(37·699 cir.
       339
       ----
        870
        791
        ----
         790
         678
         ----
         1120
         1017
         ----
          103
```

```
As 1 : 3·14159 :: 12
              12
       ----------
       37·69908 cir.
```

Note. 3·14159 may be contracted to 3·1416 without any sensible difference.

Art. 13. *The Circumference of a Circle being given, to find the Diameter.*

Rule.—As 22 is to 7; or 355 to 113; or as 1 to ·31831, so is the circumference of a circle to the diameter.

Examp. The circumference of a circle being 326, to find the diameter.

```
As 22 : 7 :: 326
               7
            ----
  22)2282(103·72 diam.
      22
      --
       82
       66
       --
       160
       154
       ---
         60
         44
         --
         16
```

```
355 : 113 :: 326
             326
            ----
             678
            226
           339
          ------
  355)36838(103·76 diam.
      355
      ----
       1338
       1065
       ----
        2730
        2485
        ----
         245
```

```
1 : ·31831 :: 326
              326
          -------
           190986
          63662
         95493
         -----------
         103·76906 = diameter. This proportion is the most accurate.
```

Art. 14. *To find the Area of a Circle.*

Rule.—Multiply half the diameter by half the circumference and the product is the area. If

For further proof of this matter: If a cylindrical pint, beer meaſure, whoſe content is 35·25 cubick inches, be beaten into a perfectly *ſquare form*, it will contain only 28·902 cubick inches, which is leſs than the truth by 6·348+; the area of the circle is 8·76158592̇88, and the area of the ſquare only 6·8813320[illegible]07662[illegible].

Hence appears the reaſon, why taking the *fourth part* of the girth in meaſuring a cylinder (or a round ſtick of timber) is falſe.

16. If the diameter of one circle be double to *that* of another, the *area* of the firſt circle will be *four times* the area of the ſecond.

If the diameter be given, find the circumference by Art. 12.

If the circumference be given, find the diameter by Art. 13.

EXAMP. A circle whose diameter is 12, and circumference is 37·7, given, to find the area?

18·85 = half the circumference.
6 = half the diameter.

113·1 = area of the given circle.

ART. 15. *The Diameter being given to find the Area of a Circle without finding the Circumference.*

RULE.—Multiply the square of the diameter by ·7854, and the product will be the area of the circle, whose diameter was given.

EXAMP. The diameter of a circle being 12, to find the area?

$$\begin{array}{r} \cdot 7854 \\ 12\times 12 = \quad 144 \\ \hline 31416 \\ 31416 \\ 7854 \\ \hline 113\cdot 0976 = \text{area.} \end{array}$$

By the Sliding Rule.

Set 1 on A to the diameter on B, then find ·7854 (which expresses the area of a circle whose diameter is 1) on A, against which on B is a 4th number, then find this 4th number on A, against which on B is the area.

By Gunter.

The extent from 1 to the length of the diameter reaches from ·7854 to a 4th number, and from that 4th number to the area.

ART. 16. *The Circumference of a Circle being given, to find the Area without finding the Diameter.*

RULE.—Multiply the square of the circumference by ·07958, and the product will be the area of the circle.

EXAMP. The circumference of a circle being 37·7, to find the area.

$$\begin{array}{r} 37\cdot 7 \\ 37\cdot 7 \\ \hline 2639 \\ 2639 \\ 1131 \\ \hline 1421\cdot 29 = \text{square.} \end{array} \qquad \begin{array}{r} 1421\cdot 29 \\ \cdot 07958 \\ \hline 1137032 \\ 710645 \\ 1279161 \\ 994903 \\ \hline 113\cdot 1062582 = \text{area of the circle.} \end{array}$$

ART. 17. *The Dimensions of any of the parts of a Circle being given, to find the side of a Square equal to the Circle.*

RULE.—If the area of the circle be given, extract the square root of the area, which will be the side of a square equal to the circle:

If

If the diameter or circumference be given, find the area by Art. 15 or 16, and then extract the square root, as before. And this is a *general rule* to find the side of a square equal to any superficial figure, regular or irregular: for the square root of the area of any figure whatever, is the side of a square equal to the given figure. But with regard to circles, if the diameter be given; multiply it by ·886, and the product will be the side of an equal square: or, as 13·545 is to 12, or 1354 to 1200: so is the diameter of a circle to the side of a square equal to the given circle. And, if the circumference be given, multiply it by ·282 for the side of an equal square. Or, divide it by 3·545, and the quotient will be the side of an equal square.

Examp. 1.

Let the diameter of a circle be 12, to find the side of a square equal to the circle?

$\cdot 886 \times 12 = 10\cdot 632 =$ side of the square.

Or, as 13·545 : 12 :: 12 : 10·631 = the side.

Examp. 2.

The circumference being 37·7 to find the side of an equal square?

$37\cdot 7 \times \cdot 282 = 10\cdot 631 =$ side of the square.

Or, $37\cdot 7 \div 3\cdot 545 = 10\cdot 634$.

Art. 18. *The Area of a Circle being given, to find the Diameter.*

Rule.—Multiply the given area by 1·2732, and the product will be the square of the diameter; then, extracting the square root of the product, you will have the diameter.

Examp. The area of a circle being 113·09, to find the diameter.

```
 1·2732          143·986188(11·999 = 12 = diameter.
 113·09          1
 ------          --
 114588        21)43
381960            21
12732             ---
12732          229)2298
--------           2061
143·986188         ----
              2389)23761
                   21501
                   -----
             23989)226088
                   215901
                   ------
                    10187 remainder.
```

Art. 19. *The Area of a Circle being given, to find the Circumference.*

Rule.—Multiply the given area by 12·566, and extract the square root of the product, which root will be the circumference required.

Examp. The area of a circle being 113·03 to find the circumference.

12·566

12·566	1420·3349(37·68 = circumference.
113·03	9
———	———
37698	67)520
376980	469
12566	———
12566	746)5133
———	4476
1420·33498	———
	7528)65749
	60224
	———
	5525 remainder.

ART. 20. *The Side of a Square being given, to find the Diameter of a Circle equal to the Square, whose Side is given.*

RULE.—Multiply the given side by 1·128, and the product will be the diameter of a circle, whose area is equal to the area of the given square. Or, if the side of the square be divided by ·886, the quotient will be the diameter. Or, as 12 to 13·54, so is the side of any square to the diameter of an equal circle.

EXAMP. The side of a square being 10·635, to find the diameter of a circle equal to that square?

10·635×1·128 = 12 nearly. Or, 10·635 ÷ ·886 = 12 = diameter.
Or, as 12 : 13·54 :: 10·635 : 12 nearly.

ART. 21. *The Side of a Square being given, to find the Circumference of a Circle equal to the given Square.*

RULE.—Multiply the given side by 3·545 and the product will be the circumference required. Or, divide it by 282, and the quotient will be the circumference.

EXAMP. The side of a square being 10·631, to find the circumference of a circle equal to that square.

10·631×3·545=37·686=circum. Or, ·282)10·631(37·698 circum.

ART. 22. *To find the Area of a Semicircle, the Diameter being given.*

RULE.—Find the area of the circle by Art. 15, and take the half of it.

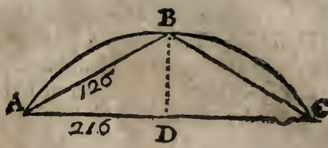
In the same manner may the area of a quadrant, or a quarter of a circle, be found, by taking a fourth part of the area of the whole circle.

But with regard to measuring a sector, or a segment of a circle, it will be necessary first to show how to find the length of the arch line of a sector, and the diameter of the circle to a given segment.

ART. 23. *A Segment of a Circle being given, to find the length of the Arch Line.*

RULE.—Divide the segment into two equal parts; then measure the chord of the half arch, from the double of which subtract the chord of the whole segment; and one third of that difference, being added to the double of the chord of the half arch, will give the length of the arch line.

EXAMP.



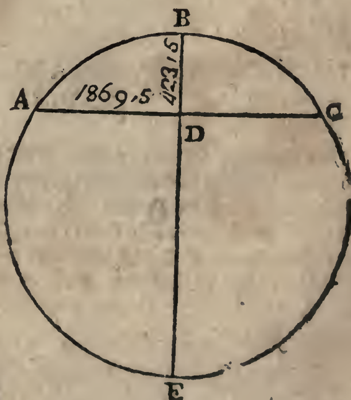
Examp. In the segment ABCD, the whole chord ADC is 216, and the chord AB or BC 126, to find the arch line ABC.

126 = chord AB or BC.
2
---
252 = double.
216 = ADC, to be subtracted.
---
3)36 = difference.
---
12 = $\frac{1}{3}$ difference.

252 = double of AB.
12 = $\frac{1}{3}$ difference added.
---
264 = length of the arch ABC.

Art. 24. *The Chord and versed Sine of a Segment being given, to find the Diameter of a Circle.*

Rule.—Multiply half the chord by itself, and divide the product by the versed sine; then add the quotient to the versed sine, and the sum will be the diameter of the circle.



Example. In the segment ABCD, the chord AC is 1869·5, and the versed sine BD 423·5, to find the diameter.

934·75 {half the chord AC
934·75

467375
654325
373900
280425
841275

423·5)873757·5625(2063·1 = DE.
8470

26757
25410

13475
12705

7706
4235

3471

423·5 = BD, add.

2486·6 = diameter BDE

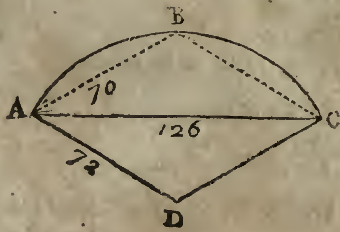
Art.

ART. 25. *To measure a Sector.*

Definition. A sector is a part of a circle, contained between an arch line, and two radii or semidiameters of the circle.

RULE.—Find the length of half the arch by Art. 23: Then multiply this by the radius or semidiameter, and the product will be the area.

EXAMP. 1. In the sector ABCD, given the radius AD or DC 72 feet, the chord AC = 126 feet, and the chord AB or BC = 70, to find the area of the sector.



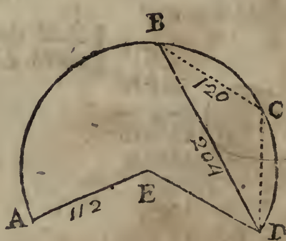
First.

70 = chord AB or BC.
2
———
140
126 = AC, subtract.
———
3)14
———
4·66
140
———
144·16 = length of the arch ABC, [by Art. 23.
———
72·33

Secondly.

72·33 = half the arch.
72 = radius.
———
14466
50631
———
5207·76 = area.

EXAMP. 2. In the sector ABCD, greater than a semicircle, given the radius AE or ED = 112, the chord BD (of half the arch ABD) = 204, and the chord BC (of half the arch BCD) = 120, to find the area of the sector.



120 = BC.
2
———
240
204 subtract.
———
3)36
———
12
240 Add.
———
252 = { Length of the arch BCD, by Art. 23.

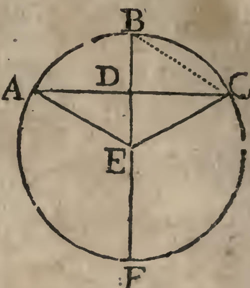
252 = half the arch ABD.
112 = radius.
———
504
252
252
———
28224 = area of the sector.

ART. 26. *To find the Area of a Segment of a Circle.*

Definition. A segment of a circle is any part of a circle cut off by a right line drawn across the circle, which does not pass through the centre, and is always greater or less than a semicircle.

EXAMP.

Examp. 1. To find the area of the segment ABC, whose chord AC is 172, the chord of half the arch ABC, viz. BC = 104, and the versed sine BD = 58·48.



Rule.—By Art. 23, find the length of the arch line ABC, and by Art. 24, the diameter FB; then multiply half the chord of the arch ABC by half the diameter, and the product will be the area of the sector ABCE: then find the area of the triangle AEC, whose base AC is 172, and perpendicular height 34, found by subtracting the versed sine BD from half the diameter; and the area of the triangle AEC, being subtracted from the area of the sector ABCE, will leave the area of the segment ABC.

104 = BC.
2
——
208
172 = AC, subtract.
——
3)36
——
12
208 add.
——
220 = arch line ABC.
——
110 = half arch.

92·475 = radius.
110
——
924750
92475
——
10172·25 = area of the sector.
86 = half the base = AD.
34 = perpendicular DE.
——
344
258
——
2924 = area of the triangle.

86 = half ADC.
86
——
516
688
——
58·48)7396·00(126·47 = DEF.
5848
——
15480
11696
——
37840
35088
——
27520
23392
——
41280
40936
——
344

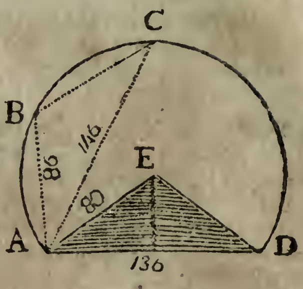
58·48 = BD, add.
——
184·95 = diameter BF.
——
92·475 = radius or semi-diameter.

10172·25 = area of the sector.
2924 = area of the triangle.
——
7248·25 = area of the segment.

Examp. 2. In the segment ABCD greater than a semicircle, given the chord of the whole segment AD = 136, the chord AC of half the

the arch ACD=146, the chord AB or BC one fourth of the arch ACD = 86, and the radius AE or ED = 80, to find the area of the segment ABCD.

First find the area of the sector ABCDE, by Art. 25, at the second Example; then find the area of the triangle AED, by Art. 6, and, adding the area of the triangle to the area of the sector, you will have the area of the segment.



86 = chord AB.
2
——
172
146 = chord AC, subtract.
——
3)26
——
8·666
172 = double of AB, add.
——
180·666 = arch line ABC.
80 = radius.
——
14453·280 = area of the sector.

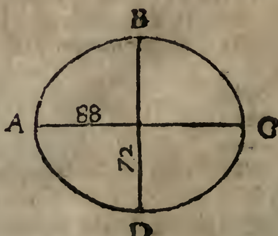
68 = half the base AD.
42 = perpendicular E 136.
——
136
272
——
2856 = area of the triangle AED.
14453·28 = area of the sector, add.
——
17309·28 = area of the segment.

Art. 27. *To find the Area of an Ellipsis.*

Definition. An ellipsis, or oval, is a curve which returns into itself like a circle, but has two diameters, one longer than the other, the longest of which is called the transverse, and the shortest the conjugate diameter.

Rule.—Multiply the two diameters of the ellipsis together; then multipling the product by ·7854, this last product will be the area of the ellipsis.

Examp. In the ellipsis ABCD, the transverse diameter AC is 88, and the conjugate diameter BD is 72, to find the area.



```
     88
     72
    ———
    176
    316
    ———
   6336
  ·7854
  —————
  25344
 31680
50688
44352
————————
4976·2944 = area.
```

The content is found by the sliding rule and Gunter, in the same way as the circle, only using the product of the two diameters as the square of the diameter of a circle.

Mensuration of Superficies is easily applied to *Surveying:* thus, take the angles of the plot with a good compass, then measure the sides with Gunter's chain, which note down in links (or chains and links, which is done by separating the two right hand figures of your links by a comma, your chain being 100 links) then cast up the contents, according to the rule of the figure, cutting off the five right hand figures of the product, and those at the left hand, if any, are acres; then multiply the five figures cut off, by 4, by 40, and by $272\frac{1}{4}$, cutting off as before, and those at the left hand, will be roods, poles and feet, respectively.

Section II. Of Solids.

Solids are measured by the solid inch, foot or yard, &c. 1728 of these inches, that is 12×12×12, make one cubick or solid foot.

The solid content of every body is found by rules adapted to their particular figures.

Art. 28. *To measure a Cube.**

Definition. A cube is a solid of six equal sides, each of which is an exact square.

The

* Here follows a Table of the Proportions, which the following *Solids* have to the *Cube* and *Cylinder*, having the same *Base* and *Altitude*.

	Solid Inches.
1. A *Cube* whose side is 12 inches, contains - - -	1728
2. A *Prism*, having an equilateral triangle, whose side is 12 inches from its *Base*, and its *Altitude* 12 inches, contains - -	784·24
3. A *Square Pyramid*, whose height and the side of its base, are each 12 inches, is $\frac{1}{3}$ of the above cube, and therefore contains	576
4. A *Triangular Pyramid*, whose height and side of its triangular base are each 12 inches, is near $\frac{1}{7}$ of the cube and contains -	249·413
5. A *Cylinder*, whose diameter and height are each 12 inches, is $\frac{11}{14}$ of the above cube, and contains - - -	1357·17
6. A *Sphere* or *Globe*, whose axis or diameter is 12 inches, equal to the side of the cube, is $\frac{11}{21}$ of it, and contains - -	904·78
7. A *Cone*, whose base and altitude are each 12 inches, equal to the side of the cube, is $\frac{5}{19}$ of it, and contains - -	452·38829

8. A

The solid foot is composed of 1728 inches; for a solid, that is 1 foot, or 12 inches every way, that is 12×12×12, contains 1728 inches.

RULE.

	Solid Inches.
8. A *Parabolick Conoid*, whose diameter at the base and height, are each 12 inches, being $\frac{1}{2}$ its circumscribing cylinder, contains -	678·583
9. A *Hyperbolick Conoid*, whose height, and diameter at the base, are each 12 inches, is $\frac{5}{12}$ of its circumscribing cylinder, and contains	565·49
10. A *Parabolick Spindle*, whose height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumscribing cylinder, and contains	723·824

Hence arises a different method of finding their contents.

General Rule. If the base of the solid, whose contents you would find, be rectilinear, consider it as *Parallelopipedon*; if curved, as a *Cylinder*, and find the content accordingly: then take such a part of the content, thus found, as is specified in the preceding Table, which if the parts be taken in inches, will be the solid content of the given figure, in inches, which, divided by 1728, will give the cubick feet.

EXAMP. 1. There is a triangular prism, the side of whose base is 48 inches, and whose perpendicular height is 108 inches: what is its solid content?

The base being right lined, I consider it as a *parallelopipedon*, the side of whose base is 48 inches, and whose length is 108 inches, and as 784·24 is contained 2·20340712 times in a cubick foot; 2·20340712 is a divisor, to divide the content of the parallelopipedon by; therefore 48×48×108÷2·20340712=112930·56 solid inches = 65·353 solid feet.

Had the dimensions been given in feet, it would have been 4×4×9÷2·20340712 =65·353 feet.

EXAMP. 2. There is a square pyramid, whose height is 12 feet, and the side of whose base is 3·5 feet; what is its content?

3·5×3·5×12÷3=49 feet, Ans.

EXAMP. 3. There is a triangular pyramid, whose height is 15 feet, and the side of whose base is 5 feet: what is its content?

5×5×15÷7=53·57 feet, Ans.

EXAMP. 4. There is a cylinder whose diameter is 2·5 feet, and whose length is 24 feet; what is its content?

Here, the diameter is to be considered as the side of the base of a parallelopipedon. Therefore, 2·5×2·5×24×11÷14=117·857 feet, Ans.

EXAMP. 5. There is a spherical balloon, whose diameter is 50 feet; how many cubick feet of air does it contain?

Here, the diameter is to be considered as the side of a cube. Therefore,

50×50×50×11÷21=65476·19 feet, Ans.

EXAMP. 6. There is a cone, whose height is 15 feet, and the diameter of whose base is 5 feet; what is its content?

Here, the diameter of the base is to be considered as the side of the base of a parallelopipedon, and its height, as the length. Therefore,

5×5×15×5÷19=98·684 feet, Ans.

EXAMP. 7. There is a parabolick conoid, whose diameter at the base is 2·9 feet, and whose height is 6 feet; what is the content?

This solid being $\frac{1}{2}$ of a cylinder; we must first find the content as of that of a cylinder, and then halve it. Therefore,

2·9×2·9×6×11÷14=39·647, and 39·647÷2=19·823, Ans.

EXAMP. 8. There is a hyperbolick conoid, whose diameter at the base is 2·9 feet, and whose height is 6 feet; what is the content?

First, find the content of a cylinder,

2·9×2·9×6×11÷14=39·647, and 39·647×$\frac{5}{12}$=16·519 feet, Ans.

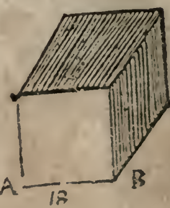
EXAMP. 9. There is a parabolick spindle, whose middle diameter is 2·9 feet and whose length is 6 feet; required the content?

First, find the content of a cylinder.

2·9×2·9×6×11÷14=39·647, and 39·647×$\frac{8}{15}$=21·145 feet, Ans.

RULE.—Multiply the side by itself and that product by the same side, and this last product will be the solid content of the cube.

EXAMP. The side of a cube AB, being 18 inches, or 1 foot and 6 inches, to find the content?



1 foot 6 inches = 1·5 foot.	18 inches.
1·5	18
75	144
15	18
2·25	324
1·5	18
1125	2592
225	324
3·375	1728)5832(3·375
	5184
	6480
	5184
	12960
	12096
	8640
	8640

In this operation, the inches are changed into the decimal parts of a foot.

I have done this two different ways, that the learner may see they come out the same. The content in inches is 5832, which being divided by 1728, the inches in a solid foot, and the division continued by annexing cyphers, it comes out the same as the decimal operation.

Note. The area of the surface, or superficial content of the cube and parallelopipedon is found by adding the areas of the several quadrilateral figures which compose them.

ART. 29. *To measure a Parallelopipedon.*

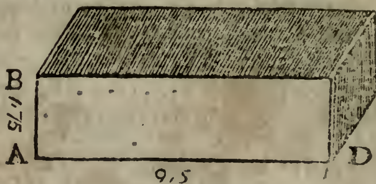
Definition. A parallelopipedon is a solid of three dimensions, length, breadth and thickness; as a piece of timber exactly squared, whose length is more than the breadth and thickness. The ends are called bases, which are equal.

RULE.—Find the area of the base, then multiply that by the length, and it will give the solid content.

EXAMP. 1. The side AB is 1·75 foot, and the length AD 9·5 feet, to find the solid content?

1·75 =

1·75 = 1 foot, 9 inches.
1·75
———
875
1225
175
———
3·0625 = area of base.
9·5
———
153125
275625
———
29·09375 = solid content.



Examp. 2. A vessel 3·5 feet each side within, and 5 feet deep, to find the content?

3·5
3·5
———
175
105
———
12·25
5
———
61·25 = the content.

If a piece of timber, or any other thing, be of an equal bigness through its whole length, though there be a difference between the breadth and thickness, if the breadth and thickness are multiplied together, and that product multiplied by the length, this last product will be the solid content.

Examp. 3. A piece of timber being 1 foot and 6 inches, or 18 inches broad, 9 inches thick, and 9 feet 6 inches, or 114 inches long, to find the content?

1 foot, 6 inches = 1·5 foot
9 inches = ·75 foot.
———
75
105
———
1·125
9 feet 6 inches = 9·5
———
5625
10125
———
10·6875 = content.

In this operation the inches are changed into the decimal fractions of a foot.

Breadth = 18 inches.
Depth = 9 inches.
———
162
Length = 114 inches.
———
648
162
162
———
1728)18468(10·6875 = content, as before.
1728
———
11880
10368
———
15120
13824
———
12960
12096
———
8640
8640

Note.

Note. When the end is given in inches and the length in feet, find the area at the end in inches, multiply that by the length in feet, and divide this product by 144 (the square inches in a foot) and the quotient will be the feet.

Take the last example.

```
 Foot.
 1·5  = 18 inches.
  ·75 =  9 inches.
         ---
        162 area in inches,
        9·5 feet = length.
        ---
        810
       1458
       ----
144)1539(10·6875 = content.
    144
    ----
      990
      864
      ----
      1260
      1152
      ----
       1080
       1008
       ----
         720
         720
```

By the Sliding Rule.

Set 12 inches on the girt line D to the side of the square end on C, then, against the length on D, you will have the answer on C.

By Gunter.

Extend the compasses from 12 inches to the length of the side of the square end; that distance, twice turned over from the length, will reach to the content.

When the side of a square solid is given, in inches, to find how much in length will make a foot solid.

Rule.—As the given side is to 12, so is 12 to a fourth number, and so is that fourth number to its required length. Or divide 1728 by the area at the end, and the quotient will be the length making a solid foot.

If the given side is in foot measure, then,

Rule.—As the given side is to 1; so is 1 to a fourth number, and so is that fourth number to the required length.

When two sides of an equal square solid (that is, of unequal breadth) are given, to find what length will make any number of solid feet.

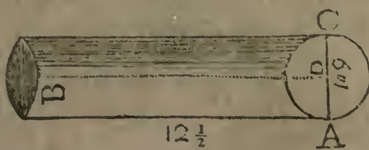
Rule.—Multiply the proposed number of feet by 144: divide that product by the product of the breadth and depth, and the quotient will be the length required.

Art. 30. *To measure a Cylinder.*

Definition. A cylinder is a round body, whose bases are circles, like a round column, or a rolling stone of a garden.

Rule.—The diameter of the base being given, find the area of the end by Art. 15, then, multiplying the area of the base by the length, that product will be the content of the cylinder.

Examp. The diameter of the base AC being 1 foot and 9 inches, and the length BD 12 feet and 6 inches, to find the content.




```
   1·75 = diam. of the base.
   1·75
   ----
    875
   1225
   175
  ------
  3·0625
   ·7854
  -------
  122500
 153125
 245000
214375
---------
2·40528750 = area of the base.
```

```
2·405 = area of the base.
 12·5 = length.
------
12025
4810
2405
------
30'0625 = content.
```

If the square of the diameter of a cylinder be multiplied by ·7854, and the solidity divided by that product, the quotient will be the length.

The learner may, for his practice, reduce all the dimensions to inches, and find the solid content in inches, which being divided by 1728, the quotient will be the solid content in feet: or, if he finds the area at the end in inches, and multiplies that by the length in feet, and divides by 144; the quotient will be feet.

This is a general rule for finding the content of any straight solid body, of equal bigness from end to end, of whatever form the bases are: for, if the area of the base be multiplied by the length, the product will be the solid content.

By the Sliding Rule.

Set 13·5, the square root of 183·34 (which is a gauge point arising from the division of 144 by ·7854) found on D, to the diameter found on C, and opposite to the length, on D, you will find the content on C.

Or,

Or, as 42·54 is to the circumference ; so is the length in feet to a fourth number, and so is that fourth number to the answer.

Note. The superficial content of a cylinder is found by multiplying the circumference of one of the bases into the length, and to the product adding the areas of the two bases, or ends.

When the diameter is given in inches, to find what length will make a solid foot.

Rule.—As the given diameter is to 13·531 : so is 12 to a fourth number, and so is that fourth number to the required length. If the diameter be given in foot measure : *Rule*, as the given diameter is to 1·128 : so is 1 to a fourth number, and so is that fourth number to the required length. Or, divide 1728 by the area at the end in inches, and the quotient will be the required length.

To find how much a Cylindrick or round Tree, that is equally thick from end to end, will hew to, when made square.

Rule.—Multiply twice the square of its semidiameter by the length, then divide the product by 144, and the quotient will be the answer.

If the diameter of a round stick of timber be 24 inches from end to end, and its length 20 feet : how many solid feet will it contain, when hewn square ; and what will be the content of the slabs which reduce it to a square ?

$$\frac{12\times12\times2\times20}{144} = 40 \text{ feet, the solidity when hewn square.}$$

$$\frac{24\times24\times\cdot7854\times20}{144} = 62\cdot8 \text{ feet, or } 2\times2\times\cdot7854\times20 = 62\cdot8 \text{ the total}$$

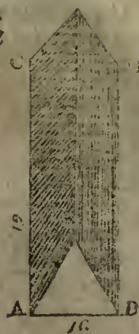
solidity, whence 62·8—40 = 22·8 feet, the solidity of the slabs.

Art. 31. *To measure a Prism.*

Definition. A prism is a body with two equal or parallel ends, either square, triangular, or polygonal, and three or more sides, which meet in parallel lines, running from the several angles at one end, to those of the other.

Rule.—Prisms of all kinds, whether square, triangular or polygonal, are measured by one general rule, viz. Find the superficial content, or area at the base (or end) by the proper rule of Sect. 1. and this multiplied by the length, or height of the prism, will give the solid content.

Examp.



Examp. The side of a stick of timber, AB, hewn three square, is 10 inches, and the length, AC, is 12 feet, to find the content?

Side = 10 inches.
½ Perpendicular = 4·33 inches.

43·3 = area at the end.
12 feet = length.

144)519·6(3·6 feet, content.
432

876
864

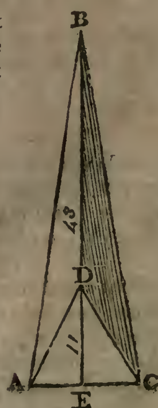
12

Note. The superficial content is found by adding the areas of the several quadrilateral and triangular figures which compose it.

Art. 32. *To measure a Pyramid.*

Definition. Solids, which decrease gradually from the base till they come to a point, are generally called pyramids, and are of different kinds, according to the figure of their bases; thus, if it has a square base, it is called a *square pyramid:* if a triangular base, a *triangular pyramid:* If the base be a circle, a *circular pyramid*, or simply a *cone.* The point, in which the top of a pyramid ends, is called a *Vertex*, and a line drawn from the vertex, perpendicular to the base, is called the height of the pyramid.

Rule—Find the area of the base, whether triangular, square, polygonal or circular, by the rules in superficial measure: then, multiply this area by one third of the height, and the product will be the solid content of the pyramid.



Examp. 1. In a triangular pyramid, the height BE, being 48, and each side of the base 13: the base being a triangle, let the perpendicular height DE be 11; to find the content.

5·5 = half ED.
13 = base AC.

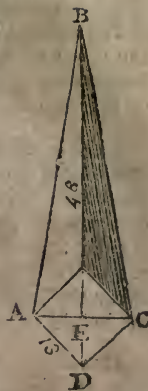
165
55

71·5 = area of the base.
16 = $\frac{1}{3}$ of the height EB.

4290
715

1144·0 = content.

EXAMP. 2. In a quadrangular pyramid, the height BE being 48, and each side of the base 13, to find the content.



$$\begin{array}{r} 13 \\ 13 \\ \hline 39 \\ 13 \\ \hline 169 \\ 16 \\ \hline 1014 \\ 169 \\ \hline 2704 \end{array}$$

169 = area of the base.
16 = $\frac{1}{3}$ of the height EB.
2704 = content.

EXAMP. 3. *To measure a Cone.*—The diameter AC being 13, and the height BD 48, to find the content.

$$\begin{array}{r} 13 \\ 13 \\ \hline 39 \\ 13 \\ \hline 169 \\ \cdot 7854 \\ \hline 676 \\ 845 \\ 1352 \\ 1183 \\ \hline 132\cdot 7326 \\ 16 \\ \hline 7963956 \\ 1327326 \\ \hline 2123\cdot 7216 \end{array}$$

132·7326 = area of the base.
16 = $\frac{1}{3}$ of the height.
2123·7216 = content,

Note. The superficial content of all pyramids is found by taking the sum of the several areas, which compose them. That of a cone, by multiplying the circumference of the base into half the line joining the vertex and any point in that circumference, and adding the area of the base to the product.

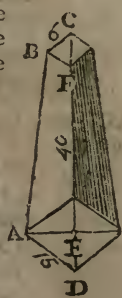
ART.

Art. 33. *To measure the Frustum of a Pyramid.*

Definition. The frustum of a pyramid is what remains after the top is cut off by a plane parallel to the base, and is in the form of a log greater at one end than the other, whether round, or hewn three or four square, &c.

Rule.—If it be the frustum of a square pyramid, multiply the side of the greater base by the side of the less; to this product add one third of the square of the difference of the sides, and the *sum* will be the mean area between the bases; but if the base be any other regular figure, multiply this *sum* by the proper multiplier of its figure in the Table, Art. 11. and the product will be the mean area between the bases: lastly, multiply this by the height, and it will give the height of the frustum.

Examp. 1. In the frustum of a square pyramid the side of the greater base AD = 15, the side of the less, BC = 6, and the height EF = 40, to find the content.



	15 = AD.	15	
	6 = BC.	6	
Prod. =	90	9	= difference.
Add	27	9	
	117	3)81	= square of the difference.
×	40		
		27	= $\frac{1}{3}$ of the square.
	4680 = content.		

Or, if it be a tapering square stick of timber, take the girth of it in the middle; square $\frac{1}{4}$ of the girth (or multiply it by itself in inches) then say, as 144 (inches) to that product; so is the length, taken in feet, to the content in feet.

Examp. 2. What is the content of a tapering square stick of timber, whose side of the largest end is 12 inches, of the least end, 8, and whose length is thirty feet.

One fourth of the girth in the middle = 10, and 10×10 = 100, the area in the middle; then, as 144 : 100 :: 30 feet : 20·83 feet the content.

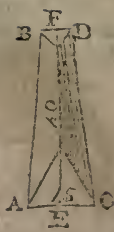
By the Sliding Rule.

Set 12 on D to $\frac{1}{4}$ of the circumference on C, and against the length on D is the answer on C.

By Gunter.

The extent from 12 to $\frac{1}{4}$ of the circumference doubled, or twice turned over, will reach from the length to the content.

Examp.



EXAMP. 3. In the frustum of a triangular pyramid, the side of the greater base AC = 15, as before, the side of the less BD = 6, and the height EF = 40, to find the content.

15 = AC. 15
6 = BD. 6
— —
9 = difference of the sides. 90
9 Add 27
— —
3)81 = square of the difference. 117
— ·433 multiplier.
27 = $\frac{1}{3}$ of the square. —
351
351
468
———
50·661 = mean area.
40 = height.
———
2026·440 = content.

Or, if it be a tapering three square stick of timber, you may find the area midway from end to end, then, as 144 is to that area, so is the length, taken in feet, to the content in feet.

EXAMP. 4. *To measure the Frustum of a Cone.*

RULE.—Multiply the diameters of the two bases together, and to the product add one third of the square of the difference of the diameters: then multiplying this sum by ·7854, it will be the mean area between the two bases, which being multiplied by the length of the frustum, will give the solid content.

Or, to the areas of the top and bottom add the square root of the product of those areas, and the sum, multiplied by *one third* of the height of the frustum, will give the solidity.

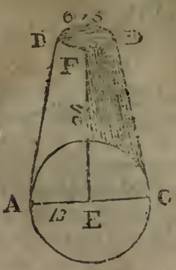
When figures run uniformly taper; but not to a point (they being considered as portions of the cone or pyramid) we may find the solidity by supplying what is wanting to complete the figure, and then deducting the part cut off.

A general rule for completing every straight sided solid, whose ends are parallel and similar.

As the difference of the top and bottom diameters is to the perpendicular height, (or depth which is the same:) so is the longest diameter to the altitude of the whole cone or pyramid.

The

The former cone in Art. 32, Examp. 3, being cut off in the middle, the greater diameter AC is 13, the less BD $6\frac{1}{2}$, and height EF 24, to find the content of the frustum.



AC = 13 inches.
BD = 6·5 inches.

65
78

84·5
Add 14·083

98·583
·7854

394332
492915
788664
690081

77·427|0882 = mean area.
24 feet = length.

309708
154854

1858·248 = content.

13
6·5

6·5 = difference.
6·5

325
390

3)42·25 = {square of the differ.

14·083 = $\frac{1}{3}$ of the square.

144)1858·248(12·9045 feet, content.
144

418
288

1302
1296

648
576

720
720

. . .

Art. 34. *To measure a Sphere or Globe.*

Definition. A sphere or globe is a round solid body, in the middle of which is a point, from which all lines drawn to the surface are equal.

Rule.—Multiply the cube of the diameter by ·5236, and the product will be the solid content.

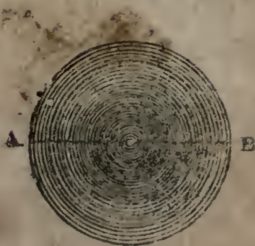
Or, multiply the circumference by the diameter, which will give the superficial content; then multiply the surface by *one sixth* of the diameter, and it will give the solidity.

Or, multiply the cube of the diameter by 11, and the product divided by 21, will give the solidity.

Examp. The diameter, AB, of a globe, is 4·5 feet; to find the solid content.

4·5

```
       4·5
       4·5
      ----
       225
      180
      ----
      20·25
       4·5
     ------
     10125
     8100
     ------
     91·125
      ·5236
     ------
     546750
    273375
   182250
  455625
  ---------
  47·7130500
```



Note. If the circumference, or greatest circle of the sphere, be given, multiply the cube of it by ·016887 for the content.

The surface of the globe may be found by multiplying the square of the diameter by 3·1416; or by multiplying the area of its greatest circle by 4, or the square of the circumference by ·3183.

When the solidity of a globe is given, the diameter may be found by dividing the solidity by ·5236, and extracting the cube root of the quotient.

Or, if the circumference be required, divide the solidity by ·016887, and the cube root of the quotient will give it.

ART. 35. *To measure the Solidity of a Frustum or Segment of a Globe.*

Definition. The frustrum of a globe is any part cut off by a plane.

RULE.—To three times the square of the semidiameter of the base, add the square of the height; then multiplying that sum by the height, and the product by ·5236, you will have the solid content.

EXAMP. The height BD being 9 inches, and the diameter of the base AC 24 inches: to find the content.

```
      12 = semidiameter.          4617
      12                          ·5236
      ---                         -----
     144 = square.               27702
   ×   3                        13851
      ---                       9234
     432                       23085
Add 9×9=81 = { square of       ---------
             { the hht.        2417·4612 = solid content.
      ---
     513
   ×   9 = height.
      ---
    4617
```

To

To measure the Surface of a Frustum or Segment of a Globe.

RULE.—Find the diameter of the globe by Art. 24, and the surface of the whole globe, by Art. 34; then, as the diameter of the globe is to the height of the frustum; so is the surface of the globe to the surface of the frustum; then, by Art. 15, find the area of the base; add these two together, and the sum will be the whole surface of the frustum.

ART. 36. *To measure the middle Zone of a Globe.*

Definition. This part of a globe is somewhat like a cask, two equal segments being wanting, one on each side of the axis.

RULE.—To twice the square of the middle diameter, add the square of the end diameter; multiply that sum by ·7854, and that product, multiplied by *one third* of the length, will give the solidity.

Or, To four times the square of the middle diameter add twice the square of the end diameter; that sum multiplied by ·7854, and that product by *one sixth* of the length, will give the solidity.

Note. This rule is applicable to the frustrum of a cone or pyramid.

If the middle diameter of a zone be 20 inches. the end diameters each 16 inches, and length 12 inches: Required its solidity?

$$\overline{20\times20\times2+16\times16}\times{\cdot}7854\times4=3317{\cdot}5296 \text{ Ans.}$$

ART. 37. *To measure a Spheroid.*

Definition. A spheroid is a solid body like an egg, only both its ends are the same.

RULE.—Multiply the square of the diameter of the greatest circle, viz. the diameter of the middle (DB in the figure) by the length AC, and that product by ·5236, and you will have the solidity.

EXAMP. The diameter BD being 20, and the length AC 30, to find the content.

$$20\times20\times30\times{\cdot}5236=6283{\cdot}2 \text{ Ans.}$$

ART. 38. *To measure the middle Frustum of the Spheroid.*

Definition. This is a cask like solid, wanting two equal segments to complete the spheroid.

RULE.—The same as in Article 36.

If the middle and end diameters of the middle frustum of a spheroid be 40 and 30 inches, and its length 50; what is its solidity?

$$50\div3=16{\cdot}6, \text{ then } \overline{40\times40\times2+30\times30}\times{\cdot}7854\times16{\cdot}6=53454{\cdot}324 \text{ Ans.}$$

ART. 39. *To measure a Segment, or Frustum, of a Spheroid.*

Definition. This is a part of a spheroid made by a plane, parallel to its greatest circular diameter.

RULE.—To four times the square of the middle diameter add the square of the base diameter, then multiply that sum by ·7854, and the product by *one sixth* of the altitude, and it will give the solidity.

If

If the base diameter of the end frustrum of a spheroid be 36, diameter at the middle of the height 30, and the height 20 inches: Required its solidity ?

$$\overline{30\times30\times4+36\times36}\times\cdot7854\times3\cdot3=12689\cdot55+,\ \text{Ans.}$$

Art. 40. *To measure a Parabolick Conoid.*

Definition. This solid may be generated by turning a semiparabola about its abscissa or altitude.

Rule.—As a parabolick conoid is half of its circumscribing cylinder, of the same base and altitude; multiply the area of the base by half the height for the solidity.

If the diameter of the base of a parabolick conoid be 40 inches, and its height 42; what is the solidity?

$$40\times40\times\cdot7854\times21=26389\cdot44\ \text{Ans.}$$

Art. 41. *To measure the lower Frustum of a Parabolick Conoid.*

Definition. This solid is made by a plane passing through the conoid, parallel to its base.

Rule.—Multiply the sum of the squares of the diameters of the bases by ·7854, and that product by half the height, for the solidity.

If the diameters of a frustum of a parabolick conoid be 40 and 30 inches, and its height 20 inches; required its solidity.

$$\overline{40\times40+30\times30}\times\cdot7854\times10=19635,\ \text{Ans.}$$

Art. 42. *To measure a Parabolick Spindle.*

Definition. This solid is formed by an obtuse parabola, turned about its greatest ordinate.

Rule.—This solid being eight fifteenths of its least circumscribing cylinder, multiply the area of its middle or greatest diameter by eight fifteenths of its perpendicular length, and it will give its solidity.

If the diameter at the middle of a parabolick spindle be 20 inches, and its length 60; required its solidity.

$$20\times20\times\cdot7854\times32\ (=60\times8\div15)=10053\cdot12\ \text{Ans.}$$

Art. 43. *To measure the middle Zone, or middle Frustum, of a Parabolick Spindle.*

Definition. This is a cask like solid, wanting two equal ends of said spindle.

Rule.—To the sum and half sum of the squares of the two diameters add three tenths of the difference of their squares, which multiply by a third of the length, and the product will be the solidity.

If the middle and end diameters of the middle frustum of a parabolick spindle be 40 and 30 inches, and its length 60; what is its solidity?

$40\times40=1600$ $\quad 1600-900=700$ the difference of the squares.
$30\times30=\ \ 900$ $\quad 700\times\cdot3=210=$ three tenths of do. then,

Sum $=2500$ $\quad 2500+1250+210\times20\ (=\frac{1}{3}$ of $60)=79200$ Ans.
Half sum $=1250$

Art.

Art. 44. *To measure a Cylinderoid, or Prismoid.*

Definition. A cylinderoid is a solid somewhat like the frustum of a cone, one base may be an ellipsis, and the other a disproportional ellipsis or circle.

A prismoid is a solid somewhat like the frustum of a pyramid, but its bases are disproportional.

Rule.—The same as for the frustum of a cone or pyramid: or, to the areas of both bases, add a mean area, that is, the square root of the product of the two bases, then multiply that sum by a third of the height or length, and it will give the solidity.

If the diameters of the greater base of a cylinderoid be 30 and 20 inches, the diameter of the less base 12, and length 60 inches; what is the solidity.

$$\left.\begin{array}{r} 30\times20=600 \\ 12\times12=144 \\ \sqrt{144\times600}=293\cdot9 \\ \hline 1037\cdot9 \end{array}\right\} \begin{array}{c} 1037\cdot9\times\cdot7854\times20\ (=60\div3)= \\ 16303\cdot33 \text{ Ans.} \end{array}$$

If the diameters of the greater base of a prismoid be 30 and 20 inches, the less base 20 by 10 inches, and length 30 inches: What is its solidity?

$$\left.\begin{array}{r} 30\times20=600 \\ 20\times10=200 \\ \sqrt{600\times200}=346\cdot4 \\ \hline 1146\cdot4 \end{array}\right\} \begin{array}{c} 1146\cdot4\times10\ (=30\div3)=11464 \text{ solidity} \\ \text{in inches.} \end{array}$$

Art. 45. *To measure a Solid Ring.*

Rule.—Measure the internal diameter of the ring, and its girth, or circumference: then multiply the girth by ·31831, and the product will be the diameter of the wire, which add to the internal diameter; multiply this sum by 3·1416, and the product will be the length of a cylinder equal to the ring of the same base. Then the area of a section of the ring multiplied by the length of the said cylinder will give the solidity of the ring.

If an iron ring be 12 inches in girth, and its internal diameter be 20 inches; what is its solidity?

$\cdot31831\times12=3\cdot8=$ring's diameter. $\overline{20+3\cdot8}\times3\cdot1416=74\cdot77$ the length of a cylinder equal to the ring: And

$3\cdot8\times3\cdot8\times\cdot7854\times74\cdot77=847\cdot97=$solidity.

Art. 46. *To measure the Solidity of any irregular Body, whose dimensions cannot be taken.*

Take any regular vessel, either square or round, and put the irregular body into it: pour so much water into the vessel as will exactly cover the body, and measure the dry part from the top of the vessel to the water, then take out the body, and measure again from the top of the vessel to the water, and subtract the first measure from the second, and the difference is the fall of the water: then, if the vessel be square, multiply the side by itself, and that product by the fall of the water, and you will have the content of the body; but if it be a long square, multiply the length by the breadth, and that product by the fall

of

of the water; or, lastly, if it be a round vessel, multiply the square of the diameter by ·7854, and that product by the fall of the water, and you will have the content.

Exam. 1. A body being put into a vessel 18 inches square, on taking out the body, the water sunk 9 inches; required the content of the body?

18 inch. = 1·5 foot.
9 inch. = ·75 foot.
1·5 × 1·5 × ·75 = 1·6875 foot, content.

Exam. 2. A body put into a cistern 4 feet by 3, on taking it out, the water fell 6 inches; required the content of the body?

4×3×·5 = 6 feet, content.

Exam. 3. A body being put into a round tub, whose diameter was 1·5 foot, on taking out the body, the water fell 1·5 foot; what was the content of the body?

1·5 × 1·5 × ·7854×1·5 = 2·65 feet, content.

Of the five Regular Bodies.

There are five solids contained under equal regular sides, which by way of distinction, are called *the five regular bodies.*

These are the *Tetraedron,* the *Hexaedron* or *Cube,* the *Octaedron,* the *Dodecaedron,* and the *Eicosiedron.* The measuring of the cube was shewn at Art. 28. I shall now show how to measure the other four by the following Table, which is the shortest method.

A Table of the solid and superficial content of each of the five bodies, the sides being unity, or 1.

Names of the Bodies.	Solidity.	Superficies.
Tetraedron.	0·11785	1·73205
Hexaedron.	1·	6·
Octaedron.	0·4714	3·464
Eicosiedron.	2·181695	8·66025
Dodecaedron.	7·663119	20·6457

All like solid bodies being in proportion to one another as the cubes of their like sides, the solid content of any of these bodies may be found by multiplying the cubes of their sides by the numbers in the second column under Solidity; and their superficies, by multiplying the squares of their sides into the numbers in the third column, under Superficies.

Of the Tetraedron.

This solid is contained under four equal and equilateral triangles, that is, it is a triangular pyramid of four equal faces, the side of whose base is equal to the slant height of the pyramid, from the angles to the vertex.

Art. 47. *The side of the Tetraedron being 3, to find the solid and superficial content.*

Cube = 3×3×3 = 27, and 27×·11785 = 3·18195 = solidity.
Square = 3×3 = 9, and 9×1·73205 = 15·58845 = superficies.

Of the Octaedron.

This solid is contained under eight equal and equilateral triangles, which may be conceived to consist of two quadrangular pyramids of equal bases joined together, the sides of whose bases are equal to the given sides of the triangles, under which it is contained.

Art.

ART. 48. *The side of an Octaedron being 3, to find the solid and superficial content.*

Cube = $3 \times 3 \times 3 = 27$, and $27 \times \cdot 4714 = 12 \cdot 7278$ = solidity.
Square = $3 \times 3 = 9$, and $9 \times 3 \cdot 464 = 31 \cdot 176$ = superficies.

OF THE DODECAEDRON.

This solid is contained under 12 equilateral pentagons, and may be conceived to consist of twelve pentagonal pyramids, of equal bases and altitude, whose vertices meet in the centre of the dodecaedron.

ART. 49. *The side of a Dodecaedron being 3, to find the solid and superficial content.*

Cube = $3 \times 3 \times 3 = 27$, and $27 \times 7 \cdot 663119 = 206 \cdot 904$.
Square = $3 \times 3 = 9$, and $9 \times 20 \cdot 6457 = 185 \cdot 8113$.

OF THE EICOSIEDRON.

This solid is contained under twenty equal and equilateral triangles, and may be conceived to consist of twenty equal triangular pyramids, whose vertices all meet in the centre.

ART. 50. *The side of an Eicosiedron being 3, to find the solid and superficial content.*

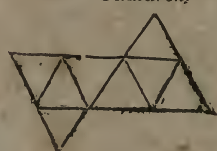
Cube = $3 \times 3 \times 3 = 27$, and $27 \times 2 \cdot 18169 = 58 \cdot 90563$ = solidity.
Square = $3 \times 3 = 9$, and $9 \times 8 \cdot 66025 = 77 \cdot 91225$ = superficies.

As the figures of some of these bodies would give but a confused idea of them, I have omitted them; but the following figures, cut out in pasteboard, and the lines cut half through, will fold up into the several bodies.

Tetraedron.

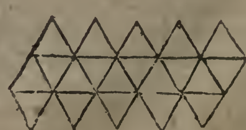
Hexaedron.

Octaedron.



Dodecaedron.

Eicosiedron.



OF CASK GAUGING.

Among the many different canons drawn from Stereometry, for Gauging *casks*, the following is as exact as any,

Take the dimensions of the cask in inches, viz. the diameter at the bung and head, and length of the cask; subtract the head diameter from the bung diameter, and note the difference.

If

If the staves of the cask be much curved or bulging *between the bung and the head*, multiply the difference by ·7; if not quite so curve, by ·65; if they bulge yet less, by ·6; and if they are almost or quite straight, by ·55, and add the product to the head diameter; the sum will be a mean diameter, by which the cask is reduced to a cylinder.

Square the mean diameter, thus found, then multiply it by the length; divide the product by 359 for ale or beer gallons, and by 294 for wine gallons.

Note 1. The length is most conveniently taken by callipers, allowing, for the thickness of both heads, 1 inch, 1½ inch, or 2 inches, according to the size of the cask; but if you have no callipers, do thus; measure the length of the stave, then take the depth of the chimes, which, with the thickness of the head, being subtracted from the length of the stave, leaves the length within.

Note 2. You must take the head diameter, close to its outside, and, for small casks, add three tenths of an inch; for casks of 30, 40, or 50 gallons, 4 tenths, and for larger casks, 5 or 6 tenths, and the sum will be very nearly the head diameter within. In taking the bung diameter, observe, by moving the rod backward and forward, whether the stave, opposite the bung, be thicker or thinner than the rest; and if it be, make allowance accordingly.

By the Sliding Rule.

On D is 18·94, the gauge point for ale or beer gallons, marked AG, and 17·14, the gauge point for wine gallons, marked WG: set the gauge point to the length of the cask on C, and against the mean diameter, on D, you will have the answer in ale or wine gallons accordingly as which gauge point you make use of.

By the Scale.

Take the extent from the gauge point to the mean diameter, set one foot of the dividers in the length, and turning them twice over, they will point out the content.

Art. 51. Required the content in ale and wine gallons, of a cask, whose bung diameter is 35 inches, head diameter, 27 inches, and length 45 inches?

Bung diameter = 35	Square of the diameter = 1062·76
Head diameter = 27	Length = 45
Difference = 8	531380
·7	425104
5·6	359)47821·20(133·21 [ale gall.
Add the head dia. = 27	294)47821·2(162·66 wine gall.
Mean diameter = 32·6	
32·6	
1956	
652	
978	
Squared 1062·76	

Art.

Art. 52. A round mash tub is 42 inches diameter at the top, within, and 36 inches at the bottom, and the perpendicular height 48 inches; required the content in beer and wine gallons?

This being the lower frustum of a cone, to the product of the diameters add $\frac{1}{3}$ of the square of their difference; multiply this sum by the length, and it will give the solidity in such parts as the dimensions are taken in. If they be taken in inches, divide by 359 for beer, and 294 for wine gallons.

$$\overline{42\times36+\frac{\overline{42-36}\times\overline{42-36}}{3}}\times48\div\begin{cases}359=203\frac{1}{2}\text{ ale gallons.}\\294=248\frac{3}{4}\text{ wine gallons.}\end{cases}$$

Art. 53. Let the difference of diameters of this tub be 6 inches, the height 48 inches, and the content $203\frac{3}{4}$ gallons, to find the diameters?

Multiply the content, if beer measure, by 359; if wine measure, by 294, and divide the product by the length: from the quotient subtract $\frac{1}{3}$ of the square of the difference of the diameters; to this remainder add the square of $\frac{1}{2}$ the difference of the diameters, and extract the square root of the sum; from the square root subtract $\frac{1}{2}$ the difference of the diameters, and it will give the least diameter to great exactness, to which add the difference of the diameters, and the sum is the greatest diameter.

$$\sqrt{\frac{203{\cdot}75\times359}{48}-\frac{6\times6}{3}+3\times3}-3=36,\text{ and }36+6=42.$$

The diameters are 36 and 42.

The content of any vessel, in gallons, &c. may be thus found: measure the inside of the vessel, according to the rule of the figure, and find the content in cubick inches; then,

Divide by	1728	and the quotient will be the content in	cubick feet.
	282		ale or beer gallons.
	231		wine gallons.
	2150·425		bushels.

Art. 54. *To ullage a Cask, lying on one side, by the Gauging Rod, when the Bung Diameter, and the Content, one, or both are greater or less than the Table on the Rod is made for.*

Rule.—As the bung diameter of the cask to be measured, is to the bung diameter that the table is made for; so are the dry inches of the cask, to a *fourth* number, which find in the table on the rod, and note the number of gallons answering to it. Then as the content of the cask that the table is made for, is to the content of the cask to be measured; so is the number of gallons answering to the aforesaid *fourth* number, to the number of gallons your cask wants of being full.

Art. 55. *To find a Ship's Burthen, or to Gauge a Ship.*

There is such a diversity in the forms of ships, that no general rule can be applied to answer all varieties; however, the following rules are practised.

Rule

RULE 1.—Multiply the breadth at the main beam, half the breadth, and length together; divide the product by 94, and the quotient is the tons.

RULE 2.—Divide the continued product of the length, breadth and depth, in feet, by 100, for ships of war, and 95 for merchant ships, in which nothing is allowed for guns, &c. and the quotient is the tons.

RULE 3.—Take the length from the stern post to the upper part of the stem; subtract two thirds of her breadth from that length: multiply the remainder by the whole breadth, and that product by half the breadth, in feet, and divide by 100 for war, and 94 for merchant tonnage.

RULE 4.—The weight of a ship's burthen is half the weight of water she can hold.

What is the tonnage of a ship, whose length is 97 feet, breadth 31 feet, and depth $15\frac{1}{2}$ feet.

By Rule 1st.

$\frac{1}{2}$ breadth 15·5
Breadth 31

155
465

480·5
Length 97

33635
43245

94)46608·5(495·83 tons.
376

900
846

548
470

785
752

330
282

48

100)46608·5(466 tons.
400

660
600

608
600

85

By Rule 2d.

Length 97
Breadth 31

97
291

3007
Depth = 15·5

15035
15035
3007

95)46608·5(490·61 tons.
380

860
855

585
570

150
95

55

By Rule 3d.

Length = 97
Subtract $\frac{2}{3}$ of breadth = 20·66

76·33
Multiply by the breadth 31

Carried over.

$$\begin{array}{r} 7633 \\ 22899 \\ \hline 2366{\cdot}23 \\ \text{Multiply by } \tfrac{1}{2} \text{ breadth} \quad 15{\cdot}5 \\ \hline 1183115 \\ 1183115 \\ 236623 \\ \hline \end{array}$$

94)36676·565(390·176 tons.

Allowing the Cubit, as it is found by modern travellers, to be 22 inches, the content of Noah's Ark is as follows, viz.

	Cubits.	
Length of the keel,	300	Its burthen as a man of war 27729 tons.
Breadth by the midship beam	50	
Depth in the hold	30	As a merchant ship, 29188·6 ts.

QUESTIONS IN MENSURATION.

1. THE largest of the Egytian pyramids is square at the base, and measures 693 feet on a side : how much ground does it cover ?

$$\frac{696 \times 393}{272{\cdot}25} = 1764 \text{ poles, and } \frac{1764}{160} = 11 \text{ acres and 4 poles, Ans.}$$

2. What difference is there between a floor 20 feet square, and two others each 10 feet square ?

$$\overline{20 \times 20} - \overline{\overline{10 \times 10} + \overline{10 \times 10}} = 200 \text{ feet, Ans.}$$

3. There is a square of 2500 yards in area : what is each side of the square, and the breadth of a walk along one side and one end, which may take up just one half of the square ?

$\sqrt{2500} = 50$ yards, each side. $\sqrt{\frac{2500}{2}} = 35{\cdot}35$, and $50 - 35{\cdot}35 = 14{\cdot}65$ yards, breadth of the walk, Ans.

4. A pine plank is 16 feet and 5 inches long, and I would have just a square yard slit off : at what distance from the edge must the line be drawn ?

A square yard = 1296 inches, and 16 feet 5 inches = 197 inches.

Therefore, $\frac{1296}{197} = 6\frac{114}{197}$ inches, Ans.

5. If the area of a triangle be 900 yards, and the perpendicular 40 yards : required the length of the base ?

$$\frac{900 \times 2}{40} = 45 \text{ yards, Ans.}$$

6. If

6. If the three sides of a plain triangle be 24, 16 and 12 perches: required its area?

$\frac{24+16+12}{2}=26$; $26-24=2$; $26-16=10$; $26-12=14$, and $\sqrt[2]{26\times14\times10\times2}=85\cdot32$ perches, = area. Again, as $24:16+12::16-12:4\cdot6+$, the difference of the segments of the base; then, $12-\frac{4\cdot6+}{2}=9\cdot6$, and $\sqrt{12\times12-9\cdot6\times9\cdot6}=7\cdot11$ the perpendicular on the longest side; whence $24\div2\times7\cdot11=85\cdot32$, area as above.

7. Required the area of a circular garden, whose diameter is 12 rods? $12\times12\times\cdot7854=113\cdot0976$ poles, Ans.

8. The wheel of a perambulator turns just once and an half in a rod: what is its diameter?

$16\cdot5\times\frac{2}{3}=11$, circumference, and $11\times\cdot31831=3\frac{1}{2}$ feet, Ans.

9. Agreed for a platform to the curb of a round well, at $7\frac{1}{2}$d. per square foot; the inward part, round the mouth of the well, is 36 inches diameter, and the breadth of the platform was to be $15\frac{1}{2}$ inches: what will it come to?

$36+\overline{15\cdot5\times2}=67$ the greatest diameter; $\overline{67\times67\times\cdot7854}-\overline{36\times36\times\cdot7854}=\frac{2507\cdot8722}{144}=17\cdot4157$ square feet, at $7\frac{1}{2}$d. per foot, $=$ 10s. $10\frac{6}{10}$d. [Ans.

10. Required the difference between the area of a circle, whose radius (or semidiameter) is 50 yards, and its greatest inscribed square?

$50\times2=100$ the diameter, and $100\times100\times\cdot7854=7854$ the area of the circle; then, $50\times50\times2=5000$ the area of the greatest inscribed square, and $7854-5000=2854$ Ans.

11. There is a section of a tree 25 inches over; I demand the difference of the areas of the inscribed and circumscribed squares, and how far they differ from the area of the section?

$\overline{25\times25}-\overline{12\cdot5\times12\cdot5\times2}=312\cdot5$ the difference of the squares. $\overline{25\times25}-\overline{25\times25\times\cdot7854}=134\cdot125$ the circumscribed square, more than the section, and $\overline{25\times25\times\cdot7854}-\overline{12\cdot5\times12\cdot5\times2}=178\cdot375$ inscribed square, less than the area of the section.

12. Four men bought a grindstone of 60 inches diameter: how much of its diameter must each grind off, to have an equal share of the stone, if one first grind his share, and then another, till the stone is ground away, making no allowance for the eye?

Rule.—Divide the square of the diameter by the number of men, subtract the quotient from the square, and extract the square root of the remainder, which is the length of the diameter after the first man has ground his share; this work being repeated by subtracting the same quotient from the remainder, for every man, to the last; extract the square root of the remainders, and subtract those roots from the diameters, one after another; the several remainders will be the answers.

60
60
4)3600
Quot. = 900

From 3600
Take 900
$\sqrt{2700}$ = 51·9615, to be taken from 60.
Subt. 900
$\sqrt{1800}$ = 42·4264, from 51·9615.
Subt. 900
$\sqrt{900}$ = 30, from 42·4264.

From 60
Take 51·9615
Remains 8·0385 = 1st. share.

From 51·9615
Take 42·4264
Rem. 9·5351 = 2d. share.

From 42·4264
Take 30·
Rem. 12·4264 = 3d. share.
And 30 inches = 4th share.

13. If a cubick foot of iron were hammered, or drawn, into a square bar, an inch about, that is, $\frac{1}{4}$ of an inch square: required its length, supposing there is no waste of metal?

$$\frac{12\times12\times12}{\cdot25\times\cdot25\times4} = 6912 \text{ inches}, = 576 \text{ feet, Ans.}$$

14. Required the axis of a globe, whose solidity may be just equal to the area of its surface?

$$\frac{\cdot7854\times4}{\cdot5236} = 6 \text{ inches, Ans.}$$

15. A joist is $7\frac{1}{2}$ inches wide, and $2\frac{1}{4}$ thick; but I want one just twice as large, which shall be $3\frac{3}{4}$ inches thick: what will be the breadth?

$$\frac{7\cdot5\times2\cdot25\times2}{3\cdot75} = 9 \text{ inches, Ans.}$$

16. I have a square stick of timber 18 inches by 14; but one of a third part of the timber in it, provided it be 8 inches deep, will serve: how wide will it be?

$$\frac{18\times14}{3} \div 8 = 10\tfrac{1}{2} \text{ inches, Ans.}$$

17. A had a beam of oak timber, 18 inches square throughout, and 25 feet long, which he bartered with B, for an equilateral triangular beam of the same length, each side 24 inches: required the balance at 1s. 4d. per foot?

$$\frac{18\times18\times25}{144} = 56\cdot95, \text{ solidity of the square beam.}$$

The perpendicular let fall on one of the sides of the triangular beam is 20·7846 inches, and the half perp. = 10·3923, then $\frac{10\cdot3923\times24}{144}$ = 1·732 foot, area at the end, and 1·732×25 = 43·3 feet, solidity of the triangular beam; therefore 56·25—43·3 = 12·95 feet, at 1s. 4d. per foot = 17s. 3·2d. balance due to A, Ans.

18. What is the difference between a solid half foot, and half a foot solid? 12×

$$\frac{12\times12\times6}{6\times6\times6}=4,$$ therefore, one is but $\frac{1}{4}$ of the other.

19. A lent B a solid stack of hay, measuring 20 feet every way; sometime afterward, B returned a quantity measuring every way 10 feet: what proportion of the hay remains due?

$20\times20\times20-10\times10\times10=7000$ feet $=\frac{7}{8}$ Ans.

20. A ship's hold is $75\frac{1}{2}$ feet long, $18\frac{1}{2}$ wide, and $7\frac{1}{4}$ deep: how many bales of goods $3\frac{1}{2}$ feet long, $2\frac{1}{4}$ deep, and $2\frac{3}{4}$ wide, may be stowed therein, leaving a gang way the whole length, of $3\frac{1}{4}$ feet wide?

$$\frac{\overline{75\cdot5\times18\cdot5\times7\cdot25}-\overline{75\cdot5\times7\cdot25\times3\cdot25}}{3\cdot5\times2\cdot25\times2\cdot75}=385\cdot44 \text{ bales, Ans.}$$

21. If a stick of timber be $20\frac{1}{2}$ feet long, 16 inches broad, and 8 inches thick, and $3\frac{1}{2}$ solid feet be sawed off one end: how long will the stick then be?

$$20\tfrac{1}{2}-\frac{1728\times3\cdot5}{16\times8}=16 \text{ feet, } 6\tfrac{3}{4} \text{ inches, Ans.}$$

22. The solid content of a square stone is found to be $136\frac{1}{2}$ feet; its length is $9\frac{1}{2}$ feet: what is the area of one end? and if the breadth be 3 feet 11 inches, what is the depth?

$$\frac{136\cdot5\times1728}{9\cdot5\times12}=\text{area } 2069\cdot0526 \text{ inches, and } \frac{2069\cdot0526}{47}=44\cdot022$$ [ins. Ans.

23. I would have a cubick box made capable of receiving just 50 bushels, the bushel containing 2150·425 solid inches: what will be the length of the side?

$$\sqrt[3]{2150\cdot4\times50}=47\cdot55 \text{ inches.}$$

24. A statute bushel is to be made 8 inches high, and $18\frac{1}{2}$ inches diameter, to contain 2176 cubick inches; (though the content of the dimensions is but 2150·125 inches) I demand what the diameter of the bushel must be, the height being 8 inches; and what the height, the diameter being $18\frac{1}{2}$ inches, to contain 2176 cubick inches?

Solidity.
Height $=8)2176$ and $\sqrt{272\times1\cdot273}=18\cdot6$ diameter. $18\cdot5\times18\cdot5\times\cdot7854=268\cdot80315=$ area, and the solidity
Area $=272$ $\quad 2176\div268\cdot8=8\cdot0956$ inches, height.

25. There is a garden rolling stone 66 inches in circumference, and $3\frac{1}{2}$ cubick feet are to be cut off from one end, perpendicular to the axis: where must the section be made?

$$\frac{1728\times3\cdot5}{412\cdot5}=14\cdot65 \text{ inches from one end, Ans.}$$
Area $=412\cdot5$

26. I would have a syringe of $1\frac{1}{2}$ inch diameter in the bore, to hold a quart, wine measure: what must be the length of the piston, sufficient to make an injection with?

$1\cdot5\times1\cdot5\times\cdot7854=1\cdot76715$, and $231\div4=57\cdot75$ the cubick inches in a quart, then, $\dfrac{57\cdot75}{1\cdot76715}=32\cdot679$ inches, Ans.

27. If a round pillar, 9 inches diameter, contain 5 feet: of what diameter is that column, of equal length, which measures 10 times as much? As 5.

As $5 : 9\times9 :: 5\times10 : 810$, and $\sqrt{810} = 28\cdot46$ inches, Ans.

28. There is a square pyramid, each side of whose base is 30 inches, and whose perperdicular height is 120 inches, to be divided by sections parallel to its base into 3 equal parts: required the perpendicular height of each part?

$30\times30\times40 = 36000$ the solidity in inches, now $\frac{2}{3}$ thereof is 24000, and $\frac{1}{3}$ is 12000. Therefore,

$$\text{As } 36000 : 120\times120\times120 :: \begin{Bmatrix} 24000 \\ 12000 \end{Bmatrix} : \begin{Bmatrix} 1152000 \\ 576000 \end{Bmatrix} \text{Then,}$$

$\sqrt[3]{1152000} = 104\cdot8$ Also, $\sqrt[3]{576000} = 83\cdot2$. Then, $120 - 104\cdot8 = 15\cdot2$ length of the thickest part, and $104\cdot8 - 83\cdot2 = 21\cdot6$ length of the middle part, consequently $83\cdot2$ is the length of the top part.

29. Suppose the diameter of the base of a conical ingot of gold to be 3 inches, and its height 9 inches; what length of wire may be expected from it, without loss of metal, the diameter of the wire being one hundredth part of an inch?

$3\times3\times\cdot7854\times3 = 21\cdot2058$ the solidity of the cone.

$$\frac{21\cdot2058}{\cdot01\times\cdot01\times\cdot7854} = 270000 \text{ inch.} = 4 \text{ miles, and } 460 \text{ yards, Ans.}$$

30. Suppose a pole to stand on an horizontal plane 75 feet in height above the surface: at what height from the ground must it be cut off, so as that the top of it may fall on a point 55 feet from the bottom of the pole, the end, where it was cut off, resting on the stump, or upright part?

As the whole length of the pole is equal to the sum of the hypothenuse and perpendicular of a triangle, (the 55 feet on the ground being the base) this, as well as the following question, may be solved by this

Rule.—From the square of the length of the pole (that is, of the sum of the hypothenuse and perpendicular) take the square of the base; divide the remainder by twice the length of the pole, and the quotient will be the perpendicular, or height at which it must be cut off.

$$\frac{\overline{75\times75} - \overline{55\times55}}{75\times2} = 17\tfrac{1}{3} \text{ feet, Ans.}$$

31. Suppose a ship sails from latitude 43°, north, between north and east, till her departure from the meridian be 45 leauges, and the sum of her distance and difference of latitude to be 135 leagues: I demand her distance sailed, and latitude come to?

$$\frac{\overline{135\times135} - \overline{45\times45}}{135\times2} = 60 \text{ leagues, and } 60\times3 = 180 \text{ miles} = 3 \text{ degrees}$$

the difference of latitude, $135 - 60 = 75$ leagues the distance. Now, as the vessel is sailing from the equator, and consequently the latitude is increasing: Therefore,

To the latitude sailed from	43°,00′N.
Add the difference of latitude	3 ,00
And the sum is the latitude come to	= 46 ,00 N.

AN

INTRODUCTION TO ALGEBRA,

DESIGNED FOR THE

USE OF ACADEMIES.

DEFINITIONS.

ALGEBRA is the art of computing by symbols.

1. *Like quantities* are those which consist of the same letters.
2. *Unlike quantities* are those which consist of different letters.
3. *Given quantities* are those whose values are known.
4. *Unknown quantities* are those whose values are unknown.
5. *Simple quantities* are those which consist of one term only.
6. *Compound quantities* are those which consist of several terms.
7. *Positive* or *affirmative quantities* are those to be added.
8. *Negative quantities* are those to be subtracted.
9. *Like signs* are all + or all —.
10. *Unlike signs* are + and —.
11. *The coefficient* of any quantity is the number prefixed to it.
12. *A binomial quantity* is one consisting of two terms; a *trinomial*, of three terms; and a *quadrinomial*, of four terms, &c.
13. *A residual quantity* is a binomial, where one of the terms is a negative.

In the computation of problems, the first letters of the alphabet are put for known quantities, and letters of the latter part of the alphabet for those which are unknown.

AXIOMS.

1. If equal quantities be added to, subtracted from, multiplied or divided by, equal quantities, the wholes, remainders, products and quotients will be respectively equal.

2. The equal powers or roots of equal quantities are equal.

3. Two quantities, respectively equal to a third, are equal to each other.

4. The whole is equal to all its parts taken together.

ADDITION.

CASE I. *To add quantities which are alike, and have like signs.**

RULE.—Add all the coefficients together, and to their sum adjoin the letters common to each term, prefixing the common sign.

5a

* When a leading quantity has no sign before it, + is always understood; and a quantity without any coefficient prefixed to it, is supposed to have unity, or 1.

$5a$	$-6br$	$8bru$	$5r^2+ru$	$7ar-u$
$7a$	$-3br$	$7bru$	$3r^2+2ru$	$8ar-3u$
$8a$	$-2br$	$3bru$	r^2+3ru	$6ar-2u$
$10a$	$-7br$	$4bru$	$7r^2+8ru$	$4ar-3u$
$2a$	$-br$	$5bru$	r^2+ru	$ar-u$
a	$-5ba$	bru	$2r^2+3ru$	$3ar-2u$
$33a$	$-24br$	$28bru$	$19r^2+18ru$	$29ar-12u$

Case II. *To add quantities which are alike, but have unlike signs.*

Rule 1. Add all the affirmative coefficients into one sum, and all the negative ones into another.

2. Subtract the least sum from the greatest, and to the diffierence prefix the sign of the greatest, with the common quantity.

$-3a$	$+8ar^2$	$+6r^{\frac{1}{3}}+8u$	$-2ru+8$	$+8r^2-u+3\sqrt{r}$
$+7a$	$+7ar^2$	$-3a^{\frac{1}{3}}+7u$	$-3ru+7$	$-10r^2-3u+2\sqrt{r}$
$+8a$	$-3ar^2$	$-13r^{\frac{1}{3}}+8u$	$+ru-10$	$-4r^2-2u+\sqrt{r}$
$-a$	$-4ar^2$	$+2r^{\frac{1}{3}}-3u$	$+5ru-7$	$+9r^2+6u-10\sqrt{r}$
$-2a$	$+4ar^2$	$+r^{\frac{1}{3}}-u$	$-ru+2$	$+r^2 * -\sqrt{r}$
$+9a$	$+12ar^2$	$-7r^{\frac{1}{3}}+19u$	*	$+4r^2 * -5\sqrt{r}$

Case III. *To add quantities which are unlike, and have unlike signs.*

Rule. Collect the like quantities together by the last rule, and set down those which are unlike, one after another, with their proper signs.

$2r$	$2r-r^2$	
$3u$	$3a-r$	$12ar-r^2-6+\sqrt{ar-r^2}$
$-a$	$2ar+6r^2$	$-6ar+r^2-r+10$
r^2	$3\sqrt{r}-2ar$	$3u-ar-4-2\sqrt{ar-r^2}$
$2r+3u-a+r^2$	$r+5r^2+3a+3\sqrt{r}$	$5ar-\sqrt{ar-r^2}-r+3u$

SUBTRACTION.

Rule.—Change the signs of all the quantities to be subtracted, and then add them together, as in Addition.

$3a^2-2b$	$6r^2-8u+2$	$35ru-2+8r-u^{\frac{1}{2}}$	$8ar-2\sqrt{ru}-10$
$2a^2-3b$	$r^2+9u-20$	$24ru-8-8r-3u$	$10r-6\sqrt{ru}-ar$
a^2+b	$5r^2-17u+22$	$11ru+6+16r-u^{\frac{1}{2}}+3u$	$9ar+4\sqrt{ru}-10-10r$

MULTIPLICATION.

Case I. *To multiply simple quantities.*

Rule.—Multiply the coefficients of the two terms together, and to the product annex all the letters in those termss.

Note:

Note. Like signs produce +, and unlike signs —.

$$\begin{array}{ccccccc} 2a & -2a & 5a & -9r & 6a^2r & -r^2u & -7ru \\ 3b & +4b & -6r & -5b & 5r & ru^2 & -ru \\ \hline 6ab & -8ab & -30ar & +45br & 30a^2r^2 & -r^3u^3 & +7r^2u^2 \end{array}$$

CASE III. *When one of the factors is a compound quantity.*

RULE.—Find the products of the multiplier and every particular term of the multiplicand separately, and place them one after another with their proper signs.

$$\begin{array}{cccc} 4a-2b & 6ru-8 & 8a^2-2r+6 & 3u-8+2ru \\ 3a & 2r & 3ru & ru \\ \hline 12a^2-6ab & 12r^2u-16r & 24a^2ru-6r^2u+18ru & 3ru^2-8ru+2r^2u^2 \end{array}$$

CASE III. *When both the factors are compound quantities.*

RULE. Multiply every particular term of the multiplier into every term of the multiplicand respectively, and set down the products one after another with their proper signs, and their sum will be the whole product.

$$\begin{array}{ll} r+u & \\ r+u & \\ \hline r^2+ru & \\ \quad +ru+u^2 & \\ \hline r^2+2ru+u^2 & \end{array} \qquad \begin{array}{l} r-u \\ r-u \\ \hline r^2-ru \\ \quad -ru+u^2 \\ \hline r^2-2ru+u^2 \end{array} \qquad \begin{array}{l} 3r^2-2ru+5 \\ r^2+2ru-3 \\ \hline 3r^4-2r^3u+5r^2 \\ \quad +6r^3u-4r^2u^2+10ru \\ \qquad -9r^2+6ru-15 \\ \hline 3r^4+4r^3u-4r^2-4r^2u^2+16ru-15 \end{array}$$

When two surd numbers are to be multiplied together, multiply them without any regard to the radical sign, and prefix the radical sign to the product. Thus,

$$\sqrt{3}\times\sqrt{2}=\sqrt{6}\,;\ \sqrt{a}\times\sqrt{b}=\sqrt{ab},\ \&c.$$

DIVISION.

CASE I. *When the divisor is a simple quantity.*

RULE 1. Place the dividend above a line, and the divisor under it, like a vulgar fraction.

2. Expunge those letters which are common to both the factors, and divide the coefficients of all the terms by any number, which will divide them without a remainder

Note. Like signs make +, and unlike signs —, as in multiplication.

$$\frac{a}{a}=1\,;\ \frac{8bc}{2b}=4c\,;\ \frac{abc}{bcd}=\frac{a}{d}\,;\ \frac{10ab+15ac}{20ad}=\frac{2b+3c}{4d}\,;\ \frac{ab+b^2}{2b}=\frac{a+b}{2}\,;\ \frac{12ru}{6r^2}$$

$$=\frac{2u}{r}\,;\ \frac{30ar-54au}{12ab}=\frac{5r-9u}{2b}\,;\ \frac{10r^2u-15u^2-5u}{5u}=2r^2-3u-1\,;$$

$$\frac{3a^2-15+6a+3b}{3a}=a-\frac{5}{a}+2+\frac{b}{a}.$$

CASE II. *When the divisor and dividend are both compound quantities.*

RULE 1. Range the terms of both the quantities according to the dimensions of some letter in them, so that the first term may have the highest power of that letter, and the second term the next highest power; and so on.

2. Divide the first term of the dividend by the first term of the divisor, and place the result in the quotient.

3. Multiply the whole divisor by the quotient term last found, and subtract the result from the dividend

4. To this remainder bring down the next term of the dividend and divide as before, and so on, as in common arithmetick.

$$\begin{array}{l}
a+r)a^3+5a^2r+5ar^2+r^3(a^2+4ar+r^2 \\
\qquad\quad a^3+a^2r \\
\qquad\quad \overline{\qquad\quad 4a^2r+5ar^2} \\
\qquad\qquad\quad 4a^2r=4ar^2 \\
\qquad\qquad\quad \overline{\qquad\qquad ar^2+r^3} \\
\qquad\qquad\qquad\quad ar^2+r^3 \\
\qquad\qquad\qquad\quad \overline{\qquad\quad *}
\end{array}$$

$$\begin{array}{l}
r-3)r^3-0r^3+27r-27(r^2-6r+9 \\
\qquad r^3-3r^2 \\
\qquad \overline{\quad -6r^2+27r} \\
\qquad\quad -6r^2+18r \\
\qquad\quad \overline{\qquad\qquad 9r-27} \\
\qquad\qquad\qquad 9r-27 \\
\qquad\qquad\qquad \overline{\qquad *}
\end{array}
\qquad
\begin{array}{l}
a-r)a^3-r^3(a^2+ar+r^2 \\
\qquad a^3-a^2r \\
\qquad \overline{\qquad a^2r-r^3} \\
\qquad\qquad a^2r-ar^2 \\
\qquad\qquad \overline{\qquad ar^2-r^3} \\
\qquad\qquad\quad ar^2-r^3 \\
\qquad\qquad\quad \overline{\qquad *}
\end{array}$$

ALGEBRAICK FRACTIONS.

PROBLEM I. *To reduce a mixed quantity to an improper fraction.*

RULE.—Multiply the integer by the denominator, and to the product add the numerator, and the denominator being placed under this sum will give the improper fraction required.

$r+\frac{r^2}{a}=\frac{ar+r^2}{a}$; $a-\frac{b}{c}=\frac{ac-b}{c}$; $1-\frac{2r}{a}=\frac{a-2r}{a}$; $a-r+\frac{a^2-ar}{r}=\frac{a^2-r^2}{r}$.

PROB. II. *To reduce an improper fraction to a whole or mixed quantity.*

RULE.—Divide the numerator by the denominator, for the integral part, and place the remainder over the denominator, for the fractional part.

ar

$\frac{ar+a^2}{r} = a + \frac{a^2}{r}$; $\frac{au+2u^2}{a+u} = u + \frac{u^2}{a+u}$; $\frac{ab-a^2}{b} = a - \frac{a^2}{b}$; $\frac{a^2+2r^2}{a-r} = a + r + \frac{a^2}{a-r}$.

PROB. III. *To reduce fractions of different denominators, to those of the same value, which shall have a common denominator.*

RULE.—Multiply every numerator separately into all the denominators but its own, for new numerators, and all the denominators together for a common denominator.

1. Reduce $\frac{a}{b}$ and $\frac{b}{c}$ to fractions of equal values, having a common denominator.

$a \times c = ac$ new nume. $\quad \frac{ac}{bc}$ and $\frac{b^2}{bc}$ = fractions required.
$b \times b = b^2$ new nume.
$b \times c = bc$ common denominator.

2. Reduce $\frac{a}{b}$, $\frac{b}{c}$ and $\frac{c}{d}$ to equivalent fractions having a common denominator.

$a \times c \times d = acd$
$b \times b \times d = b^2 d$ $\quad \frac{acd}{bcd}$, $\frac{b^2 d}{bcd}$ and $\frac{c^2 b}{bcd}$, Ans.
$c \times b \times c = c^2 b$
$b \times c \times d = bcd$

PROB. IV. *To find the greatest common measure of a fraction.*

RULE. 1. Range the quantities according to the dimensions of some letter, as was shewn in division.

2. Divide the greater term by the less, and the last divisor by the last remainder, and so on, till nothing remain, and the divisor last used, will be the common measure required.

Note. All the letters or figures, which are common to the divisor, and dividend, must be cancelled in the divisor before they are used in the operation.

To find the greatest common measure of $\frac{cr+r^2}{ca^2a+r}$

* $cr+r^2)ca^2+a^2r$
Or, $c+r)ca^2+a^2r(a^2$
ca^2+a^2r
*

Therefore the greatest common measure is $c+r$.

2. To

* Here I find that r is common to both divisor and dividend, I therefore cancel r in the divisor, that is, I divide $cr+r^2$ by r, and $c+r$ is the quotient: Thus, $r)cr+r^2(c+r$, for the divisor.

cr
$+r^2$
$+r^2$
*

2. To find the greatest common measure of $\frac{b^3-b^2r}{r^2+2br+b^2}$

$r^2+2br+b^2)r^3-b^2r(r$
$r^3+2br^2+b^2r$

$*-2br^2-2b^2r)r^2+2br+b^2$
Or, $r+b\ )r^2+2br+b^2(r+b$
r^2+br

$br+b^2$
$br+b^2$

Therefore $r+b$ is the greatest common divisor.

PROB. V. *To reduce a fraction to its lowest terms.*

RULE. 1.–Find the greatest common measure, as in the last problem.

2. Divide both of the terms of the fraction by the common measure thus found.

1. Reduce $\frac{cr+r^2}{ca^2+a^2r}$ to its lowest terms.

$cr+r^2)ca^2+a^2r$
Or, $c+r\ )ca^2+a^2r(a^2$
ca^2+a^2r

*

Therefore, $c\times r$ is the greatest common measure,

and $c+r$ $\Big)\frac{cr+r^2}{ca^2+a^2r}\Big(\frac{r}{a^2}$ = fraction required.

2. Reduce $\frac{r^3-b^2r}{r^2+2br+b^2}$ to its lowest terms.

$r^2+2br+b^2)r^3-b^2r(r$
$r^3+2br^2+b^2r$

$-2br^2-2b^2r)r^2+2br+b$
Or, $r+b\ )r^2+2br+b^2(r+b$
r^2+br

$br+b^2$
$br+b^2$

Therefore $r+b$ is the greatest common measure, and $\frac{r^3-b^2r}{r^2+2br+b^2}$ *

$r+b$

* Here $-br$ is common to the divisor and dividend; I therefore first divide $-2br^2-2b^2r$ by r, and the quotient is $-2br-2b^2$, thus,

$r)-2br^2-2b^2r(-2br-2b^2$
$-2br^2$

$-2b^2r$
$-2b^2r$

*

I then divide $-2br-2b^2$ by $-2b$, and the quotient is $r+b$, thus,

$-2b)-2br-2b^2(r+b$, for the divisor.
$-2br$

$-2b^2$
$-2b^2$

*

$$r+b\Big)\frac{r^3-b^2r}{r^2+2br+b^2}\Big(\frac{r^2-br}{r+b} = \text{fraction required.}$$

PROB. VI. *To add fractional quantities together.*

RULE.—Reduce the fractions to a common denominator.

2. Add all the numerators together, and under their sum write the common denominator.

1. Add $\frac{r}{2}$ and $\frac{r}{3}$.

$$\begin{aligned} r+3 &= 3r \\ r+2 &= 2r \\ \hline 2r+3 &= 6 \end{aligned}$$

$$\frac{3r}{6}+\frac{2r}{6}=\frac{5r}{6}=\text{sum.}$$

2. Add $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$.

$$\begin{aligned} a\times d\times f &= adf \\ c\times b\times f &= cbf \\ e\times b\times d &= ebd \\ \hline b\times d\times f &= bdf. \end{aligned} \qquad \frac{adf}{bdf}+\frac{cbf}{bdf}+\frac{ebd}{bdf}=\frac{adf+cbf+ebd}{bdf}=\text{sum.}$$

3. Add $a-\frac{3r^2}{b}$ and $b+\frac{r-2b}{c}$.

$$\begin{aligned} 3r^2\times c &= 3cr^2 \\ r-2b\times b &= br-2b^2 \\ \hline b\times c &= bc \end{aligned}$$

$$\begin{aligned} &a-\frac{3cr^2}{bc} \\ &b+\frac{br-2b^2}{bc} \\ \hline &a+b+\frac{br-3cr^2-2b^2}{br}=\text{sum.} \end{aligned}$$

PROB. VII. *To subtract one fractional quantity from another.*

RULE 1.—Reduce the fractions to a common denominator.

2. Subtract the numerators, and under their difference write the common denominator.

1. Required the difference of $\frac{r}{3}$ and $\frac{2r}{11}$?

$$\begin{aligned} r\times 11 &= 11r \\ 2r\times 3 &= 6r \\ \hline 3\times 11 &= 33 \end{aligned} \qquad \frac{11r}{33}-\frac{6r}{33}=\frac{5r}{33}=\text{difference.}$$

2. What is the difference of $\frac{r-a}{3b}$ and $\frac{2a-4r}{5c}$?

$r-a$

$\overline{r-a}\times 5c=5cr-5ac$

$\overline{2a-4r}\times 3b=6ab-12br$

$3b\times 5c=15bc$

$\frac{5cr-5ac}{15bc}-\frac{6ab-12br}{15bc}=\frac{5cr-5ac-6ab+12br}{15bc}=$ difference.

PROB. VIII. *To multiply fractional quantities.*

RULE.—Multiply the numerators together for a new numerator, and the denominators, for a new denominator.

1. Multiply $\frac{r}{6}$ and $\frac{2r}{9}$ together.

$\left.\begin{matrix} r\times 2r \\ 6\times 9 \end{matrix}\right\}=\frac{2r^2}{54}=\frac{r^2}{27}=$ product.

2. Find the product of $\frac{r}{2}$, $\frac{4r}{5}$ and $\frac{10r}{21}$.

$\left.\begin{matrix} r\times 4r\times 10r \\ 2\times 5\times 21 \end{matrix}\right\}=\frac{40r^3}{210}=\frac{4r^3}{21}=$ product.

3. Find the product of $\frac{r}{a}$ and $\frac{a+r}{a+c}$.

$\left.\begin{matrix} r\times\overline{r+a} \\ a\times\overline{a+c} \end{matrix}\right\}=\frac{r^2+ar}{a^2+ac}=$ product.

PROB. IX. *To divide one fractional quantity by another.*

RULE.—Invert the divisor, and proceed as in multiplication.

1. Divide $\frac{r}{3}$ by $\frac{2r}{9}$. $\frac{r}{3}\times\frac{9}{2r}=\frac{9r}{6r}=\frac{3}{2}=1\frac{1}{2}=$ quotient.

2. Divide $\frac{2a}{b}$ by $\frac{4c}{d}$. $\frac{2a}{b}\times\frac{d}{4c}=\frac{2ad}{4bc}=\frac{ad}{2bc}=$ quotient.

3. Divide $\frac{r+a}{2r-2b}$ by $\frac{r+b}{5r+a}$.

$\frac{r+a}{2r-2b}\times\frac{5r+a}{r+b}=\frac{5r^2+6ar+a^2}{2r^2-2b^2}=$ quotient.

INVOLUTION.

Involution is the raising of powers from any proposed root; or the method of finding the square, cube, biquadrate, &c. of any given quantity.

RULE.—Multiply the quantity into itself as often as is denoted by the index, and the last product will be the power required.

Or,

Or, Multiply the index of the quantity by the index of the power, and the result will be the same as before.

Note. When the sign of the root is +, all the powers of it will be +; and when the sign is —, all the odd powers will be —, and all the even powers +.

$$\text{Root} = a \begin{cases} a^2 = \text{square.} \\ a^3 = \text{cube.} \\ a^4 = \text{4th power.} \\ a^5 = \text{5th power.} \end{cases} \qquad \text{Root} = a^2 \begin{cases} a^4 = \text{square.} \\ a^6 = \text{cube.} \\ a^8 = \text{4th power.} \\ a^{10} = \text{5th power.} \end{cases}$$

$$\text{Root} = -3a \begin{cases} + \ 9a^2 = \text{square.} \\ - \ 27a^3 = \text{cube.} \\ + \ 81a^4 = \text{4th power.} \\ - \ 243a^5 = \text{5th power.} \end{cases}$$

$$\text{Root} = -2ar^2 \begin{cases} + \ 4a^2r^4 = \text{square.} \\ - \ 8a^3r^6 = \text{cube.} \\ + \ 16a^4r^8 = \text{4th pow.} \\ - 32a^5r^{10} = \text{5th pow.} \end{cases} \qquad \text{Root} = -\frac{r}{a} \begin{cases} \frac{r^2}{a^2} = \text{squa} \\ -\frac{r^3}{a^3} = \text{cube.} \\ \frac{r^4}{a^4} = \text{biqd.} \end{cases}$$

$$\text{Root} = -\frac{2ar^2}{36} \begin{cases} +\frac{4a^2r^4}{9b^2} = \text{square.} \\ -\frac{8a^3r^6}{27b^3} = \text{cube.} \\ +\frac{16a^4r^8}{81b^4} = \text{biquadrate.} \end{cases}$$

$$\begin{array}{l}
r+a \\
r+a \\
\hline
r^2+ar \\
\quad +ar+a^2 \\
\hline
r^2+2ar+a^2 = \text{square.} \\
r + a \\
\hline
r^3+2ar^2+a^2r \\
\quad + ar^2+2a^2r+a^3 \\
\hline
r^3+3ar^2+3a^2r+a^3 = \text{cube.} \\
r + a \\
\hline
r^4+3ar^3+3a^2r^2+ a^3r \\
\quad + ar^3+3a^2r^2+3a^3r+a^4 \\
\hline
r^4+4ar^3+6a^2r^2+4a^3r+a^4 = \text{biquadrate.} \\
\hline
\end{array}$$

Of

Of the Composition and Resolution of a Square, raised from a Binomial Root.

A binomial is a quantity consisting of two parts or members, connected together by the sign +, or —, as $r+a$, $r-a$, $r+\frac{b}{2}$, $r-\frac{b}{2}$, and a square raised from a binomial root is nothing else but the square of such a quantity; thus the square of $r+\frac{b}{2}$ is $r^2+br+\frac{b^2}{4}$, and that of $r-\frac{b}{2}$ is $r^2-br+\frac{b^2}{4}$.

$$\begin{array}{ll}
r+\frac{b}{2} & \qquad r-\frac{b}{2} \\
r+\frac{b}{2} & \qquad r-\frac{b}{2} \\
\hline
r^2+\frac{br}{2} & \qquad r^2-\frac{br}{2} \\
\quad +\frac{br}{2}+\frac{b^2}{4} & \qquad \quad -\frac{br}{2}+\frac{b^2}{4} \\
\hline
r^2+\frac{2br}{2}+\frac{b^2}{4}=r^2+br+\frac{b^2}{4} & \qquad r^2-\frac{2br}{2}+\frac{b^2}{4}=r^2-br+\frac{b^2}{4}
\end{array}$$

The difference between these two squares arises from the different sign of b, and that only affects the second member; for the third member $\frac{bb}{4}$ will be the same, whether the quantity b be affirmative or negative; therefore, if those cases be thrown into one, it will stand thus: the square of $r\pm\frac{b}{2}$; viz. $+br$ when the root is $r+\frac{b}{2}$, and $-br$ when the root is $r-\frac{b}{2}$. Now, of the three numbers, which compose this square, the first, r^2 is the square of r, the second, $\pm br$ is the root of that square multiplied into the coefficient $\pm b$, and the third member $\frac{b^2}{4}$ is the square of $\pm\frac{b}{2}$, that is, the square of half the coefficient of the second member; whence may be deduced the following observations.

1. Any

1. Any quantity consisting of two members, as $r^2 \pm br$, whereof one, as r^2 is a square, and the other $\pm br$ is the root of that square multiplied into some given coefficient $\pm b$, it may be considered as an imperfect square raised from a binomial root, and may easily be completed by adding $\frac{b^2}{4}$, that is, by adding the square of half the coefficient of r in the second term; thus r^2+6r, when completed is r^2+6r+9; r^2+3r becomes $r^2+3r+\frac{9}{4}$, because half the coefficient 3 is $\frac{3}{2}$, Again, $r^2+\frac{2r}{3}$ becomes $r^2 \times \frac{2r}{3}+\frac{1}{9}$, because half the coefficient is $\frac{1}{3}$, the square of which is $\frac{1}{9}$: Lastly, $r^2 - \frac{br}{a}$ becomes $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$: For the coefficient is $-\frac{b}{a}$, its half $-\frac{b}{2a}$, and the square $\frac{b^2}{4a^2}$.

2. The root of such a square, when completed, that is, the root of $r^2 \pm br + \frac{b^2}{4}$ will always be $r \pm \frac{b}{2}$, that is, it will always be the square root of the first, together with half the coefficient of the second: thus, the square root of r^2+6r+9 will be $r+3$, that of $r^2+3r+\frac{9}{4}$ will be $r+\frac{3}{2}$, that of $r^2+\frac{2r}{3}+\frac{1}{9}$ will be $r+\frac{1}{3}$, and lastly, that of $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$ will be $r - \frac{b}{2a}$.

Sir ISAAC NEWTON's Rule *for raising a binomial or residual quantity to any power whatever.*

1. *To find the terms without the coefficients.*

The index of the first, or leading quantity, begins with that of the given power, and decreases continually by 1, in every term to the last; and in the following quantity the indices of the terms are 0, 1, 2, 3, 4, &c.

2. *To find the unciæ or coefficients.*

The first is always 1, and the second is the index of the power: and, in general, if the coefficient of any term be multiplied by the index of the leading quantity, and the product be divided by the number of terms to that place, it will give the coefficient of the term next following.

Note. The whole number of terms will be one more than the index of the given power; and when both terms of the root are +, all the terms of the powers will be +; but if the second term be —, then all the odd terms will be +, and the even terms —.

1. Let $a+r$ be involved to the fifth power.

The terms without the coefficients will be a^5, a^4r, a^3r^2, a^2r^3, ar^4, r^5, and the coefficients will be $1, 5, \frac{5\times4}{2}, \frac{10\times3}{3}, \frac{10\times2}{4}, \frac{5\times1}{5}$, or 1, 5, 10, 10, 5, 1, and therefore the 5th power is $a^5+5a^4r+10a^3r^3+10a^2r^3+5ar^4+r^5$.

2. Let $r-a$ be involved to the 6th power.

The terms without the coefficients will be r^6, r^5a, r^4a^2, r^3a^3, r^2a^4, ra^5, a^6, and the coefficients will be $1, 6, \frac{6\times5}{2}, \frac{15\times4}{3}, \frac{20\times3}{4}, \frac{15\times2}{5}, \frac{6\times1}{6}$ or 1, 6, 15, 20, 15, 6, 1; and therefore the 6th power of $r-a$ is $r^6-6r^5a+15r^4a^2-20r^3a^3+15r^2a^4-6ra^5+a^6$.

EVOLUTION.

Evolution is the reverse of Involution, and teaches to find the roots of any given powers.

Case I. *To find the roots of simple quantities.*

Rule.—Extract the root of the coefficient, for the numerical part, and divide the index of the letters by the index of the power, and it will give the root required.

1. The square root of $9r^2=3r^{\frac{2}{2}}=3r$.
2. The cube root of $8r^3=2r^{\frac{3}{3}}=2r$.
3. The square root of $3a^2r^6=a^{\frac{2}{2}}r^{\frac{6}{2}}\sqrt{3}=ar^3\sqrt{3}$.
4. The cube root of $-125a^3r^6=-5a^{\frac{3}{3}}r^{\frac{6}{3}}=-5ar^2$.
5. The biquadrate root of $16a^4r^8=2a^{\frac{4}{4}}r^{\frac{8}{4}}=2ar^2$.

Case II. *To find the square root of a compound quantity.*

Rule.—1. Range the quantities according to the dimensions of some letter, and set the root of the first term in the quotient.

2. Subtract the square of the root, thus found, from the first term, and bring down the two next terms to the remainder for a dividend.

3. Divide the dividend by double the root, and set the result in the quotient.

4. Multiply the divisor and quotient by the term last put in the quotient, and subtract the quotient from the dividend, and so on, as in common arithmetick.

1. Extract

1. Extract the square root of $4a^4+12a^3r+13a^2r^2+6ar^3+r^4$.

$$4a^4+12a^3r+13a^2r^2+6ar^3+r^4\,(2a^2+3ar+r^2$$
$$4a^4$$

$$4a^2+3ar)\,12a^3r+13a^2r^2$$
$$12a^3r+\ 9a^2r^2$$

$$4a^2+6ar+r^2)\,4a^2r^2+6ar^3+r^4$$
$$4a^2r^2+6ar^3+r^4$$

*

2. Extract the square root of $r^4-4r^3+6r^2-4r+1$.

$$r^4-4r^3+6r^2-4r+1\,(r^2-2r+1$$
$$r^4$$

$$2r^2-2r)-4r^3+6r^2$$
$$-4r^3+4r^2$$

$$2r^2-4r+1)\,2r^2-4r+1$$
$$2r^2-4r+1$$

*

CASE III. *To find the roots of powers in general.*

RULE.—1. Find the root of the first term, and place it in the quotient.

2. Subtract the power, and bring down the second term for a dividend.

3. Involve the root, last found, to the next lowest power, and multiply it by the index of the given power, for a divisor.

4. Divide the dividend by the divisor, and the quotient will be the next term of the root.

5. Involve the whole root, and subtract and divide as before; and so on till the whole be finished.

1. Required the square root of $a^4-2a^3r-2a^3r+3a^2r^2-2ar^3+r^4$.

$$a^4-2a^3r+3a^2r^2-2ar^3+r^4\,(a^2-ar+r^2$$
$$a^4$$

$$2a^2)-2a^3r$$

$$a^4-2a^3r+a^2r^2$$

$$2a^2)\,2a^2r^2$$

$$a^4-2a^3r+3a^2r^2-2ar^3+r^4$$

*

2. Extract

2. Extract the cube root of $r^6+6r^5-40r^3+96r-64$.

$$
\begin{array}{l}
r^6+6r^5-40r^3+96r-64)r^2+2r-4 \\
r^6 \\
\hline
3r^4)6r^5 \\
\hline
r^6+6r^5+12r^4+8r^3 \\
\hline
3r^4)-12r^4 \\
\hline
r^6+6r^5-40r3+96r-64 \\
\hline
*
\end{array}
$$

INFINITE SERIES.

An *Infinite Series* is formed from a vulgar fraction, having a compound denominator, or by extracting the root of a surd quantity; and is such, as, being continued, would run on *ad infinitum*, in the manner of a decimal fraction. And, by omitting a few of the first terms, the law of the progression will be manifest, so that the series may be continued, without actually performing the whole operation.

PROBLEM I. *To reduce fractional quantities into infinite series.*

RULE.—Divide the numerator by the denominator, and the operation continued, as far as shall be thought necessary, will give the series required.

1. Let $\frac{1}{1+r}$ be thrown into an infinite series.

$$
\begin{array}{l}
1+r)1 \dots (1-r+r^2-r^3+r^4, \&c. \\
1+r \\
\hline
-r \\
-r-r^2 \\
\hline
+r^2 \\
+r^2+r^3 \\
\hline
-r^3 \\
-r^3-{}^4 \\
\hline
+r^4, \&c.
\end{array}
$$

2. Let $\frac{1}{1-r}$ be thrown into an infinite series.

$$
\begin{array}{l}
1-r(1 \dots (1+r+r^2+r^3+r^4, \&c. \\
1-r \\
\hline
+r \\
+r-r^2 \\
\hline
+r^2 \\
+r^3-r^4 \\
\hline
+r^4, \&c.
\end{array}
$$

3. Let

3. Let $\frac{ar}{a-r}$ be proposed.

$$a-r)\ ar\ \ldots\ (r+\frac{r^2}{a}+\frac{r^3}{a^2}+\frac{r^4}{a^3},\ \&c.$$

$$\frac{ar-r^2}{\quad r^2}$$

$$\frac{r^5-\frac{r^3}{a}}{\frac{r^3}{a}}$$

$$\frac{\frac{r^3}{a}-\frac{r^4}{a^2}}{\frac{r^4}{a^2}}$$

$$\frac{\frac{r^4}{a^2}-\frac{r^5}{a^3}}{\frac{r^5}{a^3},\ \&c.}$$

4. Let $\frac{a^2}{a^2+2ar+r^2}$ be proposed.

$$a^2+2ar+r^2)\ a^2\ \ldots\ (1-\frac{2r}{a}+\frac{3r^2}{a^2}-\frac{4r^3}{a^2}$$

$$\frac{a^2+2ar+r^2}{-2ar-r^2}$$

$$\frac{-2ar-4r^2-\frac{2r^3}{a}}{3r^2+\frac{2r^3}{a}}$$

$$\frac{3r^2+\frac{6r^3}{a}+\frac{3r^4}{a^2}}{-\frac{4r^3}{a}-\frac{3r^4}{a^2}}$$

$$\frac{-\frac{4a^3}{a}-\frac{8r^4}{a^2}-\frac{4r^5}{a^3}}{\frac{5r^4}{a^2}+\frac{4r^5}{a^3}\ \&c.}$$

PROBLEM II. *To reduce a compound surd into an infinite series.*

RULE.—Extract the root to such degree of exactness as shall be thought necessary.

Extract the square root of a^2+r^2 in an infinite series.

$$a^2+r^2)\ a+\frac{r^2}{2a}-\frac{r^4}{8r^3}+\frac{r6}{16a^3}-\frac{5r^8}{128a^7},\ \&c.$$

$$a^2$$

$$2a+\frac{r^2}{2a})\ r^2$$

$$\frac{r^2+\frac{r^4}{4a^2}}{}$$

$$2a+\frac{r^2}{a}-\frac{r^4}{8a^3})\ -\frac{r^4}{4a^2}$$

Carried over.

Brought over. $$\frac{r^4}{4a^2} - \frac{r^6}{8a^4} + \frac{r^8}{64a^6}$$

$$2a + \frac{r^2}{a} - \frac{r^4}{4a^3}, \text{ \&c. } \Big) \; \frac{r^6}{8a^4} - \frac{r^8}{64a^6}$$

$$- \frac{r^6}{8a^4} + \frac{r^8}{16a^6}, \text{ \&c.}$$

$$\frac{5r^8}{64a^6}, \text{ \&c.}$$

ARITHMETICAL PROPORTION.

A Series in Arithmetical Proportion is thus expressed, $a, a+b, a+2b, a+3b, a+4b$, &c. Here the common difference is b. See page 198, &c.

Note. The most useful part of Arithmetical Proportion is contained in the 1st, 3d, and 4th Theorems.

GEOMETRICAL PROPORTION.

A Series in Geometrical Proportion is thus expressed, a, ar, ar^2, ar^3, ar^4, &c. Here r is the ratio. See page 215, &c.

Note. The most useful part of Geometrical Proportion is contained in the 1st, 3d, 5th and 8th Theorems.

SIMPLE EQUATIONS.

An *Equation* is the comparing of two equal quantities which are differently expressed, together, by means of the sign = placed between them.

Thus, $12—7=5$ is an equation, expressing the quality of the quantities $12—7$ and 5.

A simple equation is that which contains only one unknown quantity, without including its power. Thus, $r—a+b=c$ is a simple equation, containing only the unknown quantity r.

Reduction of equations is the method of finding the value of the unknown quantity, which is shewn in the following rules.

Rule 1. Any quantity may be transposed from one side of the equation to the other, by changing its sign.

Thus, if $r+3=7$, then will $r=7—3=4$. And, if $r—4+6=8$, then will $r=8+4—6=6$. Also, if $r—a+b=c—d$, then will $r=c—d+a—b$. And, if $4r—8=3r+20$, then will $4a—3r=20+8$, or $r=28$.

Rule 2. If the unknown term be multiplied by any quantity, it may be taken away by dividing all the other terms of the equation by it.

Thus, if $ar=ab—a$, then will $r=b—1$. If $2r+4=16$, then will $r+2=8$, and $r=8—2=6$. Also, if $ar+2ba=3c^2$, then will $r+2b=\frac{3c^2}{a}$, and $r=\frac{3c^2}{a}—2b$.

Rule

Rule 3.—If the unknown term be divided by any quantity, it may be taken away by multiplying all the terms of the equation by it.

Thus, if $\frac{r}{2}=5+3$, then will $r=10+6=16$. If $\frac{r}{a}=b+c-d$, then will $r=ab+ac-ad$. Also, if $\frac{2r}{3}-2=6+4$, then will $2r-6=18+12$, and $2r=18+12+6=36$, or $r=\frac{36}{2}=18$.

Rule 4.—The unknown quantity in any equation may be made free from surds, by transposing the rest of the terms according to the rule, and then involving each side to such a power as is denoted by the index of the said surd.

Thus, if $\sqrt{r}-2=6$, then will $\sqrt{r}=6+2=8$, and $r=8^2=64$ If $\sqrt{4r+16}=12$, then will $4r+16=144$, and $4r=144-16=128$, or $r=\frac{128}{4}=32$. Also, if $\sqrt[3]{2r+3}+4=8$; then will $\sqrt[3]{2r+3}=8-4=4$, and $2r+3=4^3=64$, and $2r=64-3=61$, or $r=\frac{61}{2}=30\frac{1}{2}$.

Rule 5.—If that side of the equation, which contains the unknown quantity, be a complete power, it may be reduced by extracting the root of said power from both sides of the equation.

Thus, if $r^2+6r+9=25$, then will $r+3=\sqrt{25}=5$, or $r=5-3=2$. If $3r^2-9=21+3$, then will $3r^2=21+3+9=33$, and $r^2=\frac{33}{3}=11$, or $r=\sqrt{11}$. Also, if $\frac{2r^2}{3}+10=20$, then will $2r^2+30=60$, and $r^2+15=30$, or $r^2=30-15$, or $r=\sqrt{15}$.

Rule 6.—Any analogy, or proportion, may be converted into an equation, by making the product of the two mean terms equal to that of the two extremes.

Thus, if $3r : 16 :: 5 : 10$, then will $3r\times10=16\times5$, and $30r=80$, or $r=\frac{80}{30}=2\frac{2}{3}$. If $\frac{2r}{3} : a :: b : c$, then will $\frac{2cr}{3}=ab$, and $2cr=3ab$, or $r=\frac{3ab}{2c}$. Also, if $12-r : \frac{r}{2} :: 4 : 1$, then will $12-r=\frac{4r}{2}=2r$, and $2r+r=12$, or $r=\frac{12}{3}=4$.

Rule 7.—If any quantity be found on both sides of the equation with the same sign, it may be taken away from them both; and if every term in an equation be multiplied or divided by the same quantity, it may be struck out of them all. Thus,

Thus, if $4r+a=b+a$, then will $4r=b$, and $r=\frac{b}{4}$. If $3ar+5ab=8ac$, then will $3r+5b=8c$, and $r=\frac{8c-5b}{3}$. Also, if $\frac{2r}{3}-\frac{8}{3}=\frac{16}{3}-\frac{8}{3}$, then will $2r=16$, and $r=8$.

Miscellaneous Examples.

1. Given $5r-15=2r+6$, to find the value of r.

First, $5r-2r=6+15$, then $3r=21$, and $r=\frac{21}{3}=7$.

2. Given $40-6r-16=120-14r$ to find r.

First, $14r-6r=120-40+16$, then $8r=96$, therefore $r=\frac{96}{8}=12$.

3. Given $5ar-3b=2dr+c$, to find r.

First, $5ar-2dr=c+3b$, or $\overline{5a-2d}\times r=c+3b$, therefore $r=\frac{c+3b}{5a-2d}$.

4. Given $3r^2-10r=8a+r^2$, to find r.

First, $3r-10=8+r$, then $3r-r=8+10$, therefore $2r=18$, and $r=\frac{18}{2}=9$.

5. Given $6ar^3-12abr^2=3ar^3+6ar^2$, to find r.

First, dividing the whole by $3ar^2$, we shall have $2r-4b=r+2$, then $2r-r=2+4b$, whence $r=2+4b$.

6. Given $\frac{r}{2}-\frac{r}{3}+\frac{r}{4}=10$, to find r.

First, $r-\frac{2r}{3}+\frac{2r}{4}=20$, then $3r-2r+\frac{6r}{4}=60$, and $12r-8r+6r=240$, therefore $10r=240$, and $r=\frac{240}{10}=24$.

7. Given $\frac{r-3}{2}+\frac{r}{3}=20-\frac{r+19}{2}$, to find r.

First, $r-3+\frac{2r}{3}=40-r-19$, then $3r-9+2r=120-3r-57$, therefore $3r+2r+3r=120-57+9$, that is, $8r=72$, or $r=\frac{72}{8}=9$.

8. Given $\sqrt{\frac{2}{3}r}+5=7$, to find r.

First, $\sqrt{\frac{2}{3}r}=7-5=2$, then $\frac{2}{3}r=2^2=4$, and $2r=12$, or $r=\frac{12}{2}=6$.

9. Given

9. Given $x+\sqrt{a^2+x^2}=\frac{2a^2}{\sqrt{a^2+x^2}}$, to find x.

1. $x+\sqrt{a^2+x^2}\times\sqrt{a^2+x^2}=x\sqrt{a^2+x^2}+a^2+x^2$, therefore
2. $x\sqrt{a^2+x^2}+a^2+x^2=2a^2$, transp. $+a^2+x^2$ then the equat. will be
3. $x\sqrt{a^2+x^2}=2a^2-a^2-x^2$, or a^2-x^2, now square both sides of [the equat.
4. $x\sqrt{a^2+x^2}\times x\sqrt{a^2+x^2}=x^2\times a^2+x^2=a^2x^2+x^4$ and
5. $a^2-x^2\times a^2-x^2=a^4-2a^2x^2+x^4$. Therefore,
6. $a^2x^2+x^4=a^4-2a^2x^2+x^4$, transpose $+x^4$
7. $a^2x^2=a^4-2a^2x^2$. transpose $-2a^2x^2$
8. $a^2x^2+2a^2x^2$ or $3a^2x^2=a^4$, consequently
9. $x^2=\frac{a^4}{3a^2}$, and
10. $x=\sqrt{\frac{a^4}{3a^2}}=a^2\sqrt{\tfrac{1}{3}}$.

PROBLEM I. *To exterminate two unknown quantities, or to reduce the two simple equations, containing them, to a single one.*

RULE 1st.—1. Observe which of the unknown quantities is the least involved, and find its value in each of the equations, by the methods already explained.

2. Let the two values thus found be made equal to each other, and there will arise a new equation with only one unknown quantity in it, whose value may be found as before.

1. Given $\left\{\begin{array}{l}2r+3u=23\\5r-2u=10\end{array}\right\}$ to find r and u.

From the first equation, $r=\frac{23-3u}{2}$, and from the second, $r=\frac{10+2u}{5}$, and consequently $\frac{23-3u}{2},=\frac{10+2u}{5}$, or $115-15u=20+4u$, or $19u=115-20=95$, and $u=\frac{95}{19}=5$, whence $r=\frac{23-15}{2}=4$.

2. Given $\left\{\begin{array}{l}r+u=a\\r-u=b\end{array}\right\}$ to find r and u.

From the first equation, $r=a-u$, and from the second, $r=b+u$, therefore $a-u=b+u$, or $2u=a-b$, consequently $u=\frac{a-b}{2}$, and $r=a-u=a-\frac{a-b}{2}=\frac{a+b}{2}$.

3. Given $\left\{\begin{array}{l}\frac{r}{2}+\frac{u}{3}=7\\[4pt]\frac{r}{3}+\frac{u}{2}=8\end{array}\right\}$ to find r and u.

From

From the first equation, $r=14-\frac{2u}{3}$, and from the second, $r=24-\frac{3u}{2}$, therefore $14-\frac{2u}{3}=24-\frac{3u}{2}$, and $42-2u=72-\frac{9u}{2}$, or $84-4u=144-9u$; whence $5u=144-84=60$, and $u=\frac{60}{5}=12$, and $r=14-\frac{2u}{3}=14-\frac{24}{3}=6$.

Rule 2d.—1. Consider which of the unknown quantities you would first exterminate, and let its value be found in that equation where it is the least involved.

2. Substitute the value, thus found, for its equal in the other equation, and there will arise a new equation, with only one unknown quantity, whose value may be found as before.

1. Given $\left\{\begin{array}{l} r+2u=17 \\ 3r-\ u=\ 2 \end{array}\right\}$ to find r and u.

From the first equation, $r=17-2u$, and this value, substituted for r in the second, gives $\overline{17-2u}\times3-u=2$, or $51-6u-u=2$, or $51-7u=2$; that is, $7u=51-2=49$; whence $u=\frac{49}{7}=7$, and $r=17-2u=17-14=3$.

2. Given $\left\{\begin{array}{l} a : b :: r : u \\ r^2+u^2=c \end{array}\right\}$ to find r and u.

The first analogy, turned into an equation, is $br=au$, or $r=\frac{au}{b}$, and this value of r, substituted in the second, gives, $\left.\overline{\frac{au}{b}}\right|^2+u^2=c$, or $\frac{a^2u^2}{b^2}+u^2=c$, or $a^2u^2+b^2u^2=cb^2$, or $u^2=\frac{cb^2}{a^2+b^2}$, therefore $u=\left.\overline{\frac{cb^2}{a^2+b^2}}\right|^{\frac{1}{2}}$, and $r=\left.\overline{\frac{ca^2}{a^2+b^2}}\right|^{\frac{1}{2}}$.

Rule 3.—Let the given equations be multiplied or divided by such numbers or quantities as will make the term, which contains one of the unknown quantities, to be the same in both equations, and then by adding or subtracting the equations, accordingly as is required, there will arise a new equation with only one unknown quantity, as before.

1. Given $\left\{\begin{array}{l} 3r+5u=40 \\ r+2u=14 \end{array}\right\}$ to find r and u.

First, multiply the 2d equation by 3, and we shall have $3r+6u=42$, then from this last equation subtract the first, and it will give $6u-5u=42-40$, or $u=2$, therefore $r=14-2u=14-4=10$.

2. Given

2. Given $\left\{\begin{array}{l} 5r-3u=9 \\ 2r+5u=16 \end{array}\right\}$ to find r and u.

Let the first equation be multiplied by 2, and the second by 5, and we shall have $\left\{\begin{array}{l} 10r-6u=18 \\ 10r+25u=80 \end{array}\right\}$ and if the former of these be subtracted from the latter it will give $31u=62$, or $u=\frac{62}{31}=2$, consequently, $r=\frac{9+6}{5}=\frac{15}{5}=3$.

Another Method.

Multiply the 1st equation by 5, and the 2d by 3, and we shall have $\left\{\begin{array}{l} 25r-15u=45 \\ 6r+15u=48 \end{array}\right\}$ Now, add these two equations, and it will give $31r=93$, or $r=\frac{93}{31}=3$, consequently $u=\frac{16-6}{5}=\frac{10}{5}=2$, as before.

PROB. II. *To exterminate three unknown quantities, or to reduce the three simple equations, containing them, to a single one.*

RULE.—1. Let r, u, and z, be three unknown quantities to be exterminated.

2. Find the value of r, from each of the three given equations.

3. Compare the first value of r with the second, and an equation will arise, involving only u and z.

4. Compare the first value of r with the third, and another equation will arise, involving only u and z.

5. Find the value of u and z from these two equations, according to the former rules, and r, u, and z, will be exterminated as required.

Note, Any number of unknown quantities may be exterminated in nearly the same manner.

1. Given $\left\{\begin{array}{l} r+u+z=29 \\ r+2u+3z=62 \\ \frac{r}{2}+\frac{u}{3}+\frac{z}{4}=10 \end{array}\right\}$ to find r, u, and z.

From the first equation, $r=29-u-z$. From the 2d $r=62-2u-3z$. From the 3d $r=20-\frac{2u}{3}-\frac{z}{2}$, whence $29-u-z=62-2u-3z$, and $29-u-z=20-\frac{2u}{3}-\frac{z}{2}$; but from the first of these equations, $u=62-29-2z=33-2z$; and from the 2d $u=27-\frac{3z}{2}$, therefore $33-2z=27-\frac{3z}{2}$, or $z=12$, and $u=62-29-2z=62-29-24=9$, and $r=29-u-z=29-12-9=8$.

2. Given

2. Given $\left\{\begin{array}{l}\frac{r}{2}+\frac{u}{3}+\frac{z}{4}=62\\ \frac{r}{3}+\frac{u}{4}+\frac{z}{5}=47\\ \frac{r}{4}+\frac{u}{5}+\frac{z}{6}=38\end{array}\right\}$ to find r, u, and z.

First, the given equations, cleared from fractions, become

$$12r+8u+6z=1488$$
$$20r+15u+12z=2820$$
$$30r+24u+20z=4560$$

Then, if the second equation be subtracted from double the first, and three times the third, from five times the second, we shall have

$$4r+u=156$$
$$10r+3u=420$$

And again, if the second of these be subtracted from three times the first, it will give $12r-10r=468-420$, or $r=\frac{48}{2}=24$, therefore $u=156-4r=60$, and $z=\frac{1488-8u-12r}{6}=120$.

Questions producing Simple Equations.

1. To find two such numbers, as that their sum shall be 40, and their difference 16.

Let r denote the least of the two numbers required, then will $r+16$ = the greater, $r+r+16=40$ by the question, that is, $2r=40-16=24$, or $r=\frac{24}{2}=12=$ least number, and $r+16=12+16=28=$ greater number required.

2. What number is that, whose $\frac{1}{3}$ part exceeds its $\frac{1}{4}$ part by 16?

Let r=number required, then will its $\frac{1}{3}$ part be $\frac{r}{3}$, and its $\frac{1}{4}$ part $\frac{r}{4}$; therefore $\frac{r}{3}-\frac{r}{4}=16$, by the question, that is $r-\frac{3r}{4}=48$, or $4r-3r=192$; whence $r=192$ the number required.

3. Divide £.1000 between A, B and C, so that A shall have £.72 more than B, and C £.100 more than A.

Let r = B's share of the given sum, then will $r+72$ = A's share, and $r+172$ =C's share; and the sum of all these shares $r+r+72+r+172$, or $3r+244=1000$, by the question, that is, $3r=1000-244=756$, or $r=\frac{756}{3}=$ £.252 = B's share, and $r+72=252+72=$ £.324 = A's share, and $r+172=252+172=$ £.424 = C's share.

Proof, $252+324+424=$ £.1000.

4. A.

4. A prize of D.1000 is to be divided between two persons, whose shares therein are in the proportion of 7 to 9: Required the share of each?

Let r = the first person's share, then will D.1000—r = 2d person's share, and $r : 1000-r :: 7 : 9$, by the question, that is, $9r = \overline{1000-r} \times 7 = 7000-7r$, or $16r = 7000$, whence $r = \frac{7000}{16}$ = D.437 50c. = 1st share, and $1000-r = 1000$—D.437 50c. = D.562 50c. = 2d share.

5. The paving of a square at 40c. per yard, cost as much as the inclosing of it, at D.1 per yard: required the side of the square?

Let r = side of the square sought, then $4r$ = yards of inclosure, and r^2 = yards of pavement; whence $4r \times 100 = 400r$ = price of inclosing, and $r^2 \times 40 = 40r^2$ = price of paving. But $40r^2 = 400r$, by the question, therefore $r^2 = 10r$, and $r = 10$ = length of the side required.

6. A labourer engaged to serve 40 days upon these conditions, that for every day he worked, he should receive 20 cents, but for every day he was absent, he should forfeit 8 cents. Now at the end of the time, there was due to him D.3 80c.: how many days did he work, and how many days was he absent?

Let r be the number of days he worked, then will $40-r$ be the number of days he was absent: also, $r \times 20 = 20r$ = sum earned, and $\overline{40-r} \times 8 = 320-8r$ = sum forfeited; whence $20r - \overline{320-8r} = 380$c. (= D.3 80c.) by the question, that is, $20r-320+8r = 380$ or $28r = 380 + 320 = 700$, and $r = \frac{700}{28} = 25$ = number of days he worked; and $40-r = 40-25 = 15$ = number of days he was absent.

7. Out of a cask of wine, which had leaked away $\frac{1}{3}$, 21 gallons were drawn; and then, being gauged, it appeared to be half full: how much did it hold?

Let it be supposed to have holden r gallons, then it would have leaked $\frac{r}{3}$ gallons, and consequently there had been taken away $21 + \frac{r}{3}$ gallons. But $21 + \frac{r}{3} = \frac{r}{2}$ by the question, that is, $63+r = \frac{3r}{2}$ or $126+2r = 3r$, hence $3r-2r = 126$, or $r = 126$, Answer.

8. What fraction is that, to the numerator of which if 1 be added, the value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$?

Let the fraction be represented by $\frac{r}{u}$ then will $\frac{r+1}{u} = \frac{1}{3}$ and $\frac{r}{u+1} = \frac{1}{4}$, or $3r+3 = u$, and $4r = u+1$; hence $4r-3r-3 = u+1-u$, that is, $r-3 = 1$, or $r = 4$, and $u = 3r+3 = 12+3 = 15$. So that $\frac{4}{15}$ = fraction required.

9. A

9. A market woman bought a certain number of eggs, at 2 for a cent, and as many, at 3 for a cent, and sold them all out again, at the rate of 5 for 2 cents, and by so doing, lost 4 cents: what number of eggs, had she?

Let r = number of eggs of each sort, then will $\frac{r}{2}$ = price of the first sort, and $\frac{r}{3}$ = price of the second sort. But $5 : 2 :: 2r$ (the whole number of eggs) : $\frac{4r}{5}$; therefore $\frac{4r}{5}$ = price of both sorts together, at 5 for 2 cents, and $\frac{r}{2}+\frac{r}{3}-\frac{4r}{5}=4$, by the question; that is, $r+\frac{2r}{3}-\frac{8r}{5}=8$; or $3r+2r-\frac{24r}{5}=24$; or $15r+10r-24r=120$; whence $r=120$ = number of eggs of each sort required.

10. A person in the afternoon being asked what o'clock it was, answered that $\frac{3}{5}$ of the time from noon was equal to $\frac{5}{8}$ of the time to midnight: required the time?

Let r = the time sought from noon, then will $12-r$ = the time to midnight, $\frac{3}{5}$ of the time from noon $=\frac{3r}{5}$, and $\frac{5}{8}$ of the time to midnight $=\frac{60-5r}{8}$, therefore $\frac{3r}{5}=\frac{60-5r}{8}$ by the question; whence, $3r=\frac{300-25r}{8}$ and $24r=300-25r$, or $24r+25r=300$ or $r=\frac{300}{49}=6h.$ $7'\ 20''\frac{40}{49}$, Answer.

11. A merchant ships goods for South Carolina, to the amount of £.700: what sum, at 5 per cent. should he get insured, to cover his adventure?

Let r = sum to be insured, then will $r-\frac{5r}{100}=700$, whence $100r-5r=70000$, and $r=\frac{70000}{95}$ = £.736 16s. $10\frac{10}{95}$d. Ans.

12. A man lays out 30 cents for apples and pears, buying his apples, at 4, and his pears, at 5 for a cent, and afterwards sold $\frac{1}{2}$ of his apples, and $\frac{1}{3}$ of his pears for 13 cents, which was the prime cost: I demand the number he bought of each?

Let r = the number of apples, and, z = the number of pears; then, if 4 apples cost a cent, r will cost $\frac{r}{4}$ cents, and for the same reason z will

z will cost $\frac{z}{5}$ cents, and we shall have $\frac{r}{4}+\frac{z}{5}=30$, for one fundamental equation. Again, the price of $\frac{r}{2}=\frac{1}{2}$ of his apples will be $\frac{r}{8}$, and the price of $\frac{z}{3}=\frac{1}{3}$ of his pears will be $\frac{z}{15}$; hence $\frac{r}{8}+\frac{z}{15}=13$, for another fundamental equation. Now, cross multiplying $\frac{r}{4}+\frac{z}{5}=30$, and then multiplying 30 by 4 and 5 we shall have the first equation $=5r+4z=600$; and doing the same by $\frac{r}{8}+\frac{z}{15}=13$, we have the second equation $=15r+8z=1560$. Subtract the second equation from three times the first, and we shall have the third equation $=4z=240$, therefore, fourth equation $=z=60$ the number of pears: Now, substitute 60 for z, that is, 240 for $4z$, in the first equation. $5r+4z=600$, we shal have $5r=240=600$, whence, equation fifth shall $r=72=$the number of apples.

QUADRATICK EQUATIONS.

A Simple Quadratick Equation is that, which involves the square of the unknown quantity only.

An Adfected Quadratick Equation is that which involves the square of the unknown quantity, together with the product, which arises from multiplying it by some known quantity.

Thus, $ar^2=b$, is a simple quadratick equation, and $ar^2+br=c$ is an adfected quadratick equation.

All adfected quadratick equations fall under the three following forms.

1st. $r^2+ar=b$. 2d. $r^2-ar=b$. 3d. $r^2-ar=-b$. And the rule for finding the value of r, in each of these equations, is as follows:

Rule* 1. Transpose all the terms, which involve the unknown quantity, to one side of the equation, and the known terms to the other side, and let them be ranged according to their dimensions.

2. When

* The square root of any quantity may be either + or —, and therefore all quadratick equations admit of two solutions. Thus the square root of $+n^2$ is $+n$, or $-n$, for $+n\times+n$, or $-n\times-n$, are each equal to $+n^2$. So in the first form, where $r+\frac{a}{2}$ is found $=\sqrt{b+\frac{a^2}{4}}$, the root may be either $+\sqrt{b+\frac{a^2}{4}}$, or $-\sqrt{b+\frac{a^2}{4}}$, since either of them being multiplied by itself will produce $b+\frac{a^2}{4}$. And this ambiguity is expressed by writing the uncertain sign $\pm$ before $\sqrt{b+\frac{a^2}{4}}$;

thus

2. When the square of the unknown quantity has any coefficient prefixed to it, let all the rest of the terms be divided by that coefficient.

3. Add the square of half the coefficient of the second term to both sides of he equation, and that side, which involves the unknown quantity will be a complete square.

4. Extract the square root from both sides of the equation, and the value of the unknown quantity will be determined.

Note, 1. The square root of one side of the equation is always equal to the unknown quantity, with half the coefficient of the second term subjoined to it.

2. All

thus $r = \pm\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$. In the firſt form, where $r=\pm\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ the firſt value of r, viz. $r=+\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ is always affirmative.

The ſecond value, viz. $r=-\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$, will always be negative, becauſe it is compoſed of two negative terms; therefore, when $r^2+ar=b$, we ſhall have $r=+\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$ for the affirmative value of r, and $r=-\sqrt{b+\frac{a^2}{4}}-\frac{a}{2}$, for the negative value of r.

In the ſecond form, where $r=\pm\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, the firſt value, viz. $r=+\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$ is always affirmative, ſince it is compoſed of two affirmative terms, and the ſecond value, viz. $r=-\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$ will always be negative; therefore when $r^2-ar=b$, we ſhall have $r=+\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, for the affirmative value of r, and $r=-\sqrt{b+\frac{a^2}{4}}+\frac{a}{2}$, for the negative value of r.

In the third form, where $r=\sqrt{\frac{a^2}{4}-b}+\frac{a}{2}$, both the values of r will be poſitive, ſuppoſing $\frac{a^2}{a}$ to be greater than b. Therefore when $r^2-ar=-b$, we ſhall have $r=+\sqrt{\frac{a^2}{4}-b}+\frac{a}{2}$, and $-\sqrt{\frac{a^2}{4}-b}+\frac{a}{2}$, both, for the affirmative value of r.

But in this third form, if b be greater than $\frac{a^2}{4}$, the ſolution of the propoſed queſtion will be impoſſible. For ſince the ſquare of any quantity is always affirmative, the ſquare root of a negative quantity is impoſſible.

2. All equations, wherein there are two terms involving the unknown quantity, and the index of one is just double that of the other, are solved like quadraticks, by completing the square.

Thus, $r^4+ar^2=b$, or $r^n+ar^{\frac{n}{2}}=b$, are the same as quadraticks, and the value of the unknown quantity may be determined accordingly.

From this rule may be formed a general theorem, with which all particular equations may be compared, and by means whereof they may be more readily resolved.

Suppose $ar^2=br+c$ be the general quadratick equation proposed to be resolved; where a, b, and c denote known integral quantities, whether affirmative, or negative, and r=the unknown quantity; to find the values of r in this equation.

Here, transposing br, we have $ar^2-br=c$, then dividing by a, in order to free r^2 the highest power of r from its coefficient, we have $r^2-\frac{br}{a}=\frac{c}{a}$; this being done, the first side, $r^2-\frac{br}{a}$ may be considered as an imperfect square raised from a binomial root, and accordingly we may complete that square by adding the square of half the coefficient of the second term: but if $\frac{bb}{4a^2}$ must be added to the first side of the equation, to complete the square, it must be also added to the other side, to preserve the equality, otherwise by an unequal addition, the equation would be destroyed; this equal addition therefore being made, the equation will stand thus, $r^2-\frac{br}{a}+\frac{b^2}{4a^2}=\frac{b^2}{4a^2}+\frac{c}{a}$; but the two fractions $\frac{b^2}{4a^2}$ and $\frac{c}{a}$ when added give $\frac{ab^2+4a^2c}{4a^3}$, which divided by a gives $\frac{b^2+4ac}{4a^2}$; therefore $r^2-\frac{br}{a}+\frac{b^2}{4a^2}=\frac{b^2+4ac}{4a^2}$; therefore the square root of one side will be equal to the square root of the other; but the square root of $\frac{b^2+4ac}{4a^2}$, as it here stands in letters, cannot be extracted, because although the denominator $4a^2$ be a square, yet there is no literal quantity whatever, which, being multiplied into itself, will produce b^2+4ac, therefore, to put this numerator into the form of a square, let us suppose $b^2+4ac=ss$, and then the equation will be $r^2-\frac{br}{a}+\frac{b^2}{4a^2}=\frac{ss}{4a^2}$; but the square

 root

root of $r^2 - \frac{br}{a} + \frac{b^2}{4a^2}$ is $r - \frac{b}{2a}$, and that of $\frac{ss}{4a^2}$ is $\pm \frac{s}{2a}$, therefore this equation will now be reduced to a simple one, and will stand thus, $r - \frac{b}{2a} = \pm \frac{s}{2a}$, therefore $r = \frac{b \pm s}{2a}$, that is, $r = \frac{b+s}{2a}$ and $r = \frac{b-s}{2a}$.

Note. When the quantity c (and consequently $4ac$) is negative, the quantity ss, or b^2+4ac must be considered as the sum of the affirmative quantity b^2 and the negative one $4ac$, when added together according to the common rules of Addition.

Examples of the resolution of Adfected Equations with and without the general Theorem.

1. Given $r^2 = 140 - 4r$, to find the values of r.

First, transposing $-4r$, it is $r^2+4r = 140$, then, $r^2+4r+4 = 144$ by completing the square; then $\sqrt{r^2+4r+4} = \sqrt{144}$, by extracting the root; or $r+2 = \pm 12$, that is, $r = -2 \pm 12 = +10$, or -14.

By the general Theorem. a, in the general Theorem, answers to 1 in the particular one, that is to the coefficent of r^2, b answers to 4, and c to 140, that is $a = 1$, $b = -4$, $c = 140$, and $4ac = 560$, therefore ss, or b^2+4ac will be the sum of 16 and $560 = 576$, therefore, $s = 24$, $\frac{b+s}{2a} = \frac{-4+24}{2} = +10$, and $\frac{b-s}{2a} = \frac{-4-24}{2} = -14$; therefore the two roots of this equation are 10 and -14.

2. Given $r^2+8 = 6r+80$, to find r.

First, $r^2-6r = 72$, by transposition; then $r^2-6r+9 = 72+9 = 81$, by completing the square, and $r-3 = \sqrt{81} = \pm 9$, by extracting the root, therefore $r = +3 \pm 9 = +12$, or -6.

By the Theorem. $a = 1$, $b = 6$, $c = 72$, and $4ac = 288$, therefore $ss = 36+288 = 324$, therefore $s = 18$, $\frac{b+s}{2a} = 12$, and $\frac{b-s}{2a} = -6$.

3. Given $2r^2-20 = 70-8r$ to find r.

First, $2r^2+8r = 70+20 = 90$, by transposition, then $r^2+4r = 45$, by dividing by the coefficient 2, and $r^2+4r+4=45+4 = 49$, by completing the square; whence $r+2 = \sqrt{49} = \pm 7$, therefore, $r = -2 \pm 7 = 5$ or -9.

By the Theorem. $a = 2$, $b = -8$, $c = 90$, $4ac = 720$, $ss = 64+720 = 784$, therefore $s = 28$, $\frac{b+s}{2a} = +5$, $\frac{b-s}{2a} = -9$, so that $+5$, and -9 are the value of r.*

As

* In a quadratick equation of this form $ar^2 = br+c$, the ſum of the roots will always be $\frac{b}{a}$, and the product of their multiplication $\frac{-c}{a}$; therefore, if $a = 1$, that is, if the equation be $r^2 = br+c$, the ſum of the roots will be b, and their product $-c$, or the ſum will be the coefficient of the unknown quantity on the ſecond ſide of the equation, and their product, what is called the abſolute term, with its ſign changed.

As the general Theorem is sufficiently exemplified in the preceding problems, the following equations will be solved by the Rule only.

4. Given $3r^2-3r+6=5\frac{1}{3}$, to find r.

Here $r^2-r+2=1\frac{7}{9}$ by dividing by 3, and $r^2-r=1\frac{7}{9}-2$, by transposition; also $r^2-r+\frac{1}{4}=1\frac{7}{9}-2+\frac{1}{4}=\frac{1}{36}$ by completing the square; and $r-\frac{1}{2}=\sqrt{\frac{1}{36}}=\pm\frac{1}{6}$, by evolution; therefore $r=+\frac{1}{2}\pm\frac{1}{6}=\frac{2}{3}$, or $\frac{1}{3}$.

5. Given $\frac{r^2}{2}-\frac{r}{3}+20\frac{1}{2}=42\frac{2}{3}$, to find r.

Here $\frac{r^2}{2}-\frac{r}{3}=42\frac{2}{3}-20\frac{1}{2}=22\frac{1}{6}$, by transposition, and $r^2-\frac{2r}{3}=44\frac{1}{3}$, by dividing by $\frac{1}{2}$, whence $r^2-\frac{2r}{3}+\frac{1}{9}=44\frac{1}{3}+\frac{1}{9}=44\frac{4}{9}$, by completing the square, and $r-\frac{1}{3}=\sqrt{44\frac{4}{9}}=\pm 6\frac{2}{3}$, therefore $r=+\frac{1}{3}\pm 6\frac{2}{3}=7$, or $-6\frac{1}{3}$.

6. Given $ar^2+br=c$, to find r.

First, $r^2+\frac{b}{a}r=\frac{c}{a}$, by division; then $r^2+\frac{b}{a}r+\frac{b^2}{4a^2}=\frac{c}{a}+\frac{b^2}{4a^2}$, by completing the square; and $r+\frac{b}{2a}=\sqrt{\frac{c}{a}+\frac{b^2}{4a^2}}=\sqrt{\frac{4ac+b^2}{4a^2}}$, by evolution, therefore $r=\pm\sqrt{\frac{4ac+b^2}{4a^2}}-\frac{b}{2a}$.

7. Given $ar^2-br+c=d$, to find r.

Here, $ar^2-br=d-c$, by transposition, and $r^2-\frac{b}{a}r=\frac{d-c}{a}$ by division.

Also, $r^2-\frac{b}{a}r+\frac{b^2}{4a^2}=\frac{d-c}{a}+\frac{b^2}{4a^2}$ by completing the square; and $r-\frac{b}{2a}=\pm\sqrt{\frac{d-c}{a}+\frac{b^2}{4a^2}}$, by evolution; therefore $r=\frac{b}{2a}\pm\sqrt{\frac{d-c}{a}+\frac{b^2}{4a^2}}$.

8. Given $r^4+2ar^2=b$, to find r.

Here, $r^4+2ar^2+a^2=b+a^2$, by completing the square, and $r^2+a=\sqrt{b+a^2}$, by evolution; whence $r^2=\sqrt{b+a^2}-a$, and consequently $r=\sqrt{:\sqrt{b+a^2}-a}$.

9. Given $ar^n-br^{\frac{n}{2}}-c=-d$, to find r.

First,

First, $ar^n - br^{\frac{n}{2}} = c - d$, by transposition, and $r^n - \frac{b}{a} r^{\frac{n}{2}} = \frac{c-d}{a}$, by division. Also, $r^n - \frac{b}{a} r^{\frac{n}{2}} + \frac{b^2}{4a^2} = \frac{c-b}{a} + \frac{b^2}{4a^2}$, by completing the square, and $r^{\frac{n}{2}} - \frac{b}{2a} = \sqrt{\frac{c-d}{a} + \frac{b^2}{4a^2}}$, by evolution; therefore $r^{\frac{n}{2}} = \frac{b}{2a} \pm \sqrt{\frac{c-d}{a} + \frac{b^2}{4a^2}}$ and consequently $r = \left|\overline{\frac{b}{2a} \pm \sqrt{\frac{c-d}{a} + \frac{b^2}{4a^2}}}\right|^{\frac{n}{2}}$.

QUESTIONS PRODUCING QUADRATICK EQUATIONS.

1. To find two numbers, whose difference is 8, and product 240.

Let r = the least number, then will $r+8$ = the greater, and $r \times \overline{r+8} = r^2 + 8r = 240$ by the question; whence $r^2+8r+16 = 240+16 = 256$ by completing the square; also $r+4 = \sqrt{256} = 16$, by evolution, and therefore $r = 16-4 = 12$ = the least number, and $12+8 = 20$ = the greater.

2. To divide the number 60 into two such parts, as that their product may be 864.

Let r = greater part, then will $60-r$ = the less, and $r \times \overline{60-r} = 60r - r^2 = 864$, by the question, that is, $r^2 - 60r = -864$; whence $r^2 - 60r + 900 = -864 + 900 = 36$, by completing the square: also $r - 30 = \sqrt{36} = 6$, by extracting the root; therefore $r = 6+30 = 36$ = greater part, and $60 - r = 60 - 36 = 24$ = the less part.

3. Sold a piece of cloth for £.24 and gained as much per cent. as the cloth cost me: what was the price of it?

Let r = pounds the cloth cost, then $24 - r$ = whole gain, but $100 : r :: r : 24 - r$, by the question, or $r^2 = 100 \times \overline{24-r} = 2400 - 100r$, that is, $r^2 + 100r = 2400$; whence $r^2 + 100r + 2500 = 2400 + 2500 = 4900$, by completing the square, and $r + 50 = \sqrt{4900} = 70$ by extraction of roots, consequently $r = 70 - 50 = 20$ = price of the cloth.

4. A person bought a number of oxen for D.320 and if he had bought four more for the same money, he would have paid D.4 less for each: how many did he buy?

Suppose he bought r oxen, then $\frac{320}{r}$ = price of each, and $\frac{320}{r+4}$ = price of each, if $r+4$ had cost D.320. But $\frac{320}{r} = \frac{320}{r+4} + 4$, by the question, or $320 = \frac{320r}{r+4} + 4r$, or $320r + 1280 = 320r + 4r^2 + 16r$, that is, $4r^2 + 16r = 1280$; whence $4r^2 + 16r + 16 = 1289 + 16 = 1296$, by completing the square, and $2r + 4 = \sqrt{1296} = 36$, by evolution, consequently $2r = 36 - 4 = 32$, and $r = \frac{32}{2} = 16$ = number of oxen required.

5. What

5. What two numbers are those, whose sum, product and difference of their squares, are all equal to each other?

Let r = greater number, and u = less; then $\left\{\begin{array}{l} r+u = ru \\ r+u = r^2-u^2 \end{array}\right\}$ by the question, and $1 = \frac{r^2-u^2}{r+u} = r-u$, or $r = u+1$, from the 2d equation: also $u+1+u = \overline{u+1}\times u$, from the first equation; or $2u+1 = u^2+u$, that is, $u^2-u = 1$; whence $u^2-u+\frac{1}{4} = 1\frac{1}{4}$, by completing the square: also $u-\frac{1}{2} = \sqrt{1\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ by evolution, consequently $u = \frac{\sqrt{5}}{2} + \frac{1}{2} = \frac{\sqrt{5}+1}{2}$, and $r = u+1 = \frac{\sqrt{5}+3}{2}$.

6. There are four numbers in Arithmetical Progression, whereof the product of the two extremes is 45, and that of the means 77: what are the numbers?

Let r = less extreme, and u = common difference, then r, $r+u$, $r+2u$, $r+3u$ will be the four numbers, and

$\left\{\begin{array}{l} r\times\overline{r+3u} = r^2+3ru = 45 \\ \overline{r+u}\times\overline{r+2u} = r^2+3ru+2u^2 = 77 \end{array}\right\}$ by the question; whence $2u^2 = 77 - 45 = 32$, and $u^2 = \frac{32}{2} = 16$, by subtraction and division, or $u = \sqrt{16} = 4$ by evolution; therefore $r^2+3ru = r^2+12r = 45$, by the first equation; also $r^2+12r+36 = 45+36 = 81$, by completing the square, and $r+6 = \sqrt{81} = 9$, by the extraction of roots, consequently $r = 9 - 6 = 3$, and the numbers are 3, 7, 11 and 15.

RECAPITULATION OF THE PRINCIPLES OF ARITHMETICK AND ALGEBRA.

Axiom 1. Since whole numbers increase in a decuple proportion, 10 is the universal ratio of any series of numbers whatever; and the reason for carrying at 10 in Addition and Multiplication is self-evident, since 10 in any place to the right is equal to 1 in the next place to the left. Hence also the reason for carrying according to the subdivisions of any integer when several denominations are to be added.

Axiom 2. If two whole numbers be equally increased, their difference is always the same. Hence the reason of borrowing 10 in one place to the right, and paying it back by carrying one to the next place. Hence likewise the reason will be evident, for placing the first figure to the right of the product of every particular multiplier directly below its own multiplier.

Axiom 3. The multiplicand will be increased or diminished in proportion to the multiplier, when the same multiplicand is used — Hence the reason why the multiplicand is increased, when it is multiplied

tiplied by any thing greater than unity, and decreased, when it is multiplied by a fraction.

Axiom 4. The dividend will be increased or diminished in proportion to the divisor, when the same dividend is used. Hence, to divide by any thing greater than unity, will quote a number less than the dividend; and, on the contrary, to divide by any thing less than unity, will quote a number greater than the dividend.

Axiom 5. The whole is equal to all its parts taken together.—Hence one sum may be made equal to several by Addition, and Subtraction may be proved by adding the difference to the least given sum.

Axiom 6. If equal quantities be added to, taken from, multiplied or divided by, equal quantities, the sums, remainders, products, and quotients, will respectively be equal. Hence the reason of reducing equations by Addition, Subtraction, Multiplication, and Division, and of abridging commensurable terms, and cancelling equal quantities and numbers.

Axiom 7. To multiply, or divide, any quantity or number by other quantities or numbers continually, is the same as to multiply by the product of those other numbers. Hence the reason of multiplying or dividing by component parts.

Axiom. 8. If four numbers or quantities be proportional, the rectangle or product of the extremes will be equal to the product of the means; and *vice versa*, if the product of the extremes be equal to that of the means, the number or quantities are proportional.

Axiom 9. The quotient of any two succeeding powers, when the next higher is divided by the next lower, exhibits the root of these powers. On the contrary, if any power be multiplied by the root of that power, the product will be the next higher power of the root: And if a higher power be divided by the root, the quotient will exhibit the next lower power. Again, if a proportional part of a higher power be divided by a proportional part of the next lower power, the quotient will exhibit a proportional part of the root. Hence the first figure or figures in the root of any power being raised to the power next lower than that whose root is wanted, and that power multiplied by a number expressing the proportion, which the given power bears to its root, produces a proportional divisor, whose ratio, compared with the dividual, is a proportional part of the root, which being annexed to the former part of the root, and raised to the full power of the given number, will be either the whole or a proportional part of the given power, discoverable by subtraction, &c. Hence we have a general rule for extracting the root of any power whatever.

AN

INTRODUCTION

TO

CONICK SECTIONS.

SECTION I.

Of the Ellipsis.

Definition 1.

Fig. 1.

IF two pins be fixed at the points F, S ; and a thread PSFP, put about them and knotted at P ; then if the thread be drawn tight, and the point P and the thread be moved about the fixed centres F, S ; the point P will describe the curve PD*p*BEAP, called an *Ellipsis*. See Fig. 1.

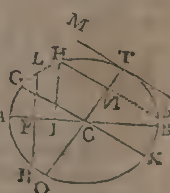
Def. 2. The points or centres F, S, are called the *foci*.

Def. 3. The line A, B, drawn through the foci to the curve, is called the *transverse axis*.

Def. 4. The point C in the middle of the axis AB, is the *centre*.

Def. 5. The line DE, (drawn through the centre C) perpendicular to the transverse AB, is called the *conjugate axis*.

Fig. 2.



Def. 6. Any line TO, drawn through the centre C to the curve, is called a *diameter*. And the extremity T (or O) its *vertex*.

Def. 7. If TO be a diameter, then the diameter GK, drawn parallel to the tangent at its vertex T, is called its *conjugate*. And the two diameters TO, GK, are said to be *conjugates* to one another.

Def. 8. The line LR (drawn through the focus F, perpendicular to the transverse axis AB,) is called the *parameter* or *latus rectum*.

Def. 9. A line drawn from any point of the curve (as HI) perpendicular to the transverse axis, is called an *ordinate* to the transverse. And, in general, any line drawn from the curve to any diameter TO, parallel to its conjugate GK, (as HN,) is an *ordinate* to that principal diameter, TO. If it go quite through the figure, as H*b*, it is called a *double ordinate*.

Def. 10. A right line meeting the ellipsis in one point T, but not cutting it, is called a *tangent* to it in that point, as TM.

Def. 11. The part of the diameter between the vertex and the ordinate, is called the *abscissa*, TN, AI. And the *vertex* is the extremity of any diameter.

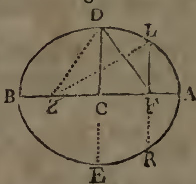
Proposition I. The sum of the lines FP, SP drawn from the foci, to any point of the curve, is equal to the transverse axis AB. See Fig. 1.

For by construction, $PF+PS=AF+AS=AF+AF+FS=2AF+FS$. And the same $PF+PS=2BS+FS$; therefore $2AF+FS=2BS+FS$, and $2AF=2BS$, or $AF=BS$. Whence $PF+PS=2AF+FS=AF+BS+FS=AB$.

Cor. The two foci are equally distant from the vertices, and also from the centre: $AF=BS$; and $FC=SC$. For it is proved that $AF=BS$; and since $AC=CB$ (Def. 4.) therefore $AC-AF=CB-BS$, or $FC=SC$.

Prop. II. A line, drawn from the end of the conjugate axis, to the focus, is equal to half the transverse; $DF=CA$. See Fig. 3.

Fig. 3.



Draw DS to the other focus. Then the two right angled triangles CDF and CDS are similar and equal. For $SC=CF$, the angles at C are right, and CD common: therefore $SD=DF$; and since the sum $SD+DF$=the transverse (Prop. 1,) one of them DF = half the transverse CA.

Cor. The distance of the foci is a mean proportional between the sum and difference of the transverse and conjugate axis, $SF^2=\overline{BA+DE}\times\overline{BA-DE}$: For $CA^2=DF^2=DC^2+CF^2$; and $CF^2=CA^2-CD^2=\overline{CA+CD}\times\overline{CA-CD}$; and $4CF^2$ or $SF^2=\overline{2CA+2CD}\times\overline{2CA-2CD}$.

Prop. III. The rectangle of the focal distances, from either vertex, is equal to the square of the semiconjugate: $AF\times FB=DC^2$. See Fig. 3.

For $DC^2=DF^2-CF^2=$ (Prop. 2.) $CA^2-CF^2=\overline{CA+CF}\times\overline{CA-CF}=\overline{BC+CF}\times\overline{CA-CF}=BF\times FA$.

Prop. IV. As the transverse axis to the conjugate, so the conjugate to the latus rectum of the transverse: $AB : DE :: DE : LR$. See Fig. 3.

For $SL+LF=BA=2CA$ (Prop.1.); and $SL=2CA-LF$, and, by squaring, $SL^2=4CA^2-4CA\times LF+LF^2$. And in the right angled triangle SLF, $SL^2=SF^2+LF^2$; whence $4AC^2-4CA\times LF+LF^2$, $=SF^2+LF^2$, and $4AC^2-4CA\times LF=SF^2=4CF^2$, and $4AC^2=4CA\times LF+4CF^2=4CA\times LF+4DF^2-4DC^2$, and $4AC^2+4DC^2=4CA\times LF+4DF^2$; but $CA^2=DF^2$ (Prop.2.); therefore $4DC^2=4CA\times LF=2CA\times 2LF$; that is, $DE^2=BA\times LR$.

Cor. 2.

Cor. 1. As the semitransverse is to the semiconjugate, so the semiconjugate to half the latus rectum; $CA : DC :: DC : LF$.

Cor. 2. As the semitransverse, to the distance of the focus from the centre; so is the same distance, to the difference between the semitransverse and half the latus rectum: $FC^2=CA\times\overline{CA-LF}$.

For $CF^2=DF^2-DC^2=CA^2-CD^2=CA^2-CA\times LF$.

Cor. 3. The rectangle BFA = half the transverse × half the latus rectum = $CA\times FL$. By Cor 1. and Prop. 3. See Fig. 3.

Scholium. Since, as the transverse axis is to the conjugate, so the conjugate to the latus rectum, of the transverse axis. Therefore, in any other diameters, the third proportional, to the diameter and its conjugate, is called the *latus rectum* of that diameter.

Prop. V. From any point M in the curve, drawing the lines MF, MS, to the two foci; and the ordinate MP perpendicular to the transverse axis BA; it will be,

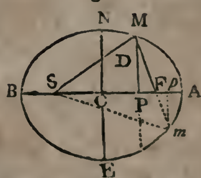
As the semitransverse CA :

To the distance of the focus from the centre, CF ::

So the distance of the ordinate from the centre, CP :

To half the difference of the lines MF, MS, or $\frac{MS-MF}{2}$.

Fig. 4.



For, make $SD=CA$, then $SM=CA+DM$, and $FM=2CA-SM=CA-DM$. In the right angled triangle SMP, SM^2 or $CA^2+2CA\times DM+DM^2=SP^2+PM^2=\overline{CF+CP}^2+PM^2=CF^2+2CF\times CP+CP^2+PM^2$, and in the right angled triangle FMP, FM^2 or $CA^2-2CA\times DM+DM^2=FP^2+PM^2=\overline{CF-CP}^2+PM^2=CF^2-2CF\times CP+CP^2+PM^2$; then subtracting the latter equation from the former, $SM^2-FM^2=4CA\times DM=4CF\times CP$, and $CF\times CP=CA\times DM$. But since $SM=CA+DM$, and $FM=CA-DM$; therefore $SM-FM=2DM$; therefore $CF\times CP=CA\times\frac{SM-FM}{2}$.

Cor. 1. If FS be the foci, MP an ordinate; then it is $CA : CF :: CP : \overline{CA-MF}$ or $\overline{SM-CA}$. See Fig. 4.

For $CF\times CP=CA\times DM$, and $DM=SM-CA=CA-FM$.

Cor. 2. If F, S, be the foci, MP an ordinate; then the difference of the squares of the lines SM, FM; that is $SM^2-FM^2=4CF\times CP$.

Cor. 3. If F, S, be the foci, MP an ordinate; then $CA\times\overline{SM-FM}=2CF\times CP$.

For $SM^2-FM^2=\overline{SM+FM}\times\overline{SM-FM}=2CA\times\overline{SM-FM}=4CF\times CP$, and $CA\times\overline{SM-FM}=2CF\times CP$.

Scholium. If PM fall on the other side of F, as pm, then $pF=Cp-CF$, and its square the same as before, and the rest of the demonstration the same.

Prop. VI. If an ordinate MP be drawn to the transverse axis; it will be,

As the square of the transverse, BA^2 :
To the square of the conjugate, NE^2 ::
So the rectangle of the segments of the transverse BPA :
To the square of the ordinate, PM^2. See Fig. 4.

For make SD=CA, then DM is half the difference of SM and MF; therefore by Prop. 5. CA : CF :: CP : DM, and CA : CA+CF or BF :: CP : CP+DM, and CA : CP :: BF : CP+DM, and CA : CA+CP or BP :: BF : BF+CP+DM. But BF=BC+CF=SD+CF; and BF+CP+DM=SD+CF+CP+DM=SM+CS+CP=SM+SP; whence CA : BP :: BF : SM+SP. Again, since CA : CF :: CP : DM; then CA : (CA—CF) AF :: CP : CP—DM; and CA : CP :: AF : CP—DM. And CA : (CA—CP) PA :: AF : AF—CP+DM. But AF=CA—CF=SD—SC; therefore AF—CP+DM=SD—SC—CP+DM=SM—SP; therefore CA : PA :: AF : SM—SP, and we had before, CA : BP :: BF : SM+SP; then multiplying these proportions together, we have $CA^2 : BP \times PA :: BF \times FA : SM^2—SP^2$.

But (Prop 3.) $BF \times FA = CN^2$; and $SM^2—SP^2=PM^2$; therefore $CA^2 : BPA :: CN^2 : PM^2$, or alternately, $CA^2 : CN^2 :: BPA : PM^2$, or BA^2 $(4CA^2) : NE^2$ $(4CN^2) :: BPA : PM^2$.

Cor. 1. $CA^2 : CN^2 :: BFA : PM^2$.

Cor. 2. As the transverse BA : to its latus rectum :: So the rectangle BPA : to square of the ordinate PM^2.

For (Prop. 4.) latus rectum $=\dfrac{NE^2}{AB}$, whence, since $BA^2 : EN^2 :: BFA : PM^2$, therefore, $BA : \dfrac{NE^2}{BA}$ or latus rectum :: $BFA : PM^2$.

Cor. 3. The rectangles of the segments of the transverse are as the squares of the ordinates.

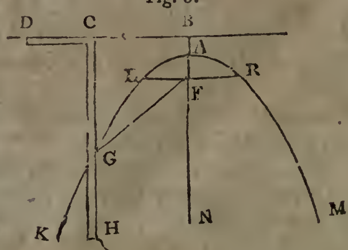
For every rectangle is to the square of its ordinate, in the given ratio of CA^2 to CN^2, or of BA to the latus rectum.

Cor. 4. As the square of the semitransverse CA^2 :
Rectangle of the focal distances from vertex BFA ::
So rectangle of the segments BPA :
To square of the ordinate PM^2.

SECTION II.

Of the Parabola.

Fig. 5.



Definition 1. If one end of a thread, equal in length to CH, be fixed at the point F, and the other end fixed at H, the end of the square DCH. And if the side CD of the square be moved along the right line BD, and always coincide with it, then if the string FGH be always kept tight, and close to the side GH of the square, the point or pin G (where it leaves the square) will describe a curve MRALGK called a Parabola, See Fig. 5. *Def.* 2.

Def. 2. The fixed point F is called the *focus*.

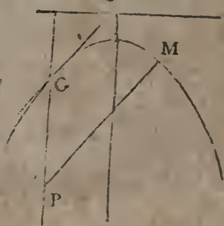
Def. 3. The right line BD is called the *directrix*.

Def. 4. If the line BN be drawn through the focus F, perpendicular to BD; then AN is called the *axis* of the parabola, and A the *vertex*.

Def. 5. A line drawn through the focus F, perpendicular to the axis, as LR, is called the *parameter* or *latus rectum*..

Def. 6. Any line drawn within the curve, parallel to the axis, as GH, is called a *diameter*. And the point G, where it cuts the curve, is the *vertex*.

Fig. 6.

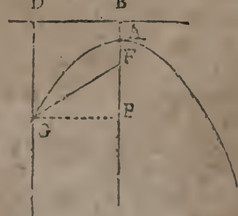


Def. 7. A right line drawn from any diameter to the curve, and parallel to the tangent at the vertex, as PM, is called an *ordinate*. If it go quite through the curve, it is called a *double ordinate*. See Fig. 6.

Def. 8. The part of the diameter between the vertex and ordinate, as GP, is called the *abscissa*.

Def. 9. A right line, meeting the curve in one point G, but not cutting it, is called a *tangent* in that point.

Fig. 7.



Proposition I. If BD be the directrix, G any point in the curve, the line GD drawn to the directrix, parallel to the axis, is equal to the line GF drawn from the same point G to the focus; GD=GF. See Fig. 7.

For HG+GF=length of the string = HD; take away GH from both, and then GD=GF.

Cor. 1. The distances of the focus, and of the directrix from the vertex are equal. AB=AF. For when D is at B, G will be at A; consequently AB=AF.

Cor. 2. If GP be an ordinate to the axis; then AP+AF=FG, For AP+AF=BP=GD.

Cor. 3. FG—FP = half the latus rectum.

Prop. II. The distance of the focus from the vertex is $\frac{1}{4}$ the latus rectum: AF=$\frac{1}{4}$LR=$\frac{1}{2}$LF. See Fig. 5.

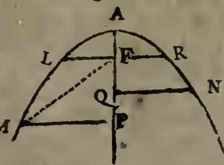
For when the pin G comes to L, then LF=FB (Prop. 1. Cor. 1.) =2FA, and AF=$\frac{1}{2}$FL. For the same reason FA=$\frac{1}{2}$FR, therefore FA=$\frac{1}{4}$LR.

Scholium.

Scholium. As the latus rectum to the axis is four times the distance of the vertex A from the focus F: So in any other diameter GH, four times the distance of its vertex from the focus, or 4FG, is called its *latus rectum*.

Prop. III. The square of any ordinate to the axis is equal to the rectangle of the latus rectum and abscissa: $PM^2 = LR \times AP$. See Fig. 8.

Fig. 8.



For $MF = AF + AP =$ (Prop. 2) $AP + \frac{1}{4}LR$, and $FP = AP - AF = AP - \frac{1}{4}LR$. And in the right angled triangle MFP, $MP^2 = MF^2 - FP^2 = \overline{MF+FP} \times \overline{MF-FP} = 2AP \times \frac{1}{2}LR = AP \times LR$.

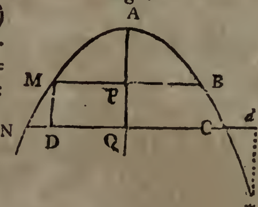
Cor. 1. If F be the focus, $MP^2 = AP \times 4AF$.

Cor. 2. The abscissas are as the squares of their ordinates. $AP : AQ :: PM^2 : QN^2$. For $AP : AQ :: AP \times LR : AQ \times LR :: PN^2 : QN^2$.

Cor. 3. The latus rectum is a third proportional to the abscissa and ordinate. $AP : PM : LR \div$.

Prop. IV. As the latus rectum to the sum of any two ordinates; so their difference, to the difference of the abscissæ. Lat. rect. : CD :: ND : PQ. See Fig. 9.

Fig. 9.



Let L = latus rectum, then (Prop. 3.) $L \times AP = PM^2$; and $L \times AQ = NQ^2$. And by subtraction, $L \times AQ - L \times AP = NQ^2 - PM^2$; therefore $L : NQ + PM :: NQ - PM : AQ - AP$; that is, $L : DC :: ND : PQ$.

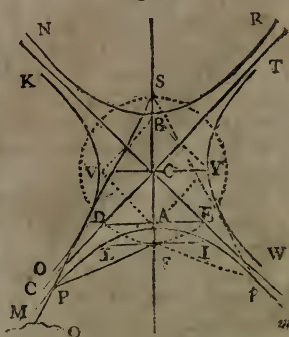
Cor. 1. If MD be the axis, NC an ordinate to it; then the rectangle NDC = MD× parameter.

Cor. 2. The rectangle NDC is every where as MD.

SECTION III.

Of the Hyperbola.

Fig. 10.



Definition 1. If the ends of two threads SPQ, FPQ, be fastened at the points S, F, and be made to pass through a small bead, or pin P, and knotted together at Q; then taking hold of Q, and drawing the threads tight; if the bead be moved along the threads, the point P, will describe the curve *mp* APM, called an *hyperbola*. See Fig. 10.

Def. 2.

Def. 2. And if the end of the long thread be fixed at F, and that of the short one at S; and the curve NBR be described after the same manner, that curve is called the *opposite hyperbola;* and both curves together, MA*m*, NBR, are called *opposite sections*, or *opposite hyperbolas.*

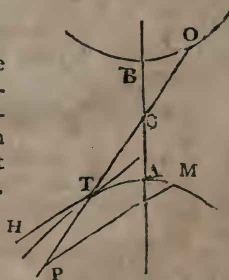
Def. 3. The two fixed points F, S, are called the *foci.*

Def. 4. The line AB (passing through the foci, when continued) contained between the two parts of the curve, is called the *transverse axis.*

Def. 5. The middle point of AB, that is, C, is called the *centre* of the hyperbola, or of the opposite sections.

Def. 6. If VY be drawn through the centre C perpendicular to AB; and with radius CF, and centre A, an arch be described, cutting VY in V, and Y; then VY is called the *conjugate axis.*

Fig. 11.



Def. 7. Any line TO drawn through the centre C, and terminated at the opposite sections, is called a diameter; and the extremity T (or O) its *vertex.* And the line drawn through the centre parallel to the tangent at the vertex, is called its *conjugate diameter.* See Fig. 10.

Def. 8. If any diameter OT be continued with the curve, the part within, TP, is called the *abscissa.*

Def. 9. Any line PM, drawn parallel to the tangent at the vertex T, and terminated at the abscissa and curve, is called an *ordinate* to that diameter TO. And if it go quite through the curve, it is called a *double ordinate.*

Def. 10. The line LI, drawn through the focus F, perpendicular to the transverse axis AB, and terminating at the curve, is called the *parameter* or *latus rectum.* See Fig. 10.

Def. 11. If the ends of the two axes be joined by the lines BY, BV; and through the centre C, two lines CH, CG, be drawn parallel to BY, BV; or (which is the same) if VY be placed at A, perpendicular to BA; and the lines CH, CG, be drawn from the centre C, through the ends E, D; these lines CH, CG, are called the *asymptotes* of the hyperbola, or of the opposite hyperbolas.

Def. 12. When the transverse and conjugate axes are equal, AC = CV or AD, the curve is called an *equilateral hyperbola* or *right angled hyperbola.*

Def. 13. A right line, which meets the hyperbola in one point T, but does not cut it, as TH, is called a *tangent* to it, in that point T. See Fig. 11.

Def. 14. If two opposite hyperbolas, KO, TW, be in like manner described to the transverse VY (= DE.) and conjugate AB; these are callen *conjugate hyperbolas*, with regard to the former.

PROPOSITIONS

Proposition I. The difference of the lines SP, FP, drawn from the foci, to any point P of the curve, is equal to the transverse axis AB. See Fig. 10.

For by construction PS—PF=AS—AF=AB+BS—AF= (because BS=AF) AB.

Cor. Hence CF=CS, or the foci are equally distant from the centre.

Prop. II. The square of the distance of the focus from the centre is equal to the sum of the squares of the semitransverse and semiconjugate. $CF^2=CA^2+CY^2$.

For, make AE equal and parallel to CY, then the radius CE=CF; and in the right angled triangle CAE, $CE^2=CA^2+AE^2$; that is, $CF^2=CA^2+AE^2=CA^2+CY^2$.

Cor. $CF^2-AE^2=CA^2$; and $CF^2-CA^2=AE^2=CY^2$.

Prop. III. The rectangle of the focal distances from either vertex is equal to the square of the semiconjugate, $FA\times SA=CY^2$.

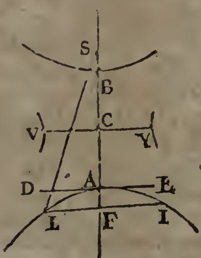
For, making AE=CY; by the property of the circle, $FA\times AS=AE^2=CY^2$.

Cor. The rectangle of the distance of either focus from the two vertices is equal to the square of the semiconjugate, $FA\times FB=AE^2=CY^2$.

For SB=FA and SA=FB, whence $SA\times FB=FA\times SA=AE^2$.

Prop. IV. As the transverse axis is to the conjugate; so the conjugate, to the latus rectum of the transverse; AB : VY :: VY : LI. See Fig. 12.

Fig. 12.



For (Prop. 1.) SL—LF=BA=2CA; and SL=2CA+LF; and $SL^2=4CA^2+4CA\times LF+LF^2$; and in the right angled triangle SLF, $SL^2=SF^2+LF^2$, and subtracting LF^2, from these two values of SL^2; then $4CA^2+4CA\times LF=SF^2=4CF^2$; and $CF^2=CA^2+CA\times LF$. But (Prop. 2.) $CF^2=CA^2+CY^2=CA^2+CA\times LF$; therefore $CY^2=CA\times LF$, and multiplying by 4, $VY^2=BA\times LI$.

Cor. 1. As the semitransverse, to the semiconjugate; so the semiconjugate to half the latus rectum, CA : CY : LF.

Cor. 2. As the semitransverse to the distance of the focus from the centre; so is the same distance, to the sum of the semitransverse and half the latus rectum, CA : CF :: CF : CA+LF.

For, (Prop. 2.) $CF^2=CA^2+CY^2=CA^2+CA\times LF=CA\times\overline{CA\times FL}$.

Cor. 3. The rectangle BFA = ½ transverse × ½ latus rectum = CA ×FL. By Cor. 1. and Prop. 3.

Scholium. Since the transverse axis is to the conjugate, as the conjugate to the latus rectum of the transverse axis; therefore, in any other diameters, the third proportional, to any diameter and its conjugate, is called the *latus rectum of that diameter*. Therefore, in a right angled hyperbola, every diameter is equal to its latus rectum.

FINIS.

ERRATA.

Page 33 note, line 14 from bottom, for $37\frac{64}{1000}$ read $37\frac{564}{1000}$.
33 note, line 8 from bottom, for 264 read 246.
33 note, line 4 from bottom, for *gold* read *ſilver.*
36 line 7, for 7960 read 7920.
36 line 31, for 575960 read 525960.
36 line 32, for 505969 read 525969.
36 line 32, for $\overset{d}{365}\ \overset{h.}{6}\ \overset{w.}{9}\ \overset{d.}{14}$ read $\overset{d.}{365}\ \overset{h.}{6}\ \overset{m.}{9}\ \overset{s.}{14}$.
36 line 33, for 365 5 48 57 read 365 5 48 57.
37 line 19, for 690 read 640.
38 line 2, for $28\frac{1}{8}$ read $28\frac{7}{8}$.
38 lines 30, 31, for $\left\{\begin{array}{l}2=1 \text{ puncheon}\\ 3=2=1 \text{ butt}\end{array}\right\}$ read $\left\{\begin{array}{l}2=1\frac{1}{3}=1 \text{ puncheon.}\\ 3=2=1\frac{1}{2}=1 \text{ butt.}\end{array}\right.$
38 lines 39, 40, for $\left\{\begin{array}{ll}2 & 1 \text{ barrel}\\ 3 & 1\frac{1}{2}\end{array}\right\}$ read $\left\{\begin{array}{l}2=1 \text{ barrel.}\\ 3=1\frac{1}{2}.\end{array}\right.$
39 line 19, for 12 read 16.
48 line 23, for $\overline{\overline{5\times3+30}}$ read $\overline{5\times3+30}$.
72 Multiplication, example 8, for $\frac{1}{2}$ read $\frac{1}{4}$.
76 line 26, after *viz.* dele =.
80 line 2 from bottom, for 76·55 read 7·654.
85 Table V, line 4 from bottom, for ·03215 read ·03125.
104 column 2, line 10 from bottom, for $\overset{\text{Liv}}{2333}\ \overset{\text{Sou. den.}}{68}$ read $\overset{\text{Liv.}}{2333}\ \overset{\text{Sou.}}{6}\ \overset{\text{den.}}{8}$.
120 example 28, for *come* read *comes.*
125 line 11, for $\frac{6}{8}$ read $\frac{6}{7}$.
149 line 22, for 56·3 yds. read 56·5 yds.
206 line 11, from bottom, for $\overline{19-1\times2}+3$ read $\overline{\overline{19-1}\times2+3}$.
207 line 6 from bottom, for $19-\overline{1\times19}$ read $\overline{19-1}\times19$.
230 Cafe 5, part 2, for $\frac{rs}{a}{}^{n-1}=\frac{s-a}{a}$ read $\frac{rs}{a}r^{n}=\frac{s-a}{a}$.
231 Cafe 8, part 1, for *n*—1 read *r*—1.
231 Cafe 8, part 2, for + read ×.
234 line 13, for 6·32 read 6·39.
252 note, two lower lines, read as follows: Thus, for 1 month $\frac{a}{1{\cdot}005}$, 2 months $\frac{a}{1{\cdot}01}$, 3 months $\frac{a}{1015}$, &c.
261 line 7, for $13\frac{1}{5}$ read $13\frac{1}{3}$.
274 Example 1, line 17, for $8\frac{3}{4}$ read $8\frac{1}{4}$.
274 Example 2, line 16, for 2 read 20.
285 note, line 18 from bottom, for *rt* read r^{t}.
301 againſt 26, under $4\frac{1}{2}$, for 15·14061 read 15·14661.
313 Example 3, for 60 read 72, and for 7s. read D.1 $37\frac{1}{2}$c.
318 Example 4, for 13— read 18—, and for 48+ read $4\frac{1}{2}$+.
325 line 8, for *of* read *to.*
325 line 9, put a comma after *increaſed.*
328 line 5, for $\sqrt{\overline{42056462+5}}\times5$ read $\sqrt{42056462+5\times5}$,
343 line 13, for 343 feet read 343 inches $=28\frac{7}{12}$ feet.
351 line 25, for 113·02×54 read 113·02+54.
355 Table, column 5, againſt 3c. for $0\frac{17}{25}$ read $1\frac{17}{25}$.
358 Table, laſt column but one, at top, for 2 read $2\frac{1}{2}$.

Page

Page 359 Table, againſt 5d. for $5\frac{5}{4}$ read 5.

369 Problem 4, example 2, line 3, for 452 read 453.

391 in the figure, for P read B.

398 line 10 from bottom, remove 1134 one place to the left.

445 laſt line under Prob. VI. for *br* read *bc*.

447 near the middle, for $\frac{2ar^2}{36}$ read $\frac{2ar^2}{3b}$.

450 line 3, for *powers* read *power*.

450 line 6, put a comma after ar^4.

470 laſt line of Axiom 8, for *number* read *numbers*.

Some other errours, of minor importance, will occur to the reader, which he is requeſted to excuſe and correct.

~~Elijah D. Harris of Brat~~

The property of James Rooney
Jun. Chester purchased of Elijah D. Har
Dec. 1829

James Rooney Jun. Chester

John C. Harris Book

James Dooney Jr Chester Cheran